AQA

A-level
FURTHER MATHEMATICS
7367/3D

Paper 3 Discrete

Mark scheme

June 2023

Version: 1.0 Final

2 3 6 A 7 3 6 7 / 3 D / M S

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from aqa.org.uk

Mark scheme instructions to examiners

General

The mark scheme for each question shows:

- the marks available for each part of the question
- the total marks available for the question
- marking instructions that indicate when marks should be awarded or withheld including the principle on which each mark is awarded. Information is included to help the examiner make his or her judgement and to delineate what is creditworthy from that not worthy of credit
- a typical solution. This response is one we expect to see frequently. However credit must be given on the basis of the marking instructions.

If a student uses a method which is not explicitly covered by the marking instructions the same principles of marking should be applied. Credit should be given to any valid methods. Examiners should seek advice from their senior examiner if in any doubt.

Key to mark types

M	mark is for method
R	mark is for reasoning
A	mark is dependent on M marks and is for accuracy
B	mark is independent of M marks and is for method and accuracy
E	mark is for explanation
F	follow through from previous incorrect result

Key to mark scheme abbreviations

CAO	correct answer only
CSO	correct solution only
ft	follow through from previous incorrect result
'their'	indicates that credit can be given from previous incorrect result
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
NMS	no method shown
PI	possibly implied
sf	significant figure(s)
dp	decimal place(s)
ISW	Ignore Subsequent Workings

Examiners should consistently apply the following general marking principles:

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Diagrams

Diagrams that have working on them should be treated like normal responses. If a diagram has been written on but the correct response is within the answer space, the work within the answer space should be marked. Working on diagrams that contradicts work within the answer space is not to be considered as choice but as working, and is not, therefore, penalised.

Work erased or crossed out

Erased or crossed out work that is still legible and has not been replaced should be marked. Erased or crossed out work that has been replaced can be ignored.

Choice

When a choice of answers and/or methods is given and the student has not clearly indicated which answer they want to be marked, mark positively, awarding marks for all of the student's best attempts. Withhold marks for final accuracy and conclusions if there are conflicting complete answers or when an incorrect solution (or part thereof) is referred to in the final answer.

AS/A-level Maths/Further Maths assessment objectives

AO		Description
AO1	AO1.1a	Select routine procedures
	AO1.1b	Correctly carry out routine procedures
	AO1.2	Accurately recall facts, terminology and definitions
AO2	AO2.1	Construct rigorous mathematical arguments (including proofs)
	AO2.2a	Make deductions
	AO2.2b	Make inferences
	AO2.3	Assess the validity of mathematical arguments
	AO2.4	Explain their reasoning
	AO2.5	Use mathematical language and notation correctly
AO3	AO3.1a	Translate problems in mathematical contexts into mathematical processes
	AO3.1b	Translate problems in non-mathematical contexts into mathematical processes
	AO3.2a	Interpret solutions to problems in their original context
	AO3.2b	Where appropriate, evaluate the accuracy and limitations of solutions to problems
	AO3.3	Translate situations in context into mathematical models
	AO3.4	Use mathematical models
	AO3.5a	Evaluate the outcomes of modelling in context
	AO3.5b	Recognise the limitations of models
	AO3.5c	Where appropriate, explain how to refine models

Q	Marking instructions	AO	Marks	Typical solution
1	Circles correct answer	1.1b	B1	3
	Question total		**1**	

Q	Marking instructions	AO	Marks	Typical solution
2	Circles correct answer	1.1b	B1	$\mathbf{J_4}$
	Question total		**1**	

Q	Marking instructions	AO	Marks	Typical solution
3	Circles correct answer	1.1b	B1	B
	Question total		**1**	

Q	Marking instructions	AO	Marks	Typical solution
4(a)	Explains that the new facility should be connected to nodes *H*, *I* and *J*	2.4	E1	The new facility, which is a supersink, should be connected to nodes *H*, *I* and *J* as these are the sinks of the network.
	States that the new facility is a supersink	3.2a	B1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
4(b)	Obtains the correct value of the cut Condone missing units	3.2a	B1	94 gallons per second.
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
4(c)	Compares their cut value from part **(b)** with Tim's cut or Correctly identifies and finds the value of a different cut which has a value lower than 106 and compares this cut with Tim's cut	1.1b	B1	Tim's cut has a value greater than 94 gallons per minute, so Tim's cut is not the minimum cut of the network. Therefore, Tim's claim is incorrect as the maximum flow through the network is equal to the minimum cut of the network, which is less than or equal to 94 gallons per minute.
	Concludes that Tim's claim is not correct based on a comparison of 106 with their cut value	2.3	B1F	
	Subtotal		**2**	

	Question total		**5**	

Q	Marking instructions	AO	Marks	Typical solution
5(a)	States that the objective function P must be maximised	1.1b	B1	To use the simplex algorithm, the objective function P must be maximised.
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
5(b)(i)	Translates problem and obtains at least one correct row in the initial simplex tableau	3.1a	M1	See below.
	Obtains a fully correct initial simplex tableau	1.1b	A1	
	Subtotal		**2**	

P	x	y	r	s	**value**
1	–4	–3	0	0	0
0	1	1	1	0	520
0	2	–3	0	1	570

Q	Marking instructions	AO	Marks	Typical solution
5(b)(ii)	Obtains at least one correct row other than the pivot row	1.1a	M1	See below.
	Obtains a fully correct simplex tableau	1.1b	A1	
	Subtotal		**2**	

P	x	y	r	s	**value**
1	0	–9	0	2	1140
0	0	$\frac{5}{2}$	1	$-\frac{1}{2}$	235
0	1	$-\frac{3}{2}$	0	$\frac{1}{2}$	285

Q	Marking instructions	AO	Marks	Typical solution
5(c)(i)	Explains correctly that the objective row only contains non-negative values	2.4	B1	There are no negative values in the objective row
	Subtotal		**1**	

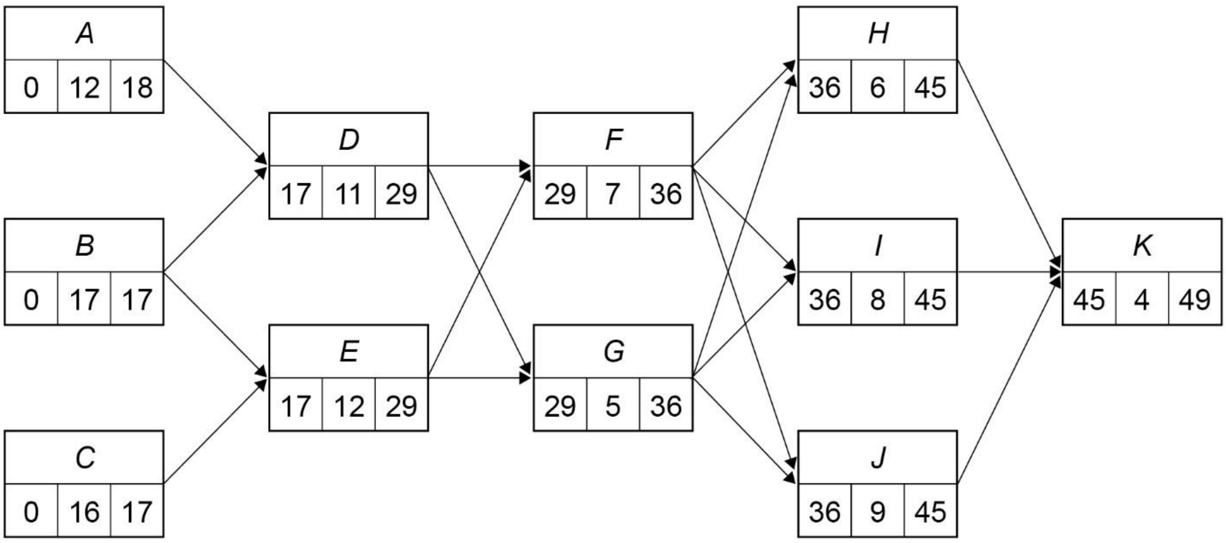
Q	Marking instructions	AO	Marks	Typical solution
5(c)(ii)	Interprets the simplex tableau and obtains the correct optimal value for P or the correct values for x and y at the optimal solution PI by correct optimal value for Q	3.1a	M1	From the simplex tableau, the maximum value of P is 1986 Therefore, the minimum value of Q is –1986
	Obtains the optimal value for Q, the objective function of the original linear programming problem	3.2a	A1	
	Subtotal		**2**	

	Question total		**8**	

Q	Marking instructions	AO	Marks	Typical solution
6(a)	Sets up a model by identifying the problem as a route inspection problem and noting that *C*, *H*, *I* and *P* are odd-degree nodes (PI)	3.3	M1	Odd degree nodes: *C*, *H*, *I*, *P* Shortest Distances *C*–*H*: 575 *I*–*P*: 850 *C*–*I*: 1000 *H*–*P*: 700 *C*–*P*: 850 *H*–*I*: 1075 Pairings (*C*–*H*)(*I*–*P*) = 1425* (*C*–*I*)(*H*–*P*) = 1700 (*C*–*P*)(*H*–*I*) = 1925 Minimum distance the traffic warden can travel whilst monitoring is 9175 + 1425 = 10 600 m The short possible time to grit all roads is $\frac{10600}{5 \times 60} = 35.\dot{3}$ minutes Therefore, the gritter truck has gritted all of the roads at least once by 7:36 pm, to the nearest minute.
	Uses the model to find at least one correct total for a pair of shortest distances	3.4	M1	
	Finds all three correct totals for the pairs of shortest distances	1.1b	A1	
	Determines their correct minimum total distance that the gritter truck will cover during the journey	1.1a	M1	
	Determines their correct least possible time from their minimum total distance	1.1a	M1	
	Determines the correct time to the nearest minute. Allow 7:35 pm or 7:36 pm, must have ‘pm’	3.2a	A1	
	Subtotal		**6**	

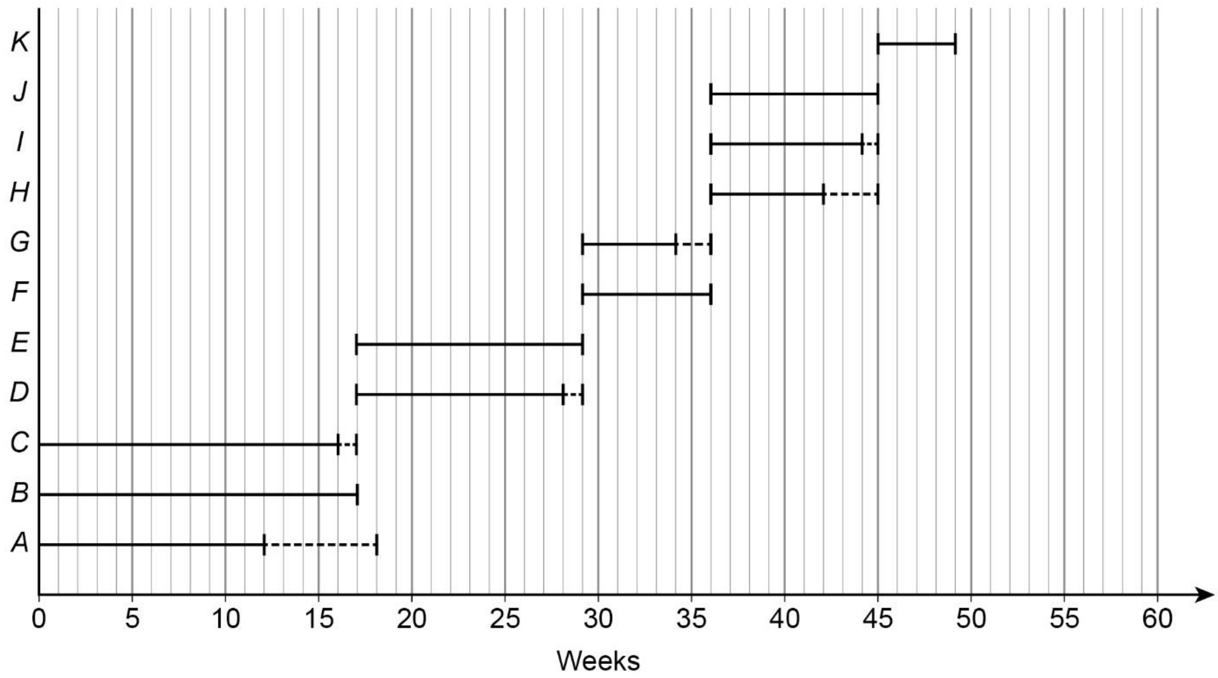
Q	Marking instructions	AO	Marks	Typical solution
6(b)	Explains that the gritter truck could start at one odd degree node and finish at a different odd degree node PI by explicit mention of two of the four odd nodes or Explains that the network should be modified to be semi-Eulerian	3.5c	E1	The gritter truck could start at one of the odd degree nodes and finish at another odd degree node. Doing this, the gritter truck would still grit all roads but cover less distance, meaning it would take less time to grit all the roads.
	Explains that the gritter truck would then need to travel less distance and therefore take less time to grit all of the roads	3.5c	E1	
	Subtotal		**2**	

	Question total		**8**	

Q	Marking instructions	AO	Marks	Typical solution
7(a)(i)	Finds the correct earliest start time for each activity	1.1b	B1	See below.
	Finds the correct latest finish time for each activity	1.1b	B1	
				
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
7(a)(ii)	Writes down the correct critical path	1.1b	B1	*BEFJK*
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
7(b)	Translates the information in the activity network into a diagram by showing and labelling all 11 activities	3.1a	M1	See below.
	All 5 critical activities shown correctly, either on individual rows or all on a single row	1.1b	A1	
	Draws a fully correct diagram, including all labelling and all floats Condone float of 5 for activity *A* if latest finish time is 17 in (a)(i)	1.1b	A1	



	Subtotal		**3**	

Q	Marking instructions	AO	Marks	Typical solution
7(c)	Explains that activity *G* now becomes critical or Explains that the earliest start time of activities *H*, *I* and *J* each decreases by 2 weeks PI by earliest start time of 34 for *H*, *I* and *J* shown on activity network	1.1b	B1	Removing activity *F* makes *G* critical, and *G* has a duration 2 weeks less than activity *F*. Therefore, the minimum completion time for the project reduces by 2 weeks.
	Deduces that the minimum completion time reduces by 2 weeks or reduces to 47 weeks	2.2a	B1	
	Subtotal		**2**	

	Question total		**8**	

Q	Marking instructions	AO	Marks	Typical solution
8(a)(i)	States that *G* does not contain any multiple edges or States that *G* does not contain any loops	1.1a	M1	*G* has no multiple edges or loops, therefore *G* is simple.
	States that *G* does not contain any multiple edges and does not contain any loops, and then deduces that *G* is simple	2.2a	A1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
8(a)(ii)	Explains that *G* is connected or Explains that *G* only has vertices with even degree	2.4	M1	*G* is a connected graph and all of its vertices have an even degree. Therefore, *G* meets the two conditions necessary for a graph to be Eulerian.
	Completes a reasoned argument with reference to *G* satisfying the two conditions necessary for a graph to be Eulerian	2.1	R1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
8(b)	States that the vertices of *G* and *H* have the same degrees	1.1a	M1	*G* also has vertices with degrees 2, 2, 4, 4, 4, 4, 4 and 4. However, this does not necessarily mean the two graphs are isomorphic.
	Infers that *H* may be isomorphic to *G* Must not be definite	2.2b	A1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
8(c)	Recalls that the formula $v - e + f = 2$ can only be used with (connected) planar graphs	1.2	B1	The formula $v - e + f = 2$ can only be used with connected, planar graphs. However, *G* contains a subgraph isomorphic to complete bipartite graph $K_{3,3}$ Hence, by Kuratowski's theorem, *G* is non-planar and so the formula $v - e + f = 2$ cannot be used with *G*
	Translates the problem into that of showing *G* is non-planar by stating that *G* contains a subgraph isomorphic to $K_{3,3}$	3.1a	M1	
	Completes a reasoned argument with reference to Kuratowski's theorem, and concludes that *G* is non-planar and so doesn't meet the conditions for the formula	2.1	R1	
	Subtotal		**3**	

	Question total		**9**	

Q	Marking instructions	AO	Marks	Typical solution
9(a)(i)	Translates the problem to that of finding a generator of *C*	3.1a	M1	If *C* is a cyclic group, it will contain at least one generator. 1 is a generator of *C* as repeated addition of 1 modulo 4 will result in all of the elements of *C*. Hence *C* is a cyclic group.
	Shows that 1 (or 3) is a generator of *C* and concludes *C* is cyclic	1.1b	A1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
9(a)(ii)	States the rotations of the square	2.5	B1	Rotations of the square.
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
9(b)(i)	Sets up the condition for an element and its inverse	1.1a	M1	$(-1, 1) \otimes (a, b) = (1, 1)$ $(-a, b) = (1, 1)$ Therefore, $a = -1$ and $b = 1$, so the inverse of $(-1, 1)$ is $(-1, 1)$
	Identifies the correct inverse of $(-1, 1)$	1.1b	A1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
9(b)(ii)	Translates the problem of determining isomorphism by stating that *V* is not a cyclic group/*V* does not contain a generator/*V* only contains self-inverse elements/there is not a one-to-one mapping between the elements that preserves the group operations	3.1a	M1	*V* does not contain a generator as all elements in *V* are self-inverse. Hence *V* is not a cyclic group. As *C* is a cyclic group of order 4, it can only be isomorphic to other cyclic groups of order 4. As *V* is not cyclic, then *C* and *V* are not isomorphic.
	Concludes that as *C* is cyclic and *V* is not cyclic then *C* is not isomorphic to *V* or Concludes that as *C* contains a generator and *V* does not contain a generator then *C* is not isomorphic to *V* or Concludes that as not all elements of *C* are self-inverse then *C* is not isomorphic to *V*	3.2a	A1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
9(c)	Infers that there may not be exactly one subgroup of *G* for every factor of the order of *G*	2.2b	M1	There could be more than one subgroup of *G* with order 2. There is not enough information about *G* to determine whether or not it has exactly 5 subgroups, so we cannot tell whether Rachel is correct or incorrect.
	Comments on the validity of Rachel's claim and concludes correctly that we cannot be sure about the number of subgroups with orders 2, 4 or 8	2.3	A1	
	Subtotal		**2**	

	Question total		**9**	

	Question Paper total		**50**	