A-level

FURTHER MATHEMATICS

7367/3S

Paper 3 Statistics

Mark scheme

June 2023

Version: 1.0 Final

2 3 6 A 7 3 6 7 / 3 S / M S

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from aqa.org.uk

Mark scheme instructions to examiners

General

The mark scheme for each question shows:

- the marks available for each part of the question
- the total marks available for the question
- marking instructions that indicate when marks should be awarded or withheld including the principle on which each mark is awarded. Information is included to help the examiner make his or her judgement and to delineate what is creditworthy from that not worthy of credit
- a typical solution. This response is one we expect to see frequently. However credit must be given on the basis of the marking instructions.

If a student uses a method which is not explicitly covered by the marking instructions the same principles of marking should be applied. Credit should be given to any valid methods. Examiners should seek advice from their senior examiner if in any doubt.

Key to mark types

M	mark is for method
R	mark is for reasoning
A	mark is dependent on M marks and is for accuracy
B	mark is independent of M marks and is for method and accuracy
E	mark is for explanation
F	follow through from previous incorrect result

Key to mark scheme abbreviations

CAO	correct answer only
CSO	correct solution only
ft	follow through from previous incorrect result
'their'	indicates that credit can be given from previous incorrect result
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
NMS	no method shown
PI	possibly implied
sf	significant figure(s)
dp	decimal place(s)
ISW	Ignore Subsequent Workings

Examiners should consistently apply the following general marking principles:

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Diagrams

Diagrams that have working on them should be treated like normal responses. If a diagram has been written on but the correct response is within the answer space, the work within the answer space should be marked. Working on diagrams that contradicts work within the answer space is not to be considered as choice but as working, and is not, therefore, penalised.

Work erased or crossed out

Erased or crossed out work that is still legible and has not been replaced should be marked. Erased or crossed out work that has been replaced can be ignored.

Choice

When a choice of answers and/or methods is given and the student has not clearly indicated which answer they want to be marked, mark positively, awarding marks for all of the student's best attempts. Withhold marks for final accuracy and conclusions if there are conflicting complete answers or when an incorrect solution (or part thereof) is referred to in the final answer.

AS/A-level Maths/Further Maths assessment objectives

AO		Description
AO1	AO1.1a	Select routine procedures
	AO1.1b	Correctly carry out routine procedures
	AO1.2	Accurately recall facts, terminology and definitions
AO2	AO2.1	Construct rigorous mathematical arguments (including proofs)
	AO2.2a	Make deductions
	AO2.2b	Make inferences
	AO2.3	Assess the validity of mathematical arguments
	AO2.4	Explain their reasoning
	AO2.5	Use mathematical language and notation correctly
AO3	AO3.1a	Translate problems in mathematical contexts into mathematical processes
	AO3.1b	Translate problems in non-mathematical contexts into mathematical processes
	AO3.2a	Interpret solutions to problems in their original context
	AO3.2b	Where appropriate, evaluate the accuracy and limitations of solutions to problems
	AO3.3	Translate situations in context into mathematical models
	AO3.4	Use mathematical models
	AO3.5a	Evaluate the outcomes of modelling in context
	AO3.5b	Recognise the limitations of models
	AO3.5c	Where appropriate, explain how to refine models

Q	Marking Instructions	AO	Marks	Typical Solution
1	Circles correct answer	1.1b	B1	0.4
	Question total		**1**	

Q	Marking Instructions	AO	Marks	Typical Solution
2	Ticks correct answer	1.1b	B1	Poisson, mean 0.8
	Question total		**1**	

Q	Marking Instructions	AO	Marks	Typical Solution
3	Obtains correct z-value **AWRT** 2.33 Condone 2.32 **PI** by a correct upper or lower limit of the confidence interval	1.1b	B1	$z = 2.33$ $401.8 \pm 2.33 \times \dfrac{4.1}{\sqrt{11}}$ = (398.9, 404.7)
	Uses formula for upper or lower limit of a confidence interval using their z-value or t-value. Condone use of $\sqrt{4.1}$ **PI**	1.1a	M1	
	Obtains correct confidence interval **AWRT** 1 d.p. Condone use of truncated z-value of 2.32	1.1b	A1	
	Question total		**3**	

Q	Marking Instructions	AO	Marks	Typical Solution
4	Obtains correct critical value **AWRT** 1.895 **PI** by correct p-value	1.1b	B1	t_7 at 95% = 1.895 $t = \dfrac{101.5 - 100}{\sqrt{\dfrac{4.8}{8}}}$ $t = 1.94$ $1.94 > 1.895$ Reject H_0
	Obtains t-test statistic with the sample mean and the standard error. Condone $z =$	3.4	M1	
	Obtains correct t-test statistic (**AWRT** 1.94) Condone $z =$ or obtains correct p-value (**AWRT** 0.0468).	1.1b	A1	
	Evaluates t model by correctly comparing their test statistic and their critical value or by comparing their p value with 0.05	3.5a	R1	
	Infers H_0 rejected as a result of carrying out a comparison using the correct test statistic and correct critical value or correct p value and 0.05	2.2b	E1	
	Question total		**5**	

Q	Marking Instructions	AO	Marks	Typical Solution
5(a)	Deduces that the test requires two rows or two columns to be merged.	2.2a	E1	Two rows or two columns need to be merged So dof = (11 – 1)(10 – 1) = 90
	Completes reasoned argument to show that the result of merging rows or columns results in the degrees of freedom equalling (11 – 1)(10 – 1) = 90	2.1	E1	
	Subtotal		**2**	

Q	Marking Instructions	AO	Marks	Typical Solution
5(b)	States both hypotheses using correct language. Variables need to be stated in at least the null hypothesis.	2.5	B1	H_0: There is no association between school and activity chosen H_1: There is an association between school and activity chosen χ^2 cv for 90 dof = 124.116 124.8 > 124.116 Reject H_0 Sufficient evidence to suggest that there is an association between school and activity chosen
	Obtains critical value for the test **AWRT** 124.1 or corresponding probability of test statistic, **AWRT** 0.009	1.1b	B1	
	Evaluates the χ^2-test statistic by correctly comparing their critical value with the test statistic.	3.5a	R1	
	Infers H_0 rejected **FT** their comparison using the χ^2 model	2.2b	E1F	
	States the conclusion in context (The conclusion must not be definite.) **FT** Must be consistent with their conclusion on whether to reject H_0 if stated or their comparison if not	3.2a	E1F	
	Subtotal		**5**	

Q	Marking Instructions	AO	Marks	Typical Solution
5(c)	Correctly identifies the strongest source of association as the group of students with the highest value of $\frac{(O-E)^2}{E}$ Condone "the highest value"	1.1b	B1	Students attending school 11 choosing activity 7 as they have the highest value of $\frac{(O-E)^2}{E}$
	Subtotal		**1**	

	Question total		**8**	

Q	Marking Instructions	AO	Marks	Typical Solution
6(a)	Uses rectangular distribution model to correctly find $P(X > 4) = 0.6$, **oe**	3.4	B1	$P(X > 4) = 0.6$
	Subtotal		**1**	

Q	Marking Instructions	AO	Marks	Typical Solution
6(b)(i)	Uses rectangular distribution model to obtain $E(X) = 5$ Condone $E(X) = \frac{9+1}{2} = 5$	3.4	B1	$E(X) = 5$ $E(Y) = 3.5$ Mean total score = 5 + 3.5 = 8.5
	Translate situation for round 2 into discrete uniform distribution model to obtain $E(Y) = 3.5$ **oe**	3.3	M1	
	Obtains the mean total score of the game = 8.5 **oe** **FT** their $E(X)$	3.2a	A1F	
	Subtotal		**3**	

Q	Marking Instructions	AO	Marks	Typical Solution
6(b)(ii)	Substitutes into correct formula for $\mathrm{Var}(X)$ or $\mathrm{Var}(Y)$ **PI** by correct $\mathrm{Var}(X)$ or $\mathrm{Var}(Y)$	1.1a	M1	$\mathrm{Var}(X) = \frac{(10-0)^2}{12} = \frac{100}{12}$ $\mathrm{Var}(Y) = \frac{6^2-1}{12} = \frac{35}{12}$ Variance of total score = $\frac{100}{12} + \frac{35}{12}$ = 11.25
	Obtains correct $\mathrm{Var}(X)$ and $\mathrm{Var}(Y)$ (may be unsimplified)	1.1b	A1	
	Obtains $\mathrm{Var}(X + Y) = 11.25$ **FT** their $\mathrm{Var}(X)$ and $\mathrm{Var}(Y)$ with answer given correct to at least two decimal places	1.1b	A1F	
	Subtotal		**3**	

	Question total		**7**	

Q	Marking Instructions	AO	Marks	Typical Solution
7(a)	Obtains the correct mean of the Poisson distribution.	1.1b	B1	$\lambda = 5^2 = 25$ H_0: $\lambda = 25$ H_1: $\lambda \neq 25$ X~Po(25) P($X \leq 16$) = 0.0377… 0.0377… > 0.025 Accept H_0 Insufficient evidence to suggest that the average number of toys per week that do not pass quality checks has changed
	States both hypotheses using correct language. **FT** ‘their’ value of λ	2.5	B1F	
	Uses Poisson model with their value of λ to find one of P($X \leq 16$), P($X < 16$), P($X \geq 16$), P($X > 16$) or P($X = 16$)	3.3	M1	
	Uses Poisson model to calculate P($X \leq 16$) = 0.0377… **AWRT** 0.04 Condone P($X < 16$) = 0.0377…	3.4	A1	
	Evaluates the Poisson model by correctly comparing their probability with 0.025 If correct probability seen, it must be used in the comparison from which conclusions are made or concludes that 16 is not in the critical region as 16 > 15	3.5a	R1	
	Infers H_0 not rejected. **FT** comparison of their p-value using the Poisson model with 0.025	2.2b	E1F	
	States the conclusion in context (The conclusion must not be definite.) Must mention average number of toys per week **FT** Must be consistent with their conclusion on whether to reject H_0 if stated or their comparison if not	3.2a	E1F	
	Subtotal		**7**	

Q	Marking Instructions	AO	Marks	Typical Solution
7(b)	Interprets Type II error in context Must mention number of toys per week	3.2a	E1	To conclude that the average number of toys per week that do not pass quality checks has not changed when it has
	Subtotal		**1**	

Q	Marking Instructions	AO	Marks	Typical Solution
7(c)	Obtains correct distribution for their value of λ, λ + 18	1.1b	B1F	$X + Y \sim \text{Po}(43)$
	Subtotal		**1**	

Q	Marking Instructions	AO	Marks	Typical Solution
7(d)	Recognises one limitation of the Poisson model relating to independence in context.	3.5b	E1	Individual toys passing quality checks may not be independent The number of toys produced over time might vary to meet demand
	Recognises limitation of the Poisson model relating to constant rate over time in context.	3.5b	E1	
	Subtotal		**2**	

	Question total		**11**	

Q	Marking Instructions	AO	Marks	Typical Solution
8(a)	Uses correct equation. $\int_0^{\frac{\pi}{6}} k\sin 2x \, dx = 1$ Condone missing dx	1.1a	M1	$\int_0^{\frac{\pi}{6}} k\sin 2x \, dx = 1$ $\left[-\frac{k}{2}\cos 2x\right]_0^{\frac{\pi}{6}} = 1$ $-\frac{k}{2}\cos\frac{\pi}{3} + \frac{k}{2} = 1$ $\frac{k}{4} = 1$ $k = 4$
	Obtains $-\frac{k}{2}\cos 2x$ **OE**	1.1b	A1	
	Substitutes in limits into their integrated function and subtracts either way round	1.1a	M1	
	Completes reasoned argument to show that $k = 4$ by solving the correct equation.	2.1	R1	
	Subtotal		**4**	

Q	Marking Instructions	AO	Marks	Typical Solution
8(b)	Uses integral of the form $\int f(y) \, dy$ to find $F(x)$ for the interval $0 \le x \le \frac{\pi}{6}$ **PI** by $2 - 2\cos 2x$ or $-2\cos 2y$ or $-2\cos 2x$ Condone $\int f(x) \, dx$	1.1a	M1	$F(x) = \int_0^x 4\sin 2y \, dy$ $= \left[-2\cos 2y\right]_0^x$ $= 2 - 2\cos 2x$ $F(x) = \begin{cases} 0 & x < 0 \\ 2 - 2\cos 2x & 0 \le x \le \frac{\pi}{6} \\ 1 & x > \frac{\pi}{6} \end{cases}$
	Obtains correct $F(x)$ for the interval $0 \le x \le \frac{\pi}{6}$	1.1b	A1	
	Obtains correct $F(x)$ for all intervals 0 and $\frac{\pi}{6}$ must each be in one interval only	1.1b	A1	
	Subtotal		**3**	

Q	Marking Instructions	AO	Marks	Typical Solution
8(c)	Sets their integrated expression for $\mathrm{F}(m)$ equal to 0.5 **PI**	1.1a	M1	$2 - 2\cos 2m = 0.5$ $\cos 2m = 0.75$ $m = 0.361$
	Obtains correct median **AWRT** 0.361	1.1b	A1	
	Subtotal		**2**	

Q	Marking Instructions	AO	Marks	Typical Solution
8(d)	Uses correct integral of the form $\int x\,\mathrm{f}(x)\,\mathrm{d}x$ with any limits	1.1a	M1	Mean = $\int_0^{\frac{\pi}{6}} 4x\sin 2x\ \mathrm{d}x$
	Integrates by parts with $u = x$ and $v' = \cos 2x$ to reach the form $\left[\pm Ax\cos 2x\right] - \int \pm A\cos 2x\ \mathrm{d}x$	1.1a	M1	$= \left[-2x\cos 2x\right]_0^{\frac{\pi}{6}} - \int_0^{\frac{\pi}{6}} -2\cos 2x\ \mathrm{d}x$ $= \left[-2x\cos 2x + \sin 2x\right]_0^{\frac{\pi}{6}}$
	Obtains the correct integrated function. May not be seen as a single expression.	1.1b	A1	$= \left(-\frac{\pi}{3}\times\frac{1}{2}+\frac{\sqrt{3}}{2}\right) - 0$ $= \frac{1}{6}\left(3\sqrt{3}-\pi\right)$
	Substitutes the correct limits into their integrated function and subtracts either way round	1.1a	M1	
	Completes reasoned argument to show that the mean is $\frac{1}{6}\left(3\sqrt{3}-\pi\right)$ by correctly simplifying the correct expression	2.1	R1	
	Subtotal		**5**	

	Question total		**14**	

	Paper total		**50**	