# AS LEVEL
# FURTHER MATHEMATICS

7366/1 Paper 1
Report on the Examination

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### General

The majority of students demonstrated a high level of knowledge and understanding in many of the questions. However, a significant minority of students didn't appreciate the required structure of a 'show that' question. Such questions should begin with either a definition or the information provided in the question and then proceed to the required result step by step. All 'show that' questions should finish with a conclusion, which can usually be copied from the question. Most students used appropriate mathematical language, although some confused factors with roots. Equals signs and imply signs are still used incorrectly by a minority of students.

### Question 1

Almost every student correctly identified the required expression.

### Question 2

The vast majority of students understood that a zero scalar product indicates that the vectors are perpendicular. Only a small number of students chose 0°.

### Question 3

Almost every student identified the correct product.

### Question 4

The vast majority of students id entified the correct sum of product-pairs, although a significant minority changed the sign, selecting $\frac{3}{5}$ instead.

### Question 5

The majority of students correctly calculated the mean in part (a). A significant minority mistakenly divided the required integral by 5, instead of by the difference between the limits. A small number of students mistakenly assumed the function could only take integer values and so calculated the mean of $3\times1^2$, $3\times2^2$, etc.

In part (b), about half realised that the function had been translated vertically by $c$ units and were able to write down the correct answer without any further work. Most other students integrated again, formed an equation to solve, and usually calculated the correct value.

**Question 6**

Parts (a) and (b) were well answered. The majority of errors were due to a poor application of the formula provided. Most students spotted the link in part (b) and substituted $-x$ for $x$, but a significant minority simply swapped all positive signs for negatives.

In part (c) many students realised they could combine their answers to parts (a) and (b), although some of these did not halve their sum. A minority of students assumed the series for cosh would be the same as for cos.

**Question 7**

In part (a), most students understood how to combine the two fractions, but only half of them could construct a complete mathematical argument.

The vast majority of students could use the method of differences to write the sum as a single fraction, although many answers were out by a factor of 2.

In part (c), a common error was to assume the sequence comprised 100 terms rather than 50.

**Question 8**

Parts (a) and (d) were answered very well.

In part (b), most incorrect responses either explained that the modulus should be negative, or that the argument should be $\frac{\pi}{3}$.

Such errors were often duplicated in part (c).

**Question 9**

The majority of students employed correct mathematical notation to show how matrix **M** could be multiplied by the position vector of the point (4, 5) to find the position vector of its image. A significant number of students did not follow this with a suitable conclusion.

Most students made good progress in part (b). However, many of the attempts which used $mx$ or $mx+c$ involved a lot of algebra and either included errors in the manipulation or stopped abruptly.

In part (c), the majority of students correctly stated that the determinant of **M** should equal zero and formed a suitable cubic equation in $p$. A significant minority then assumed the cubic expression factorised to $(p-3)^3$. Only a third of students completed a convincing argument.

**Question 10**

Parts (a), (b) and (c) were well answered. A common wrong answer to part (a) was $y = 3x$.

In part (d) only half of the students produced a fully correct proof of the required result. Common errors included the omission of '= 0' from the quadratic 'equation' generated by equating $\mathrm{f}(x)$ to –1 or $k$. Of those who opted for the latter, many confused themselves with the extra algebraic manipulation required.

**Question 11**

The vast majority of students were able to answer parts (a) and (b) well, but only half were able to make good progress in the first part of (c). The most common errors occurred when removing the square root sign from the required Cartesian equation. A minority of students struggled to present their answer in the required form.

The majority of students identified a correct transformation in part (c)(ii). A reflection in $y = x$ was the most common response. Common wrong answers included reflection in $y = -x$ and a clockwise 90° rotation.

**Question 12**

Most students were able to demonstrate the techniques required in parts (a) and (b)(i), but only half were able to set them out as a mathematical argument, using the correct mathematical language. For example, in part (a) it was not uncommon for students to correctly show that $(1 + \mathrm{i})^2$ is $2\mathrm{i}$, and also provide a correct proof that $(2\mathrm{i})^2$ is –4. However, the reader was then left to draw the conclusion about the link with $(1+\mathrm{i})^4$ for themselves.

A common error in part (b)(i) was to confuse factors with roots.

Almost all students could identify a second root of $\mathrm{f}(z) = 0$ in part (b)(ii). Most students could use this root to find a linear factor and a quadratic factor in (b)(iii), and many of these went on to find a second quadratic factor in (b)(iv). The final part proved to be a stumbling block for many students, many of whom were confused by the change of variable from $z$ to $x$. However, most students understood that the explanation was related to the absence of real roots.

**Question 13**

The majority of students understood the mechanics of a proof by induction, but only a minority completed it with a suitable conclusion.

In part (b), most students were able to use the sum of squares to find an expression for the sum of the first $2n$ squares, but most struggled to find the sum of the first $n$ even squares and the first $n$ odd squares. Some ignored the previous results from parts (b) and (c), and instead calculated a sum of odd squares by considering a general expression for an odd number. Unfortunately, many of these used $\sum_{r=1}^{n}(2r+1)^2$ rather than $\sum_{r=1}^{n}(2r-1)^2$.

### Question 14

Most students were able to make some progress with this question, but only half successfully found both of the required values. Sign errors were not uncommon. Some responses included an expansion of the two quadratic expressions, but many of these made little progress beyond the expansion. Some also expanded the linear factors $(x+9)(x+3)(x-2)(x-b)$ and compared coefficients, but there were often errors in the algebraic manipulation.

**Mark Ranges and Award of Grades**

Grade boundaries and cumulative percentage grades are available on the Results Statistics page of the AQA Website.