

Please write clearly in block capitals.

Centre number

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Candidate number

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Surname

Forename(s)

Candidate signature

I declare this is my own work.

AS FURTHER MATHEMATICS

Paper 1

Monday 11 May 2020

Afternoon

Time allowed: 1 hour 30 minutes

Materials

- You must have the AQA formulae and statistical tables booklet for A-level Mathematics and A-level Further Mathematics.
- You should have a scientific calculator that meets the requirements of the specification. (You may use a graphical calculator.)

Instructions

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you require extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do **not** write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work you do not want to be marked.

Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

For Examiner's Use	
Question	Mark
1	
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18	
TOTAL	



Answer **all** questions in the spaces provided.

1 Express the complex number $1 - i\sqrt{3}$ in modulus-argument form.

Tick (✓) **one** box.

[1 mark]

$$2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$$

☐

$$2\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$$

☐

$$2\left(\cos\left(-\frac{\pi}{3}\right) + i\sin\left(-\frac{\pi}{3}\right)\right)$$

☐

$$2\left(\cos\left(-\frac{2\pi}{3}\right) + i\sin\left(-\frac{2\pi}{3}\right)\right)$$

☐

2 Given that $1 - i$ is a root of the equation $z^3 - 3z^2 + 4z - 2 = 0$, find the other two roots.

Tick (✓) **one** box.

[1 mark]

$$-1 + i \text{ and } -1$$

☐

$$1 + i \text{ and } 1$$

☐

$$-1 + i \text{ and } 1$$

☐

$$1 + i \text{ and } -1$$

☐


3 Given $(x - 1)(x - 2)(x - a) < 0$ and $a > 2$

Find the set of possible values of x .

Tick (✓) **one** box.

[1 mark]

$$\{x : x < 1\} \cup \{x : 2 < x < a\}$$

☐

$$\{x : 1 < x < 2\} \cup \{x : x > a\}$$

☐

$$\{x : x < -a\} \cup \{x : -2 < x < -1\}$$

☐

$$\{x : -a < x < -2\} \cup \{x : x > -1\}$$

☐

Turn over for the next question

Turn over ►



$$\mathbf{A} = \begin{bmatrix} 2 & a & 3 \\ 0 & -2 & 1 \end{bmatrix} \quad \text{and} \quad \mathbf{B} = \begin{bmatrix} 1 & -3 \\ -2 & 4a \\ 0 & 5 \end{bmatrix}$$

[2 marks]

[illegible]

[1 mark]

[illegible]

4 (c) Show that **AB** is singular when $a = -1$

[2 marks]

Turn over for the next question

Turn over ►



5 (a) Show that

$$r^2(r+1)^2 - (r-1)^2r^2 = pr^3$$

where p is an integer to be found.

[1 mark]



$$\sum_{r=1}^n r^3 = \frac{1}{4}n^2(n+1)^2$$
[illegible]

Anna has been asked to describe the transformation given by the matrix

She writes her answer as follows:

The transformation is a rotation about the x -axis through an angle of θ , where

Identify and correct the error in Anna's work.

This image shows a blank sheet of white paper with horizontal ruling lines. The lines are evenly spaced and extend across the width of the page. There are no margins, text, or other markings on the paper.



Prove by induction that, for all integers $n \geq 1$, the expression $7^n - 3^n$ is divisible by 4
[4 marks]

[illegible]

$$\tanh^{-1} x = \frac{1}{2} \ln \left(\frac{1+x}{1-x} \right)$$

[5 marks]

[illegible]

8 (b) Prove that the graphs of

$$y = \sinh x \quad \text{and} \quad y = \cosh x$$

do **not** intersect.

[3 marks]

Turn over ►



[1 mark]

[1 mark]

[2 marks]

[illegible]

[4 marks]

[illegible]

10 (a) Show that the equation

$$y = \frac{3x - 5}{2x + 4}$$

can be written in the form

$$(x + a)(y + b) = c$$

where a , b and c are constants to be found.

[3 marks]

10 (b) Write down the equations of the asymptotes of the graph of

$$y = \frac{3x - 5}{2x + 4}$$

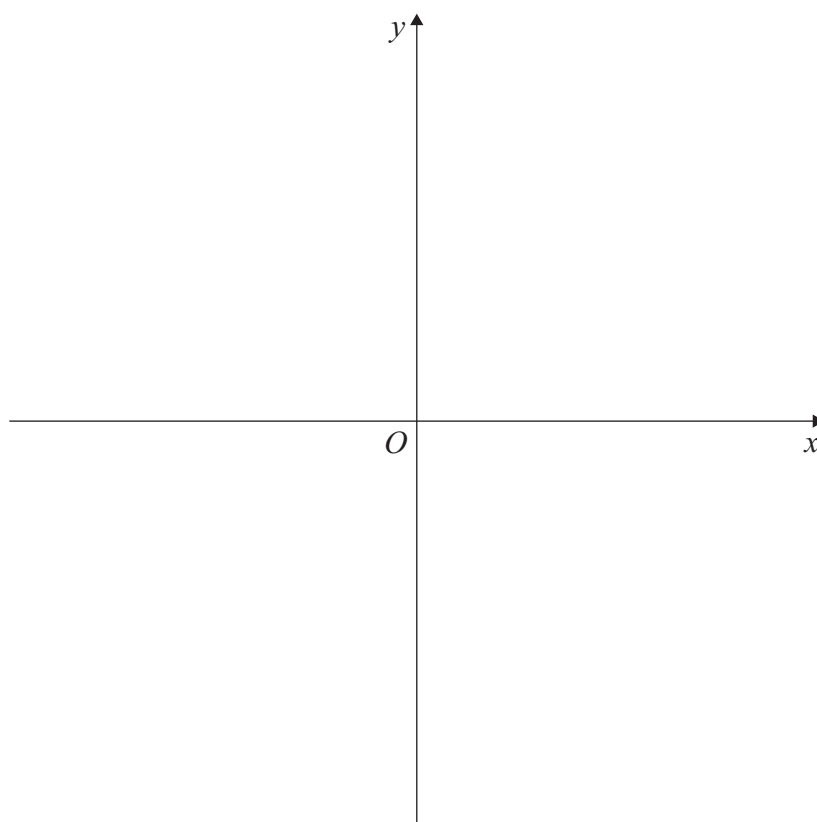
[2 marks]



10 (c) Sketch, on the axes provided, the graph of

$$y = \frac{3x - 5}{2x + 4}$$

[3 marks]



Turn over ►



11 Sketch the polar graph of

$$r = \sinh \theta + \cosh \theta$$

for $0 \leq \theta \leq 2\pi$

[3 marks]

O —————→ Initial line



This image shows a blank sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

13

and line l_2 has equation

$$\mathbf{r} = \begin{bmatrix} -7 \\ 4 \\ -2 \end{bmatrix} + \mu \begin{bmatrix} 12 \\ a+3 \\ 2b \end{bmatrix}$$

13 (a)

[illegible]

[5 marks]

[illegible]

[2 marks]

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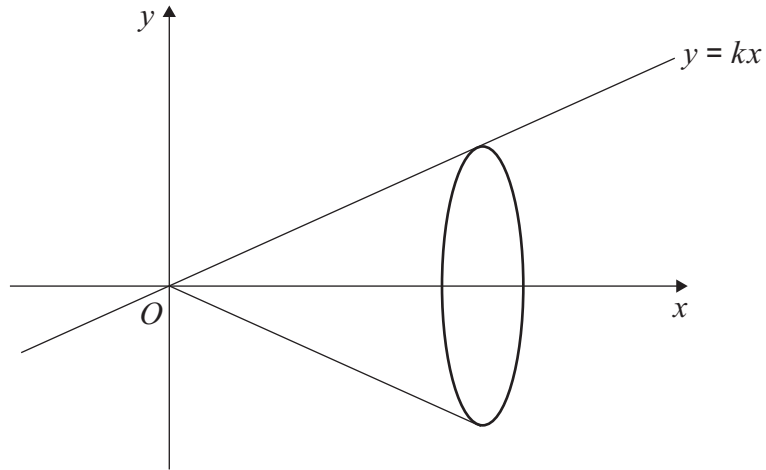
Turn over ►



- 15** A segment of the line $y = kx$ is rotated about the x -axis to generate a cone with vertex O .

The distance of O from the centre of the base of the cone is h .

The radius of the base of the cone is r .



- 15 (a)** Find k in terms of r and h .

[1 mark]



$$\frac{1}{3}\pi r^2 h$$
[illegible]

[1 mark]

[3 marks]

This image shows a single sheet of white paper with horizontal blue or grey ruling lines. The lines are evenly spaced and run across the width of the page. There are approximately 20 lines visible. The paper has a slight shadow on the right side, suggesting it's resting on a surface.

The polar equation of the circle C is

Find, in terms of a , the radius of C .

Fully justify your answer.

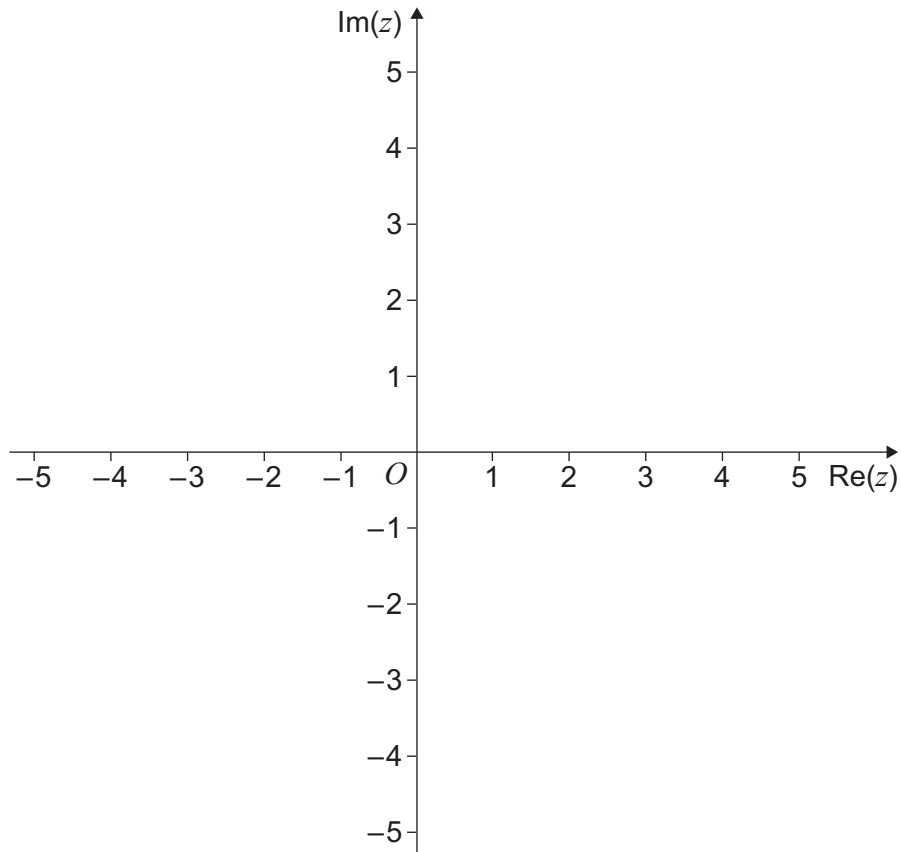
[illegible]

18 The locus of points L_1 satisfies the equation $|z| = 2$

The locus of points L_2 satisfies the equation $\arg(z + 4) = \frac{\pi}{4}$

18 (a) Sketch L_1 on the Argand diagram below.

[1 mark]



18 (b) Sketch L_2 on the Argand diagram above.

[1 mark]



There are no questions printed on this page

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ANSWER IN THE SPACES PROVIDED**



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[illegible]

[illegible]

Question number	Additional page, if required. Write the question numbers in the left-hand margin.
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