AQA

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname _____

Forename(s) _____

Candidate signature _____

I declare this is my own work.

A-level
FURTHER MATHEMATICS

Paper 1

Wednesday 22 May 2024 Afternoon Time allowed: 2 hours

Materials
- You must have the AQA Formulae and statistical tables booklet for A-level Mathematics and A-level Further Mathematics.
- You should have a graphical or scientific calculator that meets the requirements of the specification.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you require extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do **not** write outside the box around each page or on blank pages.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 100.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

For Examiner's Use	
Question	Mark
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TOTAL	

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Answer **all** questions in the spaces provided.

1 The roots of the equation $20x^3 - 16x^2 - 4x + 7 = 0$ are α, β and γ

Find the value of $\alpha\beta + \beta\gamma + \gamma\alpha$

Circle your answer.

[1 mark]

$-\frac{4}{5}$ $-\frac{1}{5}$ $\frac{1}{5}$ $\frac{4}{5}$

2 The complex number $z = e^{\frac{i\pi}{3}}$

Which **one** of the following is a real number?

Circle your answer.

[1 mark]

z^4 z^5 z^6 z^7

Do not write outside the box

3 The function f is defined by

$$\mathrm{f}(x) = x^2 \qquad (x \in \mathbb{R})$$

Find the mean value of $\mathrm{f}(x)$ between $x = 0$ and $x = 2$

Circle your answer.

[1 mark]

$\frac{2}{3}$ $\frac{4}{3}$ $\frac{8}{3}$ $\frac{16}{3}$

4 Which **one** of the following statements is correct?

Tick (✓) **one** box.

[1 mark]

$\lim_{x \to 0}(x^2 \ln x) = 0$ ☐

$\lim_{x \to 0}(x^2 \ln x) = 1$ ☐

$\lim_{x \to 0}(x^2 \ln x) = 2$ ☐

$\lim_{x \to 0}(x^2 \ln x)$ is not defined. ☐

Turn over for the next question

Do not write outside the box

5 The points A, B and C have coordinates $A(5, 3, 4)$, $B(8, -1, 9)$ and $C(12, 5, 10)$

The points A, B and C lie in the plane Π

5 (a) Find a vector that is normal to the plane Π

[3 marks]

Do not write outside the box

5 (b) Find a Cartesian equation of the plane Π

[2 marks]

Do not write outside the box

6 The sequence $u_1, u_2, u_3, \ldots$ is defined by

$$u_1 = 1$$

$$u_{n+1} = u_n + 3n$$

Prove by induction that for all integers $n \geq 1$

$$u_n = \frac{3}{2}n^2 - \frac{3}{2}n + 1$$

[4 marks]

Do not write outside the box

Turn over for the next question

7 The complex numbers z and w satisfy the simultaneous equations

$$z + w^* = 5$$

$$3z^* - w = 6 + 4\mathrm{i}$$

Find z and w

[5 marks]

Do not write outside the box

8 The ellipse *E* has equation

$$x^2 + \frac{y^2}{9} = 1$$

The line with equation $y = mx + 4$ is a tangent to *E*

Without using differentiation show that $m = \pm\sqrt{7}$

[4 marks]

9 (a) It is given that

$$p = \ln\left(r + \sqrt{r^2 + 1}\right)$$

Starting from the exponential definition of the sinh function, show that $\sinh p = r$

[4 marks]

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9 (b) Solve the equation

$$\cosh^2 x = 2\sinh x + 16$$

Give your answers in **logarithmic form**.

[4 marks]

Do not write outside the box

10 The complex numbers z and w are defined by

$$z = \cos\frac{\pi}{4} + \mathrm{i}\sin\frac{\pi}{4}$$

and $$w = \cos\frac{\pi}{6} + \mathrm{i}\sin\frac{\pi}{6}$$

By evaluating the product zw, show that

$$\tan\frac{5\pi}{12} = 2 + \sqrt{3}$$

[6 marks]

Do not write outside the box

11 (a) Find $\frac{\mathrm{d}}{\mathrm{d}x}(x^2\tan^{-1}x)$

[1 mark]

11 (b) Hence find $\int 2x\tan^{-1}x\,\mathrm{d}x$

[4 marks]

12 The line L_1 has equation

$$\mathbf{r} = \begin{bmatrix} 4 \\ 2 \\ 1 \end{bmatrix} + \lambda \begin{bmatrix} 1 \\ 3 \\ -1 \end{bmatrix}$$

The transformation T is represented by the matrix

$$\begin{bmatrix} 2 & 1 & 0 \\ 3 & 4 & 6 \\ -5 & 2 & -3 \end{bmatrix}$$

The transformation T transforms the line L_1 to the line L_2

12 (a) Show that the angle between L_1 and L_2 is 0.701 radians, correct to three decimal places.

[4 marks]

Do not write outside the box

12 (b) Find the shortest distance between L_1 and L_2

Give your answer in an exact form.

[6 marks]

Do not write outside the box

13 (a) Use de Moivre's theorem to show that

$$\cos 3\theta = 4\cos^3\theta - 3\cos\theta$$

[3 marks]

13 (b) Use de Moivre's theorem to express $\sin 3\theta$ in terms of $\sin\theta$

[2 marks]

Do not write outside the box

13 (c) Hence show that

$$\cot 3\theta = \frac{\cot^3\theta - 3\cot\theta}{3\cot^2\theta - 1}$$

[4 marks]

14 Solve the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} + y\tanh x = \sinh^3 x$$

given that $y = 3$ when $x = \ln 2$

Give your answer in an exact form.

[7 marks]

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Turn over for the next question

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15 A curve is defined parametrically by the equations

$$x = \frac{3}{2}t^3 + 5$$

$$y = t^{\frac{9}{2}} \qquad (t \geq 0)$$

Show that the arc length of the curve from $t = 0$ to $t = 2$ is equal to 26 units.

[5 marks]

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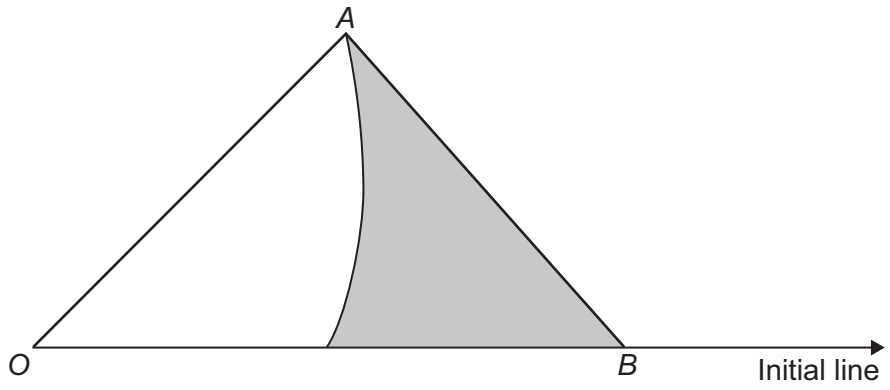
Turn over for the next question

16 The curve C has polar equation $r = 2 + \tan\theta$

The curve C meets the line $\theta = \frac{\pi}{4}$ at the point A

The point B has polar coordinates (4, 0)

The diagram shows part of the curve C, and the points A and B



16 (a) Show that the area of triangle OAB is $3\sqrt{2}$ units.

[2 marks]

16 (b) Find the area of the shaded region.

Give your answer in an exact form.

[7 marks]

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Turn over for the next question

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17 By making a suitable substitution, show that

$$\int_{-2}^{1} \sqrt{x^2 + 6x + 8}\ \mathrm{d}x = 2\sqrt{15} - \frac{1}{2}\cosh^{-1}(4)$$

[7 marks]

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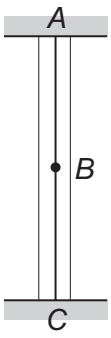
Turn over for the next question

18 **In this question use $g = 9.8$ m s^{-2}**

Two light elastic strings each have one end attached to a small ball B of mass 0.5 kg

The other ends of the strings are attached to the fixed points A and C, which are 8 metres apart with A vertically above C

The whole system is in a thin tube of oil, as shown in the diagram below.



The string connecting A and B has natural length 2 metres, and the tension in this string is $7e$ newtons when the extension is e metres.

The string connecting B and C has natural length 3 metres, and the tension in this string is $3e$ newtons when the extension is e metres.

18 (a) Find the extension of each string when the system is in equilibrium.

[3 marks]

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18 (b) It is known that in a large bath of oil, the oil causes a resistive force of magnitude $4.5v$ newtons to act on the ball, where v m s^{-1} is the speed of the ball.

Use this model to answer part **(b)(i)** and part **(b)(ii)**.

18 (b) (i) The ball is pulled a distance of 0.6 metres downwards from its equilibrium position towards C, and released from rest.

Show that during the subsequent motion the particle satisfies the differential equation

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} + 9\frac{\mathrm{d}x}{\mathrm{d}t} + 20x = 0$$

where x metres is the displacement of the particle below the equilibrium position at time t seconds after the particle is released.

[3 marks]

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18 (b) (ii) Find x in terms of t

[5 marks]

18 (c) State one limitation of the model used in part **(b)**

[1 mark]

END OF QUESTIONS

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