AQA

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname _____

Forename(s) _____

Candidate signature _____

I declare this is my own work.

A-level
FURTHER MATHEMATICS

Paper 2

Monday 3 June 2024 Afternoon Time allowed: 2 hours

Materials
- You must have the AQA Formulae and statistical tables booklet for A-level Mathematics and A-level Further Mathematics.
- You should have a graphical or scientific calculator that meets the requirements of the specification.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you require extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do **not** write outside the box around each page or on blank pages.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 100.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

For Examiner's Use	
Question	Mark
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Answer **all** questions in the spaces provided.

1 It is given that

$$\begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix} \bullet \begin{bmatrix} 5 \\ \lambda \\ -6 \end{bmatrix} = 0$$

where λ is a constant.

Find the value of λ

Circle your answer.

[1 mark]

–28 –8 8 28

2 The movement of a particle is described by the simple harmonic equation

$$\ddot{x} = -25x$$

where x metres is the displacement of the particle at time t seconds, and $\ddot{x}$ m s^{-2} is the acceleration of the particle.

The maximum displacement of the particle is 9 metres.

Find the maximum speed of the particle.

Circle your answer.

[1 mark]

15 m s^{-1} 45 m s^{-1} 75 m s^{-1} 135 m s^{-1}

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3 The function g is defined by

$$g(x) = \text{sech } x \qquad (x \in \mathbb{R})$$

Which one of the following is the range of g?

Tick (✓) **one** box.

[1 mark]

$-\infty < g(x) \leq -1$ ☐

$-1 \leq g(x) < 0$ ☐

$0 < g(x) \leq 1$ ☐

$1 \leq g(x) \leq \infty$ ☐

4 The function f is a quartic function with real coefficients.

The complex number $5i$ is a root of the equation $f(x) = 0$

Which **one** of the following **must** be a factor of $f(x)$?

Circle your answer.

[1 mark]

$(x^2 - 25)$ $(x^2 - 5)$ $(x^2 + 5)$ $(x^2 + 25)$

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5 The first four terms of the series S can be written as

$$S = (1 \times 2) + (2 \times 3) + (3 \times 4) + (4 \times 5) + ...$$

5 (a) Write an expression, using $\sum$ notation, for the sum of the first n terms of S

[1 mark]

5 (b) Show that the sum of the first n terms of S is equal to

$$\frac{1}{3}n(n + 1)(n + 2)$$

[2 marks]

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6 The cubic equation

$$x^3 + 5x^2 - 4x + 2 = 0$$

has roots α, β and γ

Find a cubic equation, with integer coefficients, whose roots are 3α, 3β and 3γ

[3 marks]

Turn over for the next question

Do not write outside the box

7 The matrices **A** and **B** are defined as follows.

$$\mathbf{A} = \begin{bmatrix} p-2 & p-1 \\ 0 & 1 \end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} 1 & 2p-1 \\ 0 & 4-p \end{bmatrix}$$

Find the values of p such that **A** and **B** are commutative under matrix multiplication.

Fully justify your answer.

[4 marks]

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8 The vectors **a**, **b,** and **c** are such that $\mathbf{a} \times \mathbf{b} = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$ and $\mathbf{a} \times \mathbf{c} = \begin{bmatrix} 0 \\ 0 \\ 3 \end{bmatrix}$

Work out $(\mathbf{a} - 4\mathbf{b} + 3\mathbf{c}) \times (2\mathbf{a})$

[4 marks]

9 A curve passes through the point (–2, 4.73) and satisfies the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{y^2 - x^2}{2x + 3y}$$

Use Euler's step by step method once, and then the midpoint formula

$$y_{r+1} = y_{r-1} + 2h\mathrm{f}(x_r, y_r), \quad x_{r+1} = x_r + h$$

once, each with a step length of 0.02, to estimate the value of y when $x = -1.96$

Give your answer to five significant figures.

[4 marks]

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Turn over for the next question

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10 The matrix **C** is defined by

$$\mathbf{C} = \begin{bmatrix} 3 & 2 \\ -4 & 5 \end{bmatrix}$$

Prove that the transformation represented by **C** has no invariant lines of the form $y = kx$

[4 marks]

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11 Latifa and Sam are studying polynomial equations of degree greater than 2, with real coefficients and no repeated roots.

Latifa says that if such an equation has exactly one real root, it must be of degree 3

Sam says that this is not correct.

State, giving reasons, whether Latifa or Sam is right.

[3 marks]

Turn over for the next question

12 The transformation S is represented by the matrix $\mathbf{M} = \begin{bmatrix} 1 & -6 \\ 2 & 7 \end{bmatrix}$

The transformation T is a reflection in the line $y = x\sqrt{3}$ and is represented by the matrix **N**

The point $P(x, y)$ is transformed first by S, then by T

The result of these transformations is the point $Q(3, 8)$

Find the coordinates of P

Give your answers to three decimal places.

[5 marks]

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Turn over for the next question

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13 (a) Use the method of differences to show that

$$\sum_{r=2}^{n} \frac{1}{(r-1)r(r+1)} = \frac{1}{4} - \frac{1}{2n} + \frac{1}{2(n+1)}$$

[5 marks]

13 (b) Find the smallest integer n such that

$$\sum_{r=2}^{n} \frac{1}{(r-1)r(r+1)} > 0.24999$$

[3 marks]

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14 The matrix **M** is defined as

$$\mathbf{M} = \begin{bmatrix} 5 & 2 & 1 \\ 6 & 3 & 2k+3 \\ 2 & 1 & 5 \end{bmatrix}$$

where k is a constant.

14 (a) Given that **M** is a non-singular matrix, find $\mathbf{M}^{-1}$ in terms of k

[5 marks]

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14 (b) State any restrictions on the value of k

[1 mark]

14 (c) Using your answer to part **(a)**, show that the solution to the set of simultaneous equations below is independent of the value of k

$$\begin{aligned} 5x + 2y + z &= 1 \\ 6x + 3y + (2k+3)z &= 4k+3 \\ 2x + y + 5z &= 9 \end{aligned}$$

[4 marks]

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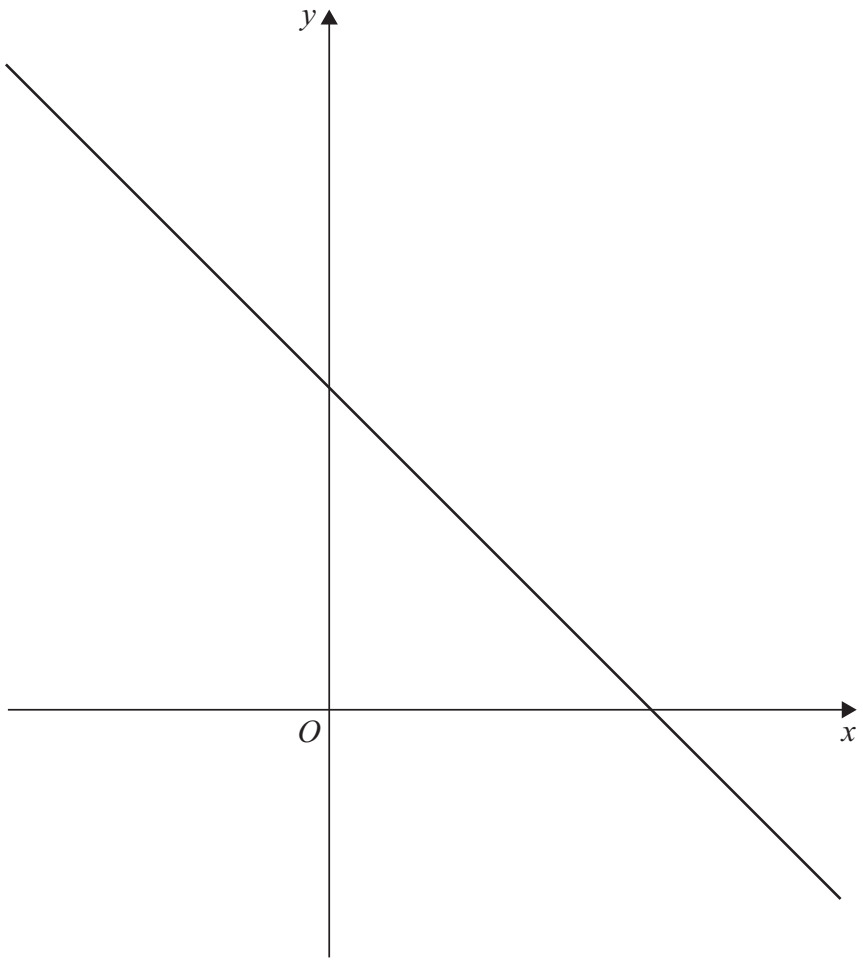
15 The diagram shows the line $y = 5 - x$



15 (a) On the diagram above, sketch the graph of $y = |x^2 - 4x|$, including all parts of the graph where it intersects the line $y = 5 - x$

(You do not need to show the coordinates of the points of intersection.)

[3 marks]

15 (b) Find the solution of the inequality

$$|x^2 - 4x| > 5 - x$$

Give your answer in an exact form.

[4 marks]

16 The function f is defined by

$$\mathrm{f}(x) = \frac{ax + 5}{x + b}$$

where a and b are constants.

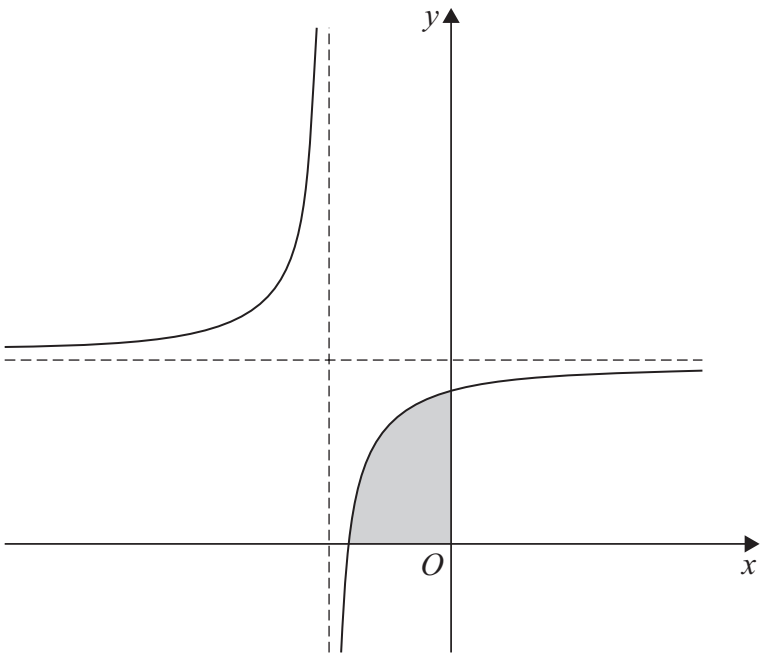
The graph of $y = \mathrm{f}(x)$ has asymptotes $x = -2$ and $y = 3$

16 (a) Write down the value of a and the value of b

[2 marks]

16 (b) The diagram shows the graph of $y = \mathrm{f}(x)$ and its asymptotes.

The shaded region R is enclosed by the graph of $y = \mathrm{f}(x)$, the x-axis and the y-axis.



16 (b) (i) The shaded region R is rotated through 360° about the x-axis to form a solid.

Find the volume of this solid.

Give your answer to three significant figures.

[3 marks]

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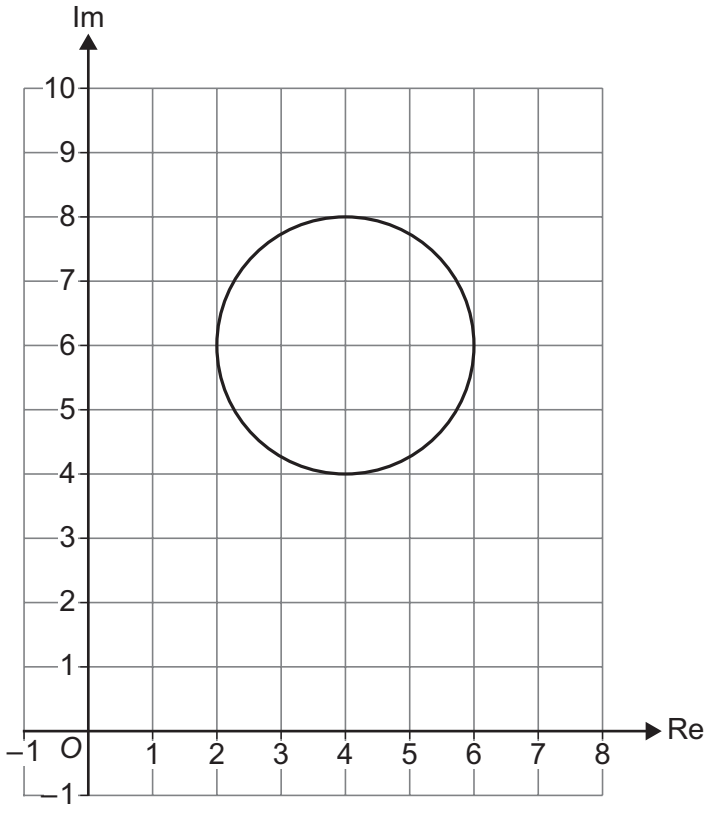
16 (b) (ii) The shaded region R is rotated through 360° about the y**-axis** to form a solid.

Find the volume of this solid.

Give your answer to three significant figures.

[4 marks]

17 The Argand diagram below shows a circle C



17 (a) Write down the equation of the locus of C in the form

$$|z - w| = a$$

where w is a complex number whose real and imaginary parts are integers, and a is an integer.

[2 marks]

17 (b) It is given that z_1 is a complex number representing a point on C. Of all the complex numbers which represent points on C, z_1 has the least argument.

17 (b) (i) Find $|z_1|$

Give your answer in an exact form.

[3 marks]

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17 (b) (ii) Show that $\arg z_1 = \arcsin\left(\frac{6\sqrt{3}-2}{13}\right)$

[4 marks]

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18 In this question you may use results from the formulae booklet without proof.

Use the binomial series for $(1 + x)^n$ and the Maclaurin's series for $\sin x$ to find the series expansion for $\dfrac{1}{(1 + \sin \theta)^4}$ up to and including the term in θ^3

[4 marks]

Turn over for the next question

19 Solve the differential equation

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + 4\frac{\mathrm{d}y}{\mathrm{d}x} - 45y = 21\mathrm{e}^{5x} - 0.3x + 27x^2$$

given that $y = \frac{37}{225}$ and $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$ when $x = 0$

[10 marks]

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Turn over ▶

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20 The integral I_n is defined by

$$I_n = \int_0^{\frac{\pi}{4}} \cos^n x \, \mathrm{d}x \qquad (n \geq 0)$$

20 (a) Show that

$$I_n = \left(\frac{n-1}{n}\right) I_{n-2} + \frac{1}{n\left(2^{\frac{n}{2}}\right)} \qquad (n \geq 2)$$

[6 marks]

20 (b) Use the result from part **(a)** to show that

$$\int_0^{\frac{\pi}{4}} \cos^6 x \, \mathrm{d}x = \frac{a\pi + b}{192}$$

where a and b are integers to be found.

[3 marks]

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