A-level

FURTHER MATHEMATICS
7367/3M

Paper 3 Mechanics

Mark scheme

June 2024

Version: 1.0 Final

246A7367/3M/MS

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

No student should be disadvantaged on the basis of their gender identity and/or how they refer to the gender identity of others in their exam responses.

A consistent use of 'they/them' as a singular and pronouns beyond 'she/her' or 'he/him' will be credited in exam responses in line with existing mark scheme criteria.

Further copies of this mark scheme are available from aqa.org.uk

Mark scheme instructions to examiners

General

The mark scheme for each question shows:

- the marks available for each part of the question
- the total marks available for the question
- marking instructions that indicate when marks should be awarded or withheld including the principle on which each mark is awarded. Information is included to help the examiner make his or her judgement and to delineate what is creditworthy from that not worthy of credit
- a typical solution. This response is one we expect to see frequently. However credit must be given on the basis of the marking instructions.

If a student uses a method which is not explicitly covered by the marking instructions the same principles of marking should be applied. Credit should be given to any valid methods. Examiners should seek advice from their senior examiner if in any doubt.

Key to mark types

M	mark is for method
R	mark is for reasoning
A	mark is dependent on M marks and is for accuracy
B	mark is independent of M marks and is for method and accuracy
E	mark is for explanation
F	follow through from previous incorrect result

Key to mark scheme abbreviations

CAO	correct answer only
CSO	correct solution only
ft	follow through from previous incorrect result
‘their’	indicates that credit can be given from previous incorrect result
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
NMS	no method shown
PI	possibly implied
sf	significant figure(s)
dp	decimal place(s)
ISW	Ignore Subsequent Workings

Examiners should consistently apply the following general marking principles:

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Diagrams

Diagrams that have working on them should be treated like normal responses. If a diagram has been written on but the correct response is within the answer space, the work within the answer space should be marked. Working on diagrams that contradicts work within the answer space is not to be considered as choice but as working, and is not, therefore, penalised.

Work erased or crossed out

Erased or crossed out work that is still legible and has not been replaced should be marked. Erased or crossed out work that has been replaced can be ignored.

Choice

When a choice of answers and/or methods is given and the student has not clearly indicated which answer they want to be marked, mark positively, awarding marks for all of the student's best attempts. Withhold marks for final accuracy and conclusions if there are conflicting complete answers or when an incorrect solution (or part thereof) is referred to in the final answer.

AS/A-level Maths/Further Maths assessment objectives

AO		Description
AO1	AO1.1a	Select routine procedures
	AO1.1b	Correctly carry out routine procedures
	AO1.2	Accurately recall facts, terminology and definitions
AO2	AO2.1	Construct rigorous mathematical arguments (including proofs)
	AO2.2a	Make deductions
	AO2.2b	Make inferences
	AO2.3	Assess the validity of mathematical arguments
	AO2.4	Explain their reasoning
	AO2.5	Use mathematical language and notation correctly
AO3	AO3.1a	Translate problems in mathematical contexts into mathematical processes
	AO3.1b	Translate problems in non-mathematical contexts into mathematical processes
	AO3.2a	Interpret solutions to problems in their original context
	AO3.2b	Where appropriate, evaluate the accuracy and limitations of solutions to problems
	AO3.3	Translate situations in context into mathematical models
	AO3.4	Use mathematical models
	AO3.5a	Evaluate the outcomes of modelling in context
	AO3.5b	Recognise the limitations of models
	AO3.5c	Where appropriate, explain how to refine models

Q	Marking Instructions	AO	Marks	Typical Solution
1	Circles 1st answer	2.2a	B1	8**i**
	Question total		**1**	

Q	Marking Instructions	AO	Marks	Typical Solution
2	Circles 3rd answer	1.1b	B1	0.9 N s
	Question total		**1**	

Q	Marking Instructions	AO	Marks	Typical Solution
3	Ticks 2nd box	2.2a	B1	$T \sin \theta = mr\omega^2$
	Question total		**1**	

Q	Marking Instructions	AO	Marks	Typical Solution
4(a)	States or uses the formula for the acceleration under circular motion. PI by correct answer.	1.1a	M1	$a = \frac{4^2}{0.8} = 20\text{ m s}^{-2}$
	Obtains 20 Condone missing units.	1.1b	A1	
	Subtotal		**2**	

Q	Marking Instructions	AO	Marks	Typical Solution
4(b)	Obtains 60 Follow through their acceleration Condone missing units.	1.1b	B1F	$T = 3 \times 20$ $= 60$ N
	Subtotal		**1**	

Q	Marking Instructions	AO	Marks	Typical Solution
4(c)	Selects and uses a method to find the angular speed for example seeing 5 (rad s^{-1}) 0.79 (revs per second) or 300 (rad per minute).	1.1a	M1	$\frac{4 \times 60}{2\pi \times 0.8} = 47.746....$ $= 48$ rpm
	Completes a reasoned argument to obtain 48. Condone missing units. AG	2.1	R1	
	Subtotal		**2**	

	Question total		**5**	

Q	Marking Instructions	AO	Marks	Typical Solution
5(a)	States MLT^{-2}	1.2	B1	MLT^{-2}
	Subtotal		**1**	

Q	Marking Instructions	AO	Marks	Typical Solution
5(b)	Uses dimensional analysis notation to form an equation. Must not use units.	3.3	M1	$[F] = [k][r]^{\alpha}[v]^{\beta}$ $MLT^{-2} = ML^{-2} \times L^{\alpha} \times L^{\beta}T^{-\beta}$ $1 = -2 + \alpha + \beta$ $\alpha + \beta = 3$ $-2 = -\beta$ $\beta = 2$ $\alpha = 1$
	Obtains either $\alpha = 1$ or $\beta = 2$ Condone use of units.	1.1a	M1	
	Completes a reasoned argument using dimensions to show that $\alpha = 1$ and $\beta = 2$	2.2a	R1	
	Subtotal		**3**	

	Question total		**4**	

Q	Marking Instructions	AO	Marks	Typical Solution
6(a)	Uses the EPE formula with their extension.	1.1a	M1	$\text{EPE} = \dfrac{18 \times 1.5^2}{2 \times 3} = 6.75\text{ J}$
	Obtains 6.75 or 6.8 Condone missing units.	1.1b	A1	
	Subtotal		**2**	

Q	Marking Instructions	AO	Marks	Typical Solution
6(b)	Uses GPE formula with their *AB*	1.1a	M1	GPE = $0.25 \times 9.8 \times 1.5$ $= 3.675$ J $= 3.7$ J (to 2sf)
	Obtains AWRT 3.7 Condone missing units.	1.1b	A1	
	Subtotal		**2**	

Q	Marking Instructions	AO	Marks	Typical Solution
6(c)	Forms a three-term energy equation using their EPE and GPE. Condone sign errors.	3.3	M1	$6.75 - 3.675 = \dfrac{1}{2} \times 0.25v^2$ $v^2 = 24.6$ $v = 5.0\text{ m s}^{-1}$
	Forms a correct equation using their energies.	1.1b	A1F	
	Obtains 5 or 5.0 Must include units.	3.2b	A1	
	Subtotal		**3**	

Q	Marking Instructions	AO	Marks	Typical Solution
6(d)	Obtains 3.6 for tension at *C*	1.1a	B1	$T = \frac{18 \times 0.6}{3} = 3.6\text{ N}$ Resultant force = $T - mg$ = 3.6 – 0.25 × 9.8 = 1.15 N Resultant force is upwards and the particle is moving upwards, so speed is increasing.
	Finds the resultant force or acceleration of the particle at *C* or compares the value of mg with the tension.	3.3	M1	
	Explains that this force is upwards and so the speed is increasing or an explanation that is consistent with their calculations.	2.4	R1	
	Subtotal		**3**	

	Question total		**10**	

Q	Marking Instructions	AO	Marks	Typical Solution
7(a)(i)	Forms two expressions for motion by resolving parallel and perpendicular to the wall.	3.3	M1	$5\cos 60° = v\cos\theta$ $0.7 \times 5\sin 60° = v\sin\theta$ $v = \sqrt{(5\cos 60°)^2 + (3.5\sin 60°)^2}$ $= 3.9$
	Obtains two correct expressions.	1.1b	A1	
	Eliminates θ to find a value for v	1.1a	M1	
	Obtains AWRT 3.9	1.1b	A1	
	Subtotal		**4**	

Q	Marking Instructions	AO	Marks	Typical Solution
7(a)(ii)	Uses their value for v or eliminates v to find an expression for $\tan\theta$, $\sin\theta$ or $\cos\theta$	3.4	M1	$\tan\theta = \dfrac{3.5\sin 60°}{5\cos 60°}$ $\theta = 50$
	Obtains AWRT 50	1.1b	A1	
	Subtotal		**2**	

Q	Marking Instructions	AO	Marks	Typical Solution
7(a)(iii)	Uses the impulse equation with components of velocities perpendicular to the wall with opposite signs.	3.4	M1	$I = 0.2 \times 3.929\sin 50.48°$ $- (-0.2 \times 5\sin 60°)$ $= 1.5$ Ns
	Obtains AWRT 1.5. Must be positive. Condone missing/incorrect units.	1.1b	A1	
	Subtotal		**2**	

Q	Marking Instructions	AO	Marks	Typical Solution
7(b)	Explains what happens to one component of the impulse.	2.4	M1	Any friction would not cause a change in the velocities perpendicular to the wall and so the magnitude of the component of the impulse perpendicular to the wall would be unchanged. There will be a component of the impulse parallel to the wall and so the magnitude of the total impulse would increase.
	Explains what will happen to the other component of the impulse and that the magnitude will increase.	2.4	R1	
	Subtotal		**2**	

	Question total		**10**	

Q	Marking Instructions	AO	Marks	Typical Solution
8(a)(i)	Sets up integral(s) to find the centre of mass. Condone lack of ρ	3.3	M1	$\pi\int_0^5 \rho(kx)^2\,\mathrm{d}x \times \bar{x} = \pi\int_0^5 \rho x(kx)^2\,\mathrm{d}x$ $\frac{125\rho\pi k^2}{3} \times \bar{x} = \frac{625\rho\pi k^2}{4}$ $\bar{x} = \frac{625\times3}{4\times125} = \frac{15}{4}$ which is independent of k
	Forms two correct integrals. Condone lack of ρ or missing dx	1.1b	A1	
	Evaluates at least one integral to obtain $\frac{125}{3}$ or $\frac{625}{4}$	1.1b	A1	
	Completes a reasoned argument to show that $\bar{x}$ is independent of k Must include ρ	2.1	R1	
	Subtotal		**4**	

Q	Marking Instructions	AO	Marks	Typical Solution
8(a)(ii)	Obtains $\frac{5}{4}$ OE Follow through their $\bar{x}$	3.4	B1F	Distance from base $= 5 - \frac{15}{4} = \frac{5}{4}$
	Subtotal		**1**	

Q	Marking Instructions	AO	Marks	Typical Solution
8(b)	States uniform	3.5b	B1	The cone is a uniform solid.
	Subtotal		**1**	

Q	Marking Instructions	AO	Marks	Typical Solution
8(c)	Obtains tanα = 0.8 or AWRT 38.7° .	3.3	B1	On the point of sliding tanα = 0.8 On the point of toppling $\tan\alpha = \frac{5k}{1.25} = 4k$ If the cone slides before it topples $0.8 < 4k$ $k > \frac{1}{5}$
	Finds an expression for tanα on the point of toppling using 5k and their $\frac{5}{4}$	3.3	M1	
	Obtains a correct expression for tanα on the point of toppling.	1.1b	A1	
	Deduces that $k > \frac{1}{5}$	2.2a	R1	
	Subtotal		**4**	

	Question total		**10**	

Q	Marking Instructions	AO	Marks	Typical Solution
9(a)	Resolves radially at the point *C*.	3.3	M1	$\frac{mv^2}{a} = mg\cos 30°$ $v^2 = ag\cos 30° = \frac{ag\sqrt{3}}{2}$ $\frac{1}{2}mU^2 = \frac{1}{2}mv^2 + mga(1+\cos 30°)$ $U^2 = v^2 + 2ga(1+\cos 30°)$ $U^2 = \frac{ag\sqrt{3}}{2} + 2ga\left(1+\frac{\sqrt{3}}{2}\right)$ $U^2 = ag\left(\frac{\sqrt{3}}{2}+2+\sqrt{3}\right)$ $U^2 = ag\left(2+3\frac{\sqrt{3}}{2}\right)$ $U^2 = \frac{ag}{2}\left(4+3\sqrt{3}\right)$
	Obtains a correct expression for the speed or speed squared at the point *C*..	1.1b	A1	
	Recognises that the initial speed is U	1.1a	B1	
	Uses conservation of energy to form a three-term equation to find the speed or speed squared of the particle at the point *C*. Must see consideration of KE at two points and change in GPE.	3.4	M1	
	Obtains a correct energy equation.	1.1b	A1	
	Completes a reasoned argument to obtain $U^2 = \frac{ag}{2}\left(4+3\sqrt{3}\right)$ AG	2.1	R1	
	Subtotal		**6**	

Q	Marking Instructions	AO	Marks	Typical Solution
9(b)	States that air resistance has been ignored.	3.5a	M1	As air resistance has been ignored, some energy would be lost so the value of U^2 must be greater than the value calculated. $U^2 > \frac{ag}{2}\left(4+3\sqrt{3}\right)$
	States $U^2 > \frac{ag}{2}\left(4+3\sqrt{3}\right)$. Condone $U^2 \geq \frac{ag}{2}\left(4+3\sqrt{3}\right)$	2.2b	R1	
	Subtotal		**2**	

	Question Total		**8**	