AS
FURTHER MATHEMATICS
7366/1

Paper 1

Mark scheme

June 2024

Version: 1.0 Final

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

No student should be disadvantaged on the basis of their gender identity and/or how they refer to the gender identity of others in their exam responses.

A consistent use of 'they/them' as a singular and pronouns beyond 'she/her' or 'he/him' will be credited in exam responses in line with existing mark scheme criteria.

Further copies of this mark scheme are available from aqa.org.uk

Mark scheme instructions to examiners

General

The mark scheme for each question shows:

- the marks available for each part of the question
- the total marks available for the question
- marking instructions that indicate when marks should be awarded or withheld including the principle on which each mark is awarded. Information is included to help the examiner make his or her judgement and to delineate what is creditworthy from that not worthy of credit
- a typical solution. This response is one we expect to see frequently. However credit must be given on the basis of the marking instructions.

If a student uses a method which is not explicitly covered by the marking instructions the same principles of marking should be applied. Credit should be given to any valid methods. Examiners should seek advice from their senior examiner if in any doubt.

Key to mark types

M	mark is for method
R	mark is for reasoning
A	mark is dependent on M marks and is for accuracy
B	mark is independent of M marks and is for method and accuracy
E	mark is for explanation
F	follow through from previous incorrect result

Key to mark scheme abbreviations

CAO	correct answer only
CSO	correct solution only
ft	follow through from previous incorrect result
‘their’	indicates that credit can be given from previous incorrect result
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
NMS	no method shown
PI	possibly implied
sf	significant figure(s)
dp	decimal place(s)
ISW	Ignore Subsequent Workings

Examiners should consistently apply the following general marking principles:

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Diagrams

Diagrams that have working on them should be treated like normal responses. If a diagram has been written on but the correct response is within the answer space, the work within the answer space should be marked. Working on diagrams that contradicts work within the answer space is not to be considered as choice but as working, and is not, therefore, penalised.

Work erased or crossed out

Erased or crossed out work that is still legible and has not been replaced should be marked. Erased or crossed out work that has been replaced can be ignored.

Choice

When a choice of answers and/or methods is given and the student has not clearly indicated which answer they want to be marked, mark positively, awarding marks for all of the student’s best attempts. Withhold marks for final accuracy and conclusions if there are conflicting complete answers or when an incorrect solution (or part thereof) is referred to in the final answer.

AS/A-level Maths/Further Maths assessment objectives

AO		Description
AO1	AO1.1a	Select routine procedures
	AO1.1b	Correctly carry out routine procedures
	AO1.2	Accurately recall facts, terminology and definitions
AO2	AO2.1	Construct rigorous mathematical arguments (including proofs)
	AO2.2a	Make deductions
	AO2.2b	Make inferences
	AO2.3	Assess the validity of mathematical arguments
	AO2.4	Explain their reasoning
	AO2.5	Use mathematical language and notation correctly
AO3	AO3.1a	Translate problems in mathematical contexts into mathematical processes
	AO3.1b	Translate problems in non-mathematical contexts into mathematical processes
	AO3.2a	Interpret solutions to problems in their original context
	AO3.2b	Where appropriate, evaluate the accuracy and limitations of solutions to problems
	AO3.3	Translate situations in context into mathematical models
	AO3.4	Use mathematical models
	AO3.5a	Evaluate the outcomes of modelling in context
	AO3.5b	Recognise the limitations of models
	AO3.5c	Where appropriate, explain how to refine models

Q	Marking instructions	AO	Marks	Typical solution
1	Circles the 1st answer.	1.1b	B1	$1+\sinh^2 x$
	Question total		**1**	

Q	Marking instructions	AO	Marks	Typical solution
2	Ticks the 2nd box.	1.1b	B1	$\pi\int_0^5 (2x+3)^2\,dx$
	Question total		**1**	

Q	Marking instructions	AO	Marks	Typical solution
3	Circles the 3rd answer.	1.1b	B1	$\frac{1}{2}$
	Question total		**1**	

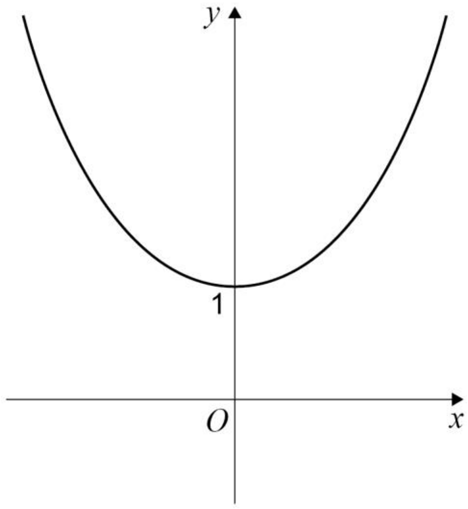
Q	Marking instructions	AO	Marks	Typical solution
4	Ticks the 2nd box.	1.1b	B1	$\frac{x-4}{-9}=\frac{y+7}{1}=\frac{z}{3}$
	Question total		**1**	

Q	Marking instructions	AO	Marks	Typical solution
5(a)	Obtains 12	1.1b	B1	$\mathbf{a.b} = 3\times2+4\times-1+-2\times-5$ $= 12$
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
5(b)	Obtains correct expression for at least one of $\lvert\mathbf{a}\rvert$ or $\lvert\mathbf{b}\rvert$ Condone missing brackets on the negative values.	1.1a	M1	$\lvert\mathbf{a}\rvert = \sqrt{3^2+4^2+(-2)^2} = \sqrt{29}$ $\lvert\mathbf{b}\rvert = \sqrt{2^2+(-1)^2+(-5)^2} = \sqrt{30}$
	Obtains $\sqrt{29}$ and $\sqrt{30}$ Condone AWRT 5.39 and 5.48	1.1b	A1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
5(c)	Writes a correct equation in θ Use of $\mathbf{a}\times\mathbf{b}$ must at least proceed to a correct calculation of $\lvert\mathbf{a}\times\mathbf{b}\rvert$ leading to an equation in θ PI by 65.99… FT their **a.b** and $\lvert\mathbf{a}\rvert$ and $\lvert\mathbf{b}\rvert$	1.1a	M1	$\cos\theta = \dfrac{12}{\sqrt{29}\times\sqrt{30}}$ $\theta = 65.9936\ldots$ $\theta = 66°$ (nearest degree)
	Obtains 66	1.1b	A1	
	Subtotal		**2**	

	Question total		**5**	

Q	Marking instructions	AO	Marks	Typical solution
6(a)	Draws the correct shape, approximately symmetrical about the y-axis and above the x-axis	1.1b	B1	
	Indicates one y-axis intercept at (0,1) Accept 1 clearly labelled on the y-axis.	1.1b	B1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
6(b)	Obtains at least one correct solution. Accept $\ln\left(2+\sqrt{3}\right)$ or $\ln\left(2-\sqrt{3}\right)$ or equivalent $\ln$ expression. Accept AWRT 1.32 or -1.32 Do not accept $\cosh^{-1}2$	1.1a	M1	$x = \pm\cosh^{-1}2$ $x = \pm1.32$ (3sf)
	Obtains AWRT ±1.32 with no other solutions given.	1.1b	A1	
	Subtotal		**2**	

	Question total		**4**	

Q	Marking instructions	AO	Marks	Typical solution
7	Writes a correct expression for the mean value, eg $\frac{1}{7-4}\int_4^7 \frac{1}{\sqrt{x}}\,dx$ PI by 0.430…	1.1a	M1	The mean of f $=\frac{1}{7-4}\int_4^7 x^{-\frac{1}{2}}\,dx$ $=\frac{1}{3}\left[2x^{\frac{1}{2}}\right]_4^7$ $=\frac{2}{3}\left(\sqrt{7}-2\right)$
	Integrates $\frac{1}{\sqrt{x}}$ to an expression of the form $ax^{\frac{1}{2}}$ where a is non-zero and substitutes 7 and 4 and subtracts. PI by 1.291… or 0.430… Note: $\frac{1}{4}\left(\frac{1}{\sqrt{4}}+\frac{1}{\sqrt{5}}+\frac{1}{\sqrt{6}}+\frac{1}{\sqrt{7}}\right)=0.433$ is M0	1.1a	M1	
	Obtains $\frac{2}{3}(\sqrt{7}-2)$ Ignore an approximated answer.	1.1b	A1	
	Question total		**3**	

Q	Marking instructions	AO	Marks	Typical solution
8(a)(i)	States $x - y\text{i}$	1.2	B1	$z^* = x - \text{i}y$
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
8(a)(ii)	Obtains a correct expansion **and** replaces i^2 with -1 PI	1.1a	M1	$zz^* = (x + \text{i}y)(x - \text{i}y)$ $= x^2 - \text{i}xy + \text{i}xy - \text{i}^2 y^2$ $= x^2 + y^2$ As x and y are both real then zz^* is real for all $z \in \mathbb{C}$
	Simplifies to $x^2 + y^2$ **and** explains zz^* (for all $z \in \mathbb{C}$) is real with a reference to x and y being real. Condone x^2 and y^2 for x and y in their explanation.	2.1	R1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
8(b)(i)	Substitutes $x + iy$ for w and $x - iy$ for w^*	3.1a	M1	Let $w = x + iy$ where $x, y \in \mathbb{R}$ $3(x + iy) + 10i = 2(x - iy) + 5$ $3x + 3iy + 10i = 2x - 2iy + 5$ Comparing real and imaginary parts: $3x = 2x + 5$ and $3y + 10 = -2y$ $x = 5$ and $5y = -10$ $y = -2$ $w = 5 - 2i$
	Obtains $\text{Re}(w) = 5$ or $\text{Im}(w) = -2$	1.1b	A1	
	Obtains $5 - 2i$	1.1b	A1	
	Subtotal		**3**	

Q	Marking instructions	AO	Marks	Typical solution
8(b)(ii)	Obtains 841 FT their w of the form $a + ib$ where a and b are non-zero	1.1b	B1F	$w^2(w^*)^2 = 841$
	Subtotal		**1**	

	Question total		**7**	

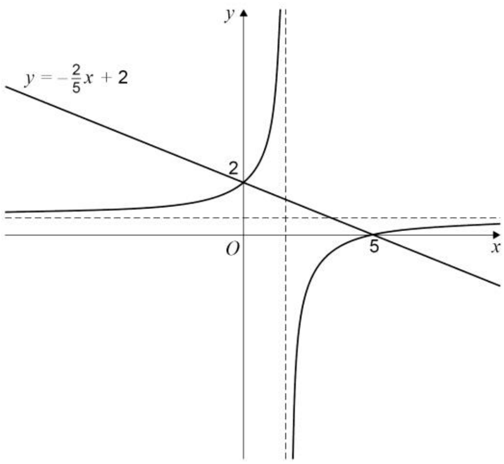
Q	Marking instructions	AO	Marks	Typical solution
9(a)	Completes a rigorous argument to show that $\frac{r+1}{r+2}-\frac{r}{r+1}=\frac{1}{(r+1)(r+2)}$ Must include the LHS, at least one intermediate step, and the RHS.	2.1	R1	$\frac{r+1}{r+2}-\frac{r}{r+1}=\frac{(r+1)^2-r(r+2)}{(r+1)(r+2)}$ $=\frac{r^2+2r+1-r^2-2r}{(r+1)(r+2)}$ $=\frac{1}{(r+1)(r+2)}$
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
9(b)	Writes the first two pairs (or last two pairs) of corresponding terms of $\frac{r+1}{r+2}$ and $\frac{r}{r+1}$	1.1a	M1	$\sum_{r=1}^{n}\frac{1}{(r+1)(r+2)}=\sum_{r=1}^{n}\left(\frac{r+1}{r+2}-\frac{r}{r+1}\right)$ $=\frac{2}{3}-\frac{1}{2}$ $+\frac{3}{4}-\frac{2}{3}$ $+\ .\ .\ .$ $+\frac{n}{n+1}-\frac{n-1}{n}$ $+\frac{n+1}{n+2}-\frac{n}{n+1}$ $=\frac{n+1}{n+2}-\frac{1}{2}$ $=\frac{2(n+1)-1(n+2)}{2(n+2)}$ $=\frac{n}{2n+4}$
	Writes at least the 1st pair **and** the nth pair of corresponding terms of $\frac{r+1}{r+2}$ and $\frac{r}{r+1}$ **and** shows the pattern of cancelling. The pattern of cancelling is possibly implied by a correct expression.	1.1a	M1	
	Completes a reasoned argument using the method of differences to obtain $\frac{n}{2n+4}$ Must include the 1st and nth terms and at least one pair of cancelling terms when completing the method of differences process.	2.1	R1	
	Subtotal		**3**	

Q	Marking instructions	AO	Marks	Typical solution
9(c)	Substitutes n = 2000 **or** n = 1000 into their $\dfrac{n}{an+b}$	1.1a	M1	$\sum_{r=1}^{2000} \dfrac{1}{(r+1)(r+2)} = \dfrac{2000}{2\times 2000+4} = \dfrac{500}{1001}$
	Substitutes n = 2000 **and** n = 1000 into their $\dfrac{n}{an+b}$ **and** subtracts.	3.1a	M1	$\sum_{r=1}^{1000} \dfrac{1}{(r+1)(r+2)} = \dfrac{1000}{2\times 1000+4} = \dfrac{250}{501}$
	Obtains the correct result. Ignore an approximated answer.	1.1b	A1	$\sum_{r=1001}^{2000} \dfrac{1}{(r+1)(r+2)} = \dfrac{500}{1001} - \dfrac{250}{501} = \dfrac{250}{501501}$
	Subtotal		**3**	

	Question total		**7**	

Q	Marking instructions	AO	Marks	Typical solution
10(a)	States $x = \frac{5}{3}$ **or** $y = \frac{2}{3}$	2.2a	M1	$x = \frac{5}{3}$ $y = \frac{2}{3}$
	States $x = \frac{5}{3}$ **and** $y = \frac{2}{3}$ and no incorrect equations seen.	2.2a	A1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
10(b)(i)	Draws a straight line with negative gradient passing through (0,2) Accept freehand if the intention is clear.	1.1a	M1	
	Draws a straight line passing through (0,2) and (5,0)	1.1b	A1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
10(b)(ii)	Deduces one of the ranges $x \le 0$ **or** $\frac{5}{3} < x \le 5$ Condone $\frac{5}{3} \le x \le 5$ or $\frac{5}{3} < x < 5$ for this mark only. Ignore any incorrect ranges.	2.2a	M1	$x \le 0$ $\frac{5}{3} < x \le 5$
	Deduces the solution $x \le 0$, $\frac{5}{3} < x \le 5$	2.2a	A1	
	Subtotal		**2**	

	Question total		**6**	

Q	Marking instructions	AO	Marks	Typical solution
11	Correct method for at least two elements of **AB**	1.1a	M1	$\mathbf{AB} = \begin{bmatrix} 3\text{i} & -2 \\ a & -\text{i} \end{bmatrix} \begin{bmatrix} 4 & 5 \\ -2\text{i} & -1 \end{bmatrix}$
	Obtains $4a-2$ in row 2 column 1 of **AB**	1.1b	B1	$= \begin{bmatrix} 12\text{i}+4\text{i} & 15\text{i}+2 \\ 4a+2\text{i}^2 & 5a+\text{i} \end{bmatrix}$
	Obtains $\begin{bmatrix} 16\text{i} & 2+15\text{i} \\ 4a-2 & 5a+\text{i} \end{bmatrix}$	1.1b	A1	$= \begin{bmatrix} 16\text{i} & 2+15\text{i} \\ 4a-2 & 5a+\text{i} \end{bmatrix}$
	Question total		**3**	

Q	Marking instructions	AO	Marks	Typical solution
12	Writes $5^1 - 2^1 = 3$ Accept $5^0 - 2^0 = 0$	2.1	B1	$5^1 - 2^1 = 3$ 3 is divisible by 3 So the rule is true for $n = 1$ If the rule is true for $n = k$ then $5^k - 2^k = 3m$ for some integer m $\Rightarrow 5^k = 3m + 2^k$ $5^{k+1} - 2^{k+1} = 5 \times 5^k - 2 \times 2^k$ $= 5(3m + 2^k) - 2 \times 2^k$ $= 15m + 5 \times 2^k - 2 \times 2^k$ $= 15m + 3 \times 2^k$ $= 3(5m + 2^k)$ $= 3 \times$ integer So the rule is also true for $n = k + 1$ Therefore, by induction $5^n - 2^n$ is divisible by 3 for all $n \in \mathbb{N}$
	Assumes $5^k - 2^k$ is divisible by 3 **and** considers $5^{k+1} - 2^{k+1}$	2.4	M1	
	Completes rigorous working to show that $5^{k+1} - 2^{k+1}$ is divisible by 3	2.2a	A1	
	Concludes a reasoned argument by stating that $5^n - 2^n$ is divisible by 3 when $n = 1$ (or $n = 0$), **and** that if $5^n - 2^n$ is divisible by 3 when $n = k$ then it is also divisible by 3 when $n = k + 1$ **and** hence (by induction) $5^n - 2^n$ is divisible by 3 for all n $(\in \mathbb{N})$ Condone reference to 'rule' / 'statement' in their final statement. Must define their m to be an integer.	2.1	R1	
	Question total		**4**	

Q	Marking instructions	AO	Marks	Typical solution
13(a)	Identifies the transformation as a translation. Do not accept alternatives such as 'move'.	3.1a	M1	Translation $\begin{bmatrix} -1 \\ 0 \end{bmatrix}$
	States Translation $\begin{bmatrix} -1 \\ 0 \end{bmatrix}$ Accept 1 to the left instead of $\begin{bmatrix} -1 \\ 0 \end{bmatrix}$ Do not accept any extra erroneous description.	3.2a	A1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
13(b)	Substitutes $y + 1$ for x Accept any letter, including x Or Writes one of: $\alpha + \beta + \gamma = 0$ $\alpha\beta + \beta\gamma + \gamma\alpha = -1$ $\alpha\beta\gamma = 7$	1.1b	B1	Let $y = x - 1$ $\Rightarrow \quad x = y + 1$ So the new equation is $(y + 1)^3 - (y + 1) - 7 = 0$ $\Rightarrow \quad y^3 + 3y^2 + 3y + 1 - y - 1 - 7 = 0$ $\Rightarrow \quad y^3 + 3y^2 + 2y - 7 = 0$ So $\mathrm{p}(x) = x^3 + 3x^2 + 2x - 7$
	Expands and simplifies their $(my + c)^3 - (my + c) - 7 = 0$ where m is non-zero Allow one incorrect simplified coefficient. Or Correctly calculates two of $\sum(\alpha - 1)$ $\sum(\alpha - 1)(\beta - 1)$ $(\alpha - 1)(\beta - 1)(\gamma - 1)$ for their values of $\alpha + \beta + \gamma$ and/or $\alpha\beta + \beta\gamma + \gamma\alpha$ and/or $\alpha\beta\gamma$ PI by two correct terms in a cubic equation other than x^3	1.1a	M1	
	Obtains $x^3 + 3x^2 + 2x - 7$	2.2a	R1	
	Subtotal		**3**	

	Question total		**5**	

Q	Marking instructions	AO	Marks	Typical solution
14(a)	Multiplies the position vector of *A* by **M** to achieve at least one correct calculation or coordinate.	1.1a	M1	$\begin{bmatrix} 3 & -1 \\ -2 & 6 \end{bmatrix}\begin{bmatrix} 4 \\ -5 \end{bmatrix} = \begin{bmatrix} 3\times 4 + -1\times -5 \\ -2\times 4 + 6\times -5 \end{bmatrix}$ $= \begin{bmatrix} 17 \\ -38 \end{bmatrix}$ Image of *A* is (17, –38)
	Obtains (17, –38) Do not accept a position vector.	1.1b	A1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
14(b)	Considers a general point (x, y) Multiplies **M** $\begin{bmatrix} x \\ y \end{bmatrix}$ to form at least one correct equation. May be unsimplified. Or Shows that the origin is invariant, eg writes **M** $\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ eg states that the origin is always invariant under a linear transformation.	3.1a	M1	$\begin{bmatrix} 3 & -1 \\ -2 & 6 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$ $3x - y = x$ and $-2x + 6y = y$ $y = 2x$ and $2x = 5y$ $x = 0$ and $y = 0$ So (0,0) is the only invariant point
	Forms two different correct equations in x and y	1.1a	M1	
	Completes a reasoned argument and concludes that the origin is the only invariant point.	2.1	R1	
	Subtotal		**3**	

Q	Marking instructions	AO	Marks	Typical solution
14(c)	Writes the product **M**$\begin{bmatrix} x \\ x+1 \end{bmatrix}$ and equates to $\begin{bmatrix} X \\ mX+c \end{bmatrix}$ or equivalent. PI by one correct equation. **Or** Multiplies **M** by a specific point on the line $y = x + 1$	3.1a	M1	
	Obtains two correct equations from **M**$\begin{bmatrix} x \\ x+1 \end{bmatrix} = \begin{bmatrix} X \\ mX+c \end{bmatrix}$ or equivalent. PI **Or** Obtains a correct specific point on the line $y = 2x + 8$	1.1b	A1	$\begin{bmatrix} 3 & -1 \\ -2 & 6 \end{bmatrix}\begin{bmatrix} x \\ x+1 \end{bmatrix} = \begin{bmatrix} X \\ mX+c \end{bmatrix}$ $3x - 1(x + 1) = X$ and $-2x + 6(x + 1) = mX + c$
	Forms a correct equation in either m or c FT their two equations **Or** Multiplies **M** by a second specific point on the line $y = x + 1$	1.1a	M1	$2x - 1 = X$ and $4x + 6 = mX + c$ So $4x + 6 \equiv m(2x - 1) + c$ $4 = 2m$ and $6 = -m + c$
	Solves two equations in m and c Accept one incorrect equation if it has clearly come from **M**$\begin{bmatrix} x \\ x+1 \end{bmatrix} = \begin{bmatrix} X \\ mX+c \end{bmatrix}$ or equivalent. **Or** Forms an unsimplified Cartesian equation for a straight line connecting their two points. Accept one incorrect point if it has clearly come from an attempt to find a point on the new line.	1.1a	M1	$m = 2$ $6 = -2 + c$ $c = 8$ The new line is $y = 2x + 8$
	Obtains $y = 2x + 8$	2.2a	A1	
	Subtotal		**5**	

	Question total		**10**	

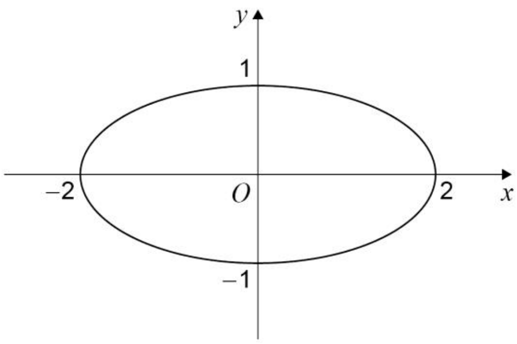
Q	Marking instructions	AO	Marks	Typical solution
15(a)	Substitutes 3x for x in the ln (1 + x) series. **and** Simplifies to $3x-\frac{9}{2}x^2+9x^3$ Condone missing LHS.	2.1	R1	$\ln(1+3x)=3x-\frac{(3x)^2}{2}+\frac{(3x)^3}{3}-....$ $\ln(1+3x)\approx 3x-\frac{9x^2}{2}+\frac{27x^3}{3}$ $\ln(1+3x)\approx 3x-\frac{9}{2}x^2+9x^3$
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
15(b)	States $(-1<)\ 3x\leq 1$ Condone $3x<1$	3.1a	M1	The expansion is only valid for $-1<3x\leq 1$ $\Rightarrow\ -\frac{1}{3}<x\leq\frac{1}{3}$ x = 1 is not in this valid range So Julia should not have substituted x = 1
	Explains that $\boldsymbol{x}$ **= 1** is not a **valid** value.	2.3	E1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
15(c)	Substitutes $x = -\frac{1}{6}$ into $3x - \frac{9}{2}x^2 + 9x^3$ **and** $\ln(1 + 3x)$ PI by $-2 \times \left(3\left(-\frac{1}{6}\right) - \frac{9}{2}\left(-\frac{1}{6}\right)^2 + 9\left(-\frac{1}{6}\right)^3\right)$	1.1a	M1	$\ln\left(1+3\left(-\frac{1}{6}\right)\right)$ $\approx 3\left(-\frac{1}{6}\right) - \frac{9}{2}\left(-\frac{1}{6}\right)^2 + 9\left(-\frac{1}{6}\right)^3$ $\ln\frac{1}{2} \approx -\frac{1}{2} - \frac{1}{8} - \frac{1}{24}$ $\ln\frac{1}{2} \approx -\frac{2}{3}$ But $\ln 4 = 2\ln 2 = -2\ln\frac{1}{2}$ $\therefore \quad \ln 4 \approx -2 \times -\frac{2}{3}$ $\ln 4 \approx \frac{4}{3}$
	Obtains $\ln\frac{1}{2} \approx -\frac{2}{3}$ Accept AWRT -0.667 PI by $\ln 4 \approx -2 \times \left(3\left(-\frac{1}{6}\right) - \frac{9}{2}\left(-\frac{1}{6}\right)^2 + 9\left(-\frac{1}{6}\right)^3\right)$ or $\ln 4 \approx -2 \times \left(-\frac{2}{3}\right)$	1.1b	A1	
	Uses $\ln 4 = -2\ln\frac{1}{2}$	1.1a	M1	
	Deduces $\ln 4 \approx \frac{4}{3}$ Accept AWRT 1.33 Condone an equals sign instead of an approximation sign for all four marks.	2.2a	R1	
	Subtotal		**4**	

	Question total		**7**	

Q	Marking instructions	AO	Marks	Typical solution
16(a)	Correctly removes the square root sign. Or Correctly isolates r^2 (or r) if moving from the Cartesian form to the polar form.	3.1a	M1	$r = \dfrac{2}{\sqrt{\cos^2\theta + 4\sin^2\theta}}$ $r^2 = \dfrac{4}{\cos^2\theta + 4\sin^2\theta}$ $r^2\cos^2\theta + 4r^2\sin^2\theta = 4$ $(r\cos\theta)^2 + 4(r\sin\theta)^2 = 4$ $x^2 + 4y^2 = 4$ $\dfrac{x^2}{4} + y^2 = 1$ $\dfrac{x^2}{2^2} + \dfrac{y^2}{1^2} = 1$
	Correctly uses $x = r\cos\theta$ **or** $y = r\sin\theta$	1.1a	M1	
	Correctly uses $x = r\cos\theta$ **and** $y = r\sin\theta$	1.1a	M1	
	Completes a reasoned argument to obtain $\dfrac{x^2}{2^2} + \dfrac{y^2}{1^2} = 1$ Accept $\dfrac{x^2}{4} + y^2 = 1$ if $a = 2$ and $b = 1$ seen.	3.2a	A1	
	Subtotal		**4**	

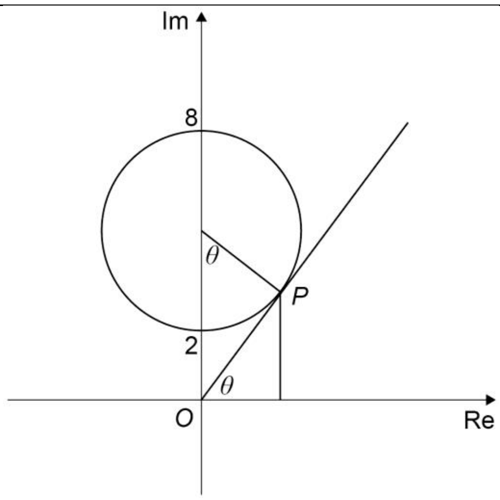
Q	Marking instructions	AO	Marks	Typical solution
16(b)	Draws an ellipse centred on the origin. Accept a circle.	1.1a	M1	
	Identifies the correct intercepts at 2 and –2 and 1 and –1 FT their a and b Accept $\pm a$ and $\pm b$ if no values found in part (a)	1.1b	A1F	
	Subtotal		**2**	

	Question total		**6**	

Q	Marking instructions	AO	Marks	Typical solution
17(a)(i)	Deduces that $a = 5$ Accept $\lvert z - 5\mathrm{i}\rvert = b$	2.2a	B1	$a = 5$
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
17(a)(ii)	Deduces that $b = 3$ Accept $\lvert z - a\mathrm{i}\rvert = 3$	2.2a	B1	$b = 3$
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
17(b)(i)	Forms a correct equation in *OP* FT their a and b	3.1a	M1	$OP^2 = 5^2 - 3^2$ $OP = 4$
	Obtains $OP = 4$ FT their a and b	1.1b	A1F	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
17(b)(ii)	Obtains a correct expression or equation for an acute angle in the 3,4,5 triangle. FT their a, b and *OP*	3.1a	M1	 $\tan\theta = \frac{4}{3}$ $k = \frac{4}{3}$
	Deduces that $k = \frac{4}{3}$	2.2a	A1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
17(b)(iii)	Forms a correct equation in the real and/or imaginary part of *P* eg $x^2+(y-5)^2=3^2$, $y=\frac{4}{3}x$ FT their a, b and *OP*	3.1a	M1	$x=\frac{3}{5}\times 4=2.4$ $y=\frac{4}{5}\times 4=3.2$ $z=2.4+3.2\mathrm{i}$
	Obtains 2.4 or 3.2	1.1b	A1	
	Obtains $2.4+3.2\mathrm{i}$	1.1b	A1	
	Subtotal		**3**	

	Question total		**9**	

	Question Paper total		**80**	