Examiners' Report
Principal Examiner Feedback

Summer 2024

Pearson Edexcel GCE

In A Level Further Mathematics (9FM0)

Paper 01 Core Pure Mathematics 1

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9FM0 01 Examiners Report

Most candidates appeared to be able to complete the paper in the time available with several producing more than one solution for certain questions when they had gone wrong. The paper gave good access throughout enabling candidates across all grades to score well, with no surprising twists in any questions to throw candidates off track.

There appeared to be a lack of space to answer question 6 in particular, with many candidates either using extra paper or completing work on the final pages, or elsewhere, in the answer booklet. This is perhaps due to a lack of confidence in proof by induction which required several attempts by some candidates, or due to the copious amounts of unnecessary working and large handwriting in the process of expanding to simplify during the inductive step.

Individual question reports

Question 1

The majority of students found this an accessible start to the paper with many fully correct solutions, and the most common error being a failure to sketch the roots correctly on their diagram for part (b).

In part (a) almost all of the candidates got the B mark for the complex conjugate as another root and the majority were able to find all four roots correctly. Only a very small number of candidates gave the wrong "conjugate" root, $-2-5\text{i}$, with omission of the question altogether being a more common reason for the loss of the B mark.

The most direct and simplest method for finding the remaining roots was to find the quadratic factor $z^2-4z+29$ from the conjugate pair of roots and then factor out by inspection to obtain the second quadratic factor before solving. However, most using this method used polynomial division to obtain the other quadratic factor, which created a lot more work and potential for error, although it is mostly done well. Only a few answer this method in the most efficient way, but holistically this was marginally the most common method of those on the scheme.

The other very common approach was using the formulae for the sum, sum of product pairs and product of roots (Alt 1). The inclusion of the manipulation of roots of polynomials on the specification seems to have made this a much more common approach. Again, this was done very well overall, usually resulting in the correct roots, though errors in manipulation were slightly more common in this approach (usually introducing a sign error in one term in the resulting quadratic). Explanations were often minimal in these approaches, sometime making it different to follow the work. Variations such as using $x\pm\text{i}y$ for the other two roots were also seen used, and generally successful.

The factor theorem approach (Alt 2) was the least popular approach, seen on occasionally, and candidates adopting this method often stopped as they were unable to cope with the resulting algebra. They tended to be the most erratic and relied on calculator assistance to expand the complex equations and solve often incorrect quadratics that resulted. A difficulty that arose was that nearly correct quadratics could sometimes lead to the "correct" roots as candidates would round to nearest integers for the real and imaginary terms, but the accuracy marks were only permitted for fully correct work.

It is also noteworthy that many candidates felt the need to find the values of *a* and *b*, which were not necessary to find in the main scheme or Alt 1 and not needed. This again created extra unnecessary work but was ignored if it was not part of the solution leading to the roots. An awareness of the demand of the question was lacking in many candidate who automatically assumed the needed to do this.

Within all methods used there were the usual numerical errors, usually focussing on incorrect multiplication of complex numbers and not using i^2 as -1.

In part (b) a surprising proportion of candidates did not draw the required diagram carefully enough, and the general standard of diagrams was quite poor. Most did plot two roots correctly (usually the first two), even if there was a slight lack of symmetry due to the quality of the diagram – being either skewed or angled (rulers were not used by most) – but the correct idea of the complex conjugates was portrayed. While the majority were able to add the second roots to the diagram correctly, many were too slack with proportions, the 1± 2i roots lying outside the sector formed by the 2 ± 5i roots being the most common fault, thus losing the final mark. The labelling was generally given if the roots were correctly placed.

Question 2

Candidates were clearly well prepared for this type of question with most achieving full marks. Many of the candidates had already exemplified competence with the sum and product of roots methods in question 1 and were able to fully correctly complete this question too.

In part (a), the only notable errors were errors in signs of the required sum, sum of product pairs and product of roots or the neglect to divide through by the 2 resulting in double the correct values, both of which happened only occasionally. The vast majority of candidates identified the correct values.

In part (b), the methods were well known, though a few candidates neglected the instruction to use the answers to part (a) and attempt transformation approaches (forfeiting the marks).

In part (b)(i) candidates usually derived the correct identity and made the appropriate substitution. Many used ∑ notation in their numerator, showing good use of notation to shorten the expression. A few lost both marks by inserting the $\frac{3}{2}$ in their denominator rather than the correct $-\frac{7}{2}$, or less commonly the $\frac{3}{2}$ was inserted as the value for $\sum\alpha\beta$. Many were able to gain the accuracy for follow through even in the rare cases that wrong values were found in part (a), while only a small number made slips simplifying from a correct initial substitution.

For part (b)(ii), in most cases there was an attempt to expand the product. Where this did not happen it was usually due to an attempt at applying a transformation $w = x \pm 1$ to find the new product of roots, but this was not permitted marks.

The expansions, when attempted, were generally done well but a common error in a number of instances was that the final term "−1" was omitted or had the incorrect sing in the end result. Either way, the final two marks were lost. Occasionally a slip in signs or incorrect introductory values resulted in the final mark being lost.

Finally, in part (b)(iii) most candidates again achieved the correct identity and the correct answer. However, it was in this part that the most inaccuracies in the identity occurred. Incorrect identities included the sum of squares of roots being $(\alpha\beta\gamma)^2 - 2\sum\alpha\beta$, $\left(\sum\alpha\right)^2 - 3\sum\alpha\sum\alpha\beta$ or $\left(\sum\alpha\right)^2 - 3\sum\alpha\beta\gamma$. The most common error, though. was using $-\frac{7}{2}$ instead of $\frac{3}{2}$ for $\sum\alpha$.

Overall the formation of the identities was therefore very good, with part (b)(ii) causing most difficulties and errors tended to be slips in substitution of simplification.

Question 3

While generally the polar work was very well done in this question, understanding the context at the start caused a few difficulties with many unable to write down the correct value for *a*.

Part (a) proved surprisingly challenging for candidates, in what was not anticipated to be a difficult part. Solutions often did not consider the extreme values of sin 3θ or tried using incorrect values of θ to proceed. Common incorrect values for a were -1.5 (which could gain the method mark) and 3.5 $(=4-0.5)$, but various other incorrect values also were seen, for instance some candidates did not take account of the either the 0.5m gap, yielding $6=4\pm a$ so $a=\pm 2$

The strategy involved in part (b) subtracting the area enclosed of the polar curve obtained by integration from the area of the circle was well understood, and the approach to the integral was carried out correctly by most. Slips in working or inappropriate limits accounted for most errors made.

There were only a small number of solutions where squaring was not seen, did not produce three terms, or where the double angle formula was not used to get to a function of 6θ. Early coefficient slips in expanding meant the A1ft was forfeit but most showed a correct method integration for integration via use of the double angle identity.

The most successful solutions then chose the limits as 0 to 2π for the whole area but other pairs of limits using symmetry were attempted, with varying levels of success – there were pairs that were not appropriate, such as 0 to π followed by doubling. Some combinations such as $-\frac{\pi}{2}$ to $\frac{\pi}{2}$ and doubling, or 0 to $\frac{2\pi}{3}$ and tripling, were possible but often only served to increase the amount of work and propensity for error. Candidates should consider whether the use of symmetry approaches make the work involved any simpler.

Most candidates went to find the area of the circle and subtract the area of the curve from it, though a few did omit this final step. With the vast majority having gained the first method, there were relatively few cases where the final M was lost despite remembering this step. It was pleasing to see candidates giving exact answers, with only very few resorting to decimal work, but careful checking was needed when correct answers were found since errors in the trigonometric terms cancelled over the full circle meaning inaccuracies in these terms would still lead to the correct answer but in fact lost both A marks.

Overall many candidates scored full marks on part (b) with some elegant and efficient mathematics, while others lost only the final A due to an incorrect answer in part (a), as follow through on intermediate marks meant the rest of the marks could be scored.

Question 4

Almost all candidates were able to make a meaningful attempt at parts or all of this question, with part (b) being the most successful overall, as a standard type proof on this topic. Notation was sometimes questionable, but the methodology was generally sound.

Part (a) was generally well approach, giving some easy access in the first mark for sorting out the negative power, so most candidates made some progress with this part. There was a variety of approaches, although most substituted directly into trigonometric terms, either from the stated *z* and Euler's identity or proceeding via De Moivre's theorem. Most who used these methods achieved a correct proof for both marks, though some missed out an *n* in their working and/or their answer, or other similar slip, losing the accuracy.

However, a few circular arguments were seen, which could score only the first mark for dealing with the negative index. These went on to use $\cos(n\theta)=\frac{1}{2}\left(e^{-in\theta}+e^{in\theta}\right)$, essentially assuming the result to be proved, to get to the result.

There were a few attempts via the alternative at combining fractions rather than applying the negative index, which were more prone to error, but often led to a successful proof.

In part (b) the vast majority of candidates scored full marks for a well-practised proof. Most started with an attempt to expand $\left(z+z^{-1}\right)^5$, whilst a few used $\left(\frac{e^{-i\theta}+e^{i\theta}}{2}\right)^5$ with equal success. Usually, the binomial coefficients were correct, and a satisfactory solution followed. On a rare occasion the terms were not grouped and the final two marks were lost. Sometimes a candidate confused $\cos^5\theta$ with cos 5θ and the solution went array.

Those who used the alternative method, by De Moivre, were largely unsuccessful, falling down when an identity for $\cos^3\theta$ was need. A correct De Moivre statement for cos5θ was not always stated or implied and the solutions usually progressed no further than obtaining a set of real terms, with or without the substitution for $\sin^2\theta$.

In part (c) the most common error was to reduce the equation incorrectly to $16\cos^5\theta=0$ scoring no marks, despite that the correct answers fortuitously followed from this. But the majority of candidates did get to the correct equation, or one of the correct form, and then most also proceeded correctly to the two solutions, to obtain the first two marks.

There was the occasional division by $\cos\theta$ that lost the method mark, but generally the method of solution was sound. Of those who factorised correctly most went on to give a suitable reason why there were no further solution and scored full marks, but there were several who did not manage to do so. Errors that did occur at this stage included: not indicating that $8\cos^4\theta=-1$ has no solutions, finding $\frac{\pi}{2}$ correctly but omitting the $\frac{3\pi}{2}$, incorrect factorisation such as $2\cos\theta\left(4\cos^4\theta+1\right)=0$ and finding incorrect solutions to $8\cos^4\theta+1=0$ (e.g. by solving $8\cos^4\theta=1$ or generating other spurious solutions).

Question 5

Despite being a context question, which often caused difficulties in comprehension, this question was well answered by the majority, who could easily see through the context to focus on the mathematics required and were able to for the most part to give answers in context at the end.

In part (a) most candidates were able to obtain full marks for fully correct solutions to the differential equation, though it was the least successfully answered part in the question overall. Very few failed to realise that an integrating factor was needed and nearly all were successful in establishing that the factor was $(t+2)^3$. If they did not then no progress was likely to be made in this part.

After finding the integrating factor, again most were able to use it correctly and attempt the integration, though a small number did not obtain either the correct left- or right-hand side. A left-hand side of $v(t+2)^2$ was seen a few times, while for the right-hand side the most common error was $\int k(t+2)^4 - 3(t+2)^3 \, \mathrm{d}t$. However, most were able to obtain a correct equation and attempt to integrate. A few opted to expand before integration, giving a much hard task to reach the given solution later, and a few attempted to use integration by parts on $(t+2)^3\left(k-\frac{3}{t+2}\right)$ without success, but most were able to obtain the most direct answer as they could see the form that was being aimed for.

The most common error in this part was to integrate correctly on both sides, including a constant but then failing to use the initial conditions $t = 0$, $v = 0$, either just stating that $c = 4(2-k)$ without justification, attempting to use $v = 4$ and $t = 2$ in their attempt to find c (whether successful or not). Omission of the constant of integration entirely was rare. Correct completion to the given equation generally followed when the correct conditions were used.

Part (b) was the best answered part of the question overall, with only a very small number of candidates unable to make any progress – usually following having tried to use the use $v = 4$ and $t = 2$ conditions in part (a) so not relating them to part (b). A few tried to use their wrong (a) to continue which led to a loss of marks, and candidates would be best advised to make use of given answers on papers rather than proceeding with their own versions.

Most were able to follow the correct procedure of substituting use $v = 4$ and $t = 2$ into the equation, usually then calculating a correct k and proceeding on to a value for v. Most often this value was correct and m/s units were stated scoring all the marks, though a few did omit the units and so lost the accuracy. There were also some who had arithmetical errors for k, with $k = 5$ arising from erroneously also substituting for t in place of the k in the $4(2-k)$ being a common such slip. But these could still proceed to a value for v to gain the method.

Also a few candidates gave an answer of 8.1 m/s (insufficient precision), while many candidates gave their answer as a fraction, which could achieve the A mark if the correct units were present.

Part (c) was generally well answered with most candidates able to give a reason and conclusion related to the context. Most common was to state that $v \to \infty$ as $t \to \infty$ and either that this is not realistic (or similar) or to mention terminal velocity, or the raindrop hitting the ground.

It was also common for candidates to mention that this would mean that the droplet would travel faster that the speed of light, or to continue increasing indefinitely, which is not possible.

But not all achieved the mark as some gave either factually incorrect or nonsensical answers (such as not being able to have infinite rain), for instance stating that v increases exponentially, or think that $v \to 0$ as t increases. There was a requirement to explicitly say that the model is invalid or similar, which some also failed to do, just stating what happens, but with no interpretation.

Question 6

Proof by induction would not at all be a surprising topic on the paper and there were very many excellent answers to this question, in which good candidates seemed well drilled. There are, of course, many who struggle with this topic who found it harder going, but overall the performance on this question was pleasing.

The need to start with proving that the inductive hypothesis was true for $n = 1$ was well known. However, the mark for this was sometimes lost because of lack of convincing detail, or inadequate working shown. In other cases excessive work via summation formulae was used to check the base case. Candidates would be well advised to show explicit initial substitution in even the simplest of cases to make the arguments convincing, but without trying to overcomplicate. Also stating explicitly "true for $n = 1$" would complete this first stage correctly and convincingly.

The next step of the proof determined those who knew the induction strategy from those who did not. Most knew that an assumption had to be made about being true for $n = k$, but not all knew what to do with this. There were numerous candidates who used the standard summation formulae to prove the result directly (whether with an assumption about $n = k$ first or not) and received no further credit as there was no induction method. Others made errors in not setting up the correct summation split, adding the k-th term to the sum to k terms, or not applying the sum to k terms at all. These cases could make no further progress.

However, most were able to apply the inductive step and split the sum to $k + 1$ appropriately. Candidates who started this induction process correctly usually scored the first 4 marks. The depth of explanation shown in progressing from a form acceptable for the first accuracy mark was very varied, with the most common route being to initially expand the cubic fully, with only the most astute candidates spotting the common factor of $(2k + 1)$ which made the working easier, and usually these progressed correctly to the required form via factorisation of the remaining quadratic and subsequent rearrangements, though slips in signs sometimes caused the loss of the last two or three accuracy marks.

Those who expanded fully again had a very mixed standard of working that followed. Some were unable to make further progress as they could not see how to find the appropriate factors, other (usually successfully) attempted long division by $(k + 1)$, knowing this factor was needed, before factorising and reaching the required result, some jumped directly to the correct expression for $n = k + 1$ with no intermediate working, losing all three accuracy marks, while others jumped directly a fully factorised form clearly from having solved to find roots on the calculator, which were in this case permitted the relevant marks. It was disappointing to see how many needed to resort to solutions by calculator to find the roots.

In all approaches sign/multiplication mistakes occurred in some responses, so that although a seemingly correct answer appeared at the end, it did not arise correctly. The process to reach the required form once the suitable quadratic factor was identified was sound in nearly all cases, though, and candidates clearly knew what they were aiming for.

There were also many candidates who used the idea of the target – that is, identifying the required answer and multiplying this out to produce a cubic that matched their forward result, with variations on the matched result. In almost all cases the candidates made enough forward progress for this method to be valid. There were also several variations on the approach working backward from the sum to $k + 1$ terms to identify it as the sum to k terms plus the $k + 1$-th term, or using the difference in sums to find the $k + 1$-th term to deduce the inductive steps. Such approached are less elegant but often did work out correctly.

For those who successfully proved the inductive step, who therefore had access to the final mark, most did make a very good attempt at the concluding statement and usually included all the notions as required. However, some of the final conclusions were borderline in terms of the "if, then" requirement and this needs emphasising in the teaching of the method. Still many give a conclusion that assumes true for $n = k$ and $n = k + 1$, without the implication connotation by giving a statement like "as true for n=1, k and $k + 1$".

Question 7

Although vectors is not a favoured topic with candidates the first three parts of this question proved accessible to most, with the final part discriminating between the better candidates. Notation throughout was generally very good, with either column vectors or the **i**, **j** and **k** forms used, or coordinates where appropriate.

For part (a) most attempted the dot product with each of the direction vectors to score the method, but it was fairly common to lose the accuracy because of an inadequate conclusion, or other slip. A few responses were seen in which candidates used the dot product with only one of the direction vectors, scoring no marks for an incomplete method.

A few candidates overcomplicated the process by finding the dot product of the normal vector with $(2\lambda+6\mu)\mathbf{i}+(\lambda+\mu)\mathbf{j}+(-4\lambda+8\mu)\mathbf{k}$ (a linear combination of the two direction vectors), reaching 0. If this was followed by a correct conclusion, it achieved both marks.

The Alt method, using the vector product, was also very common by those who have studied the further units. The product was usually correct, but occasionally a sign error was made, and the most common error in this approach was not showing the factor of 4 correctly. The second alternative, however, was much more rare to see and often went wrong during the process of solving equations when it was used.

For part (b) the method mark was almost always scored by those who made some attempt at the part, though a few did use one of the direction vectors instead of a point on the line. There was the occasional arithmetic error, for example reaching 26 rather than 20, but usually the calculation was correct. The accuracy was also often lost by a candidate leaving their answer in vector rather than Cartesian form, not adhering to the instruction in the question.

It was rare to see errors in part (c), with most able to get the correct value of p, even in some instances where an incorrect equation had been found because they went back to use a correct vector. Occasionally a candidate would fail to solve a correct linear equation in p, but this was rare. There were also some who managed to correctly find p from work in part (d), without having attempted the plane equation at all.

Part (d) was more challenging, and many were unable to see the way forward. Those who knew the process required usually carried it out correctly, but many only proceeded as far as finding the coordinates of point A. Indeed, most were able to complete the process to find the point A, wisely using the first and third ordinates to solve, and usually obtained the correct point, though some mixed up the variables after having found λ and μ correctly, while others used their incorrect p to find the coordinates instead of realising the other equation did not depend on p so would still yield a correct answer.

Having obtained the point A most then realised AB was needed but did not make further progress, having not recalled the method for finding the angle between the line and plane. Those who did know the method would generally score the next two method marks and score at least three marks. However, it was fairly common for a candidate to lose the accuracy mark despite an otherwise correct process. Common slips were: arithmetic errors in calculating the coordinates of point A; arithmetic errors in calculating the vector AB; getting an angle of 50° using arccos, but then forgetting to then find θ by subtracting from 90°.

Candidates who recalled the formula involving arcsin were more successful in attaining the correct answer rounding to 40°.

Question 8

Once again candidates coped well with this contextual connected derivatives question, and other than the last 3 marks this was well answered by the majority of the candidates. Most were able to complete what they could in the time available, and only a few did not reach the end, indicating there was no lack of time on the paper.

In part (a), most candidates attempted the question as expected, with minor variations in the process of which equations they differentiated first, and most were successful, although there were a number who pushed boundaries in terms of the steps shown given that it was a proof/show that question along with some poor organisation of their work. A small number took the option of finding expressions for the first and second derivatives of *x* and substituting them into the given differential equation and showing that all terms cancelled to give zero. Mostly this approach was also successful.

Part (b) was very well answered, with nearly all candidates demonstrating a good knowledge of how to solve second order differential equations, and the marks were scored very quickly. Only a small minority of candidates failed to achieve the correct auxiliary equation with correct roots, and most were then able to deduce the correct general solution. Even when some made a sign error in the auxiliary equation, they were able to score the method mark for a correct form of the complementary function for their roots. Also, there were occasional cases where the real and imaginary parts were mixed up in the exponential and trigonometric parts of the solution. And again, only a small number of candidates made errors of incorrect variables, giving *x* as a function of *x* or θ instead of *t*, or neglecting the "$x =$" or calling it θ.

Part (c) was less well answered but still usually done well. Most candidates used the equation $y = \frac{\mathrm{d}x}{\mathrm{d}t} - x$ with a valid attempt at differentiating *x*, and this was usually done well with appropriate use of the product rule. Occasionally sign errors were made in the subtraction of *x* giving terms such as $3A - B$ in the coefficients, but the differentiation itself was done well. A few candidates forgot to subtract the *x* after a successful differentiation despite having quoted the derivative.

The greater problems arose for the candidates who started over from the initial equations to find *y*, although this was a less common method. There was mixed success in whether these reached the correct differential equation for *y*, with it not being given, and so some entirely different equations were formed. Additionally, even when correct, these usually used the same lettering for their constants as for part (b), which meant progress could not be made sensibly in part (d). Those who realised different lettering was needed had a much harder time trying to reconcile them in part (d), although it was possible via extra work. Some who did this realised that they could not make progress and reverted to the more direct approach.

In another longer winded method, some candidates substituted their *x* into the original equation for $\frac{\mathrm{d}y}{\mathrm{d}t}$ to produce a linear differential equation in *y* and *t* which they attempted to solve using an integrating factor – a lot of work for two marks and not usually successfully completed.

Part (d) was generally started suitably by candidates who had found expressions for x and *y*, who generally attempted to use the initial conditions to find *A* and *B*. Those who had started again in part (c) and used the same constants were usually unable to get any further, while those who had four constants needed to proceed to use derivative statements to find the constant and so often simply gave up.

For those who had suitable equation and found value for *A* and *B*, the next step was straightforward, equating their *x* and *y* equations and proceeding to find a value for tan *t*. Only a small number of such candidates failed to reach an expression for tan *t*, though errors rearranging, such as from $175\cos t = 100\sin t$ to $\tan t = \frac{100}{175}$, occurred in some candidates work, but the majority of candidates were able to reach the correct vale for tan *t*. Very few proceeded by sine or cosine approaches.

The last two marks in this mark did proved discriminating, with only a minority of candidates proceeding to the correct final answer. The most common answer by far was $T = 1.05$, with correct arctan process, but neglecting to multiply by 24 to change from days to hours. Use of degrees also occurred relatively frequently, but again generally without the conversion to hours anyway, but losing the final two marks regardless of if they did this. A few cases occurred where the correct process was carried out following an incorrect tan *t* value but generally those who did convert into hours had fully correct solutions to this point.

The mark in part (e) was scored by only a small minority of candidates. It was the worst answered question part in the entire paper, with many candidates leaving it blank or using rote answers about the values tending to infinity with no indication of an investigation of what actually happens.

The rote answers used did not achieve the mark as candidates needed to show understanding that the key issue was that the equations produce negative values (after a relatively short period of time). Instead, candidates wrote comments about predators – clearly using rote predator-prey model answer, which were not relevant to bacteria – or external factors without understanding that models can include these when the parameters are calculated. Many candidates referred to the populations growing exponentially which would make it an unsuitable model, while the concept of limited space, bacteria consuming each other, not considering the number of bacteria dying and other environmental factors were also frequently cited. However, constraints on space and so on for a model studying behaviour of bacteria, which are microscopic and potentially have, for all intents and purposes of the model, unlimited space to grow, are not acceptable. Answers about continual growth do not consider the periods of contraction, which are indeed what invalidates the model in this instance as it breaks down when the population becomes negative. This is another answer that candidates would be well to be armed with in future and be able to determine when each of these types of answer is appropriate to the model in hand.

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