Examiners' Report

Principal Examiner Feedback

Summer 2024

Pearson Edexcel GCE

A Level Further Mathematics (9FM0)

Paper 3A Further Pure Mathematics 1

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9FM0 3A Summer 2024 Examiners Report

The paper contained numerous questions with standard processes which the well-prepared candidate was able to answer efficiently and accurately, meaning many marks were on offer to the higher grades. The biggest pinch point was question 9(ii), which tested a niche part of the specification that candidates are not yet well learned in and had the most mixed responses of the paper. Elsewhere, there was very little to trouble the better, well-prepared entrants.

At the lower end of the spectrum of ability, for those who were less prepared or less well trained there were still plenty of access points, but also questions where care and thought was needed. The coordinate geometry questions both had aspects that needed thinking through (but once done so were approachable), while use of l'Hopital's Rule and Leibnitz theorem tested the ability to apply some of the results encountered in the specification.

Question 7 was noteworthy in the amount of space candidates used to answer it, with many response for it written on other pages of the examination booklet, and a larger than expected number of non-standard scripts produced. A memorisation of the half angle identities would be recommended to save having to derive them from scratch in a given question.

Individual question report

Question 1

This question proved an accessible start to the paper with most candidates achieving full marks.

Very few errors were made in identifying correctly the value of the unknown ordinate in part (a), with failure to do this likely to indicate not more marks would be scored on the paper, especially since answers rounding to it were accepted.

Part (b) was again well attempted by the majority, but various of the expected errors were made by weaker candidates, such as an incorrect value for the strip width such as $\frac{3}{7}$, or confusion of the “odd” and “even” ordinates roles in the use of Simpson’s Rule. As usual, there were also cases of missing brackets, and usually these were not recovered. Even at this level a lack of care when it comes to bracketing costs marks. A lack of showing working was also noted in some students, presumably obtaining the answer from a calculator (which gave the same answer to 3 significant figures) and so scoring no marks.

The final mark of part (b) tested accuracy and so candidates who did not round their answer to the required three significant figure accuracy lost the final mark for otherwise correct work. This affected only a minority of candidates and most dutifully rounded to the required accuracy throughout the paper.

Most candidates understood the relationship between the estimated integral and the integral asked for in (c) and thus halved their value from (b), though some had to carry out algebraic manipulation of the integral before they could see it. The follow through allowed the mark to be scored even following misapplication of Simpson’s Rule, and sometimes this was the only mark scored in the question.

Question 2

The majority of candidates seemed able to make a good attempt at this question to gain at least 2 or 3 marks. By far the most common approach was forming and solving two separate equations/inequalities for the different cases of the modulus function. Fewer candidates took the approach of squaring both sides to form and then solve a quartic equation and occasionally mistakes were made when interpreting the modulus function which resulted in the incorrect equation $x^2 + 2x = x$ occasionally appearing.

Once the correct equations were formed, all three critical values were usually determined correctly with the division by x resulting in the loss of $x = 0$ as a critical value being only occasionally seen. The final mark proved more elusive as quite a few candidates were not able to piece together their previous values/solutions to give an overall correct range. The most common error here was the omission of $x = 0$.

Although the mark scheme was quite generous in condoning incorrect inequalities for the method marks, the question revealed some confusion with regards to inequalities. It was very common for candidates to use the wrong symbols and give incorrect ranges during the intermediate stages of work, even if their final answer was ultimately correct. It was pleasing to see that many candidates drew accurate sketches of the graphs of both $y = |x^2 - 2x|$ and of $y = x$ and when this was done it very often helped them to include $x = 0$ and to reach the correct final solution set.

Different notations for the final answer were seen, but stating the inequalities was the most common, with set notation and interval notation only used in a small number of cases.

Question 3

This question again provided good opportunity for the well-prepared candidate to score well, with the final mark being a bit of a discriminator for showing completeness of work.

Most candidates realised the necessity combine to a single fraction to be able to use l'Hopital's Rule, though a few did attempt to apply on the separate terms and so score no marks. Similarly, a few attempted the quotient rule, rather than differentiating numerator and denominator, and so score no marks.

The majority, however, realised also the need to apply the rule twice as the $\lim_{x\to 0}\frac{x-\sin x}{x\sin x}$ and $\lim_{x\to 0}\frac{1-\sin x}{\sin x + x\cos x}$ are not in determinate form, but again, some faltered at the first derivative stage think that $\frac{0}{0}=0$ and so failing to complete the method. Where error led to a determinate for at the first derivative the final method was permitted for demonstrating the use of l'Hopital's rule, but such cases were rare.

However, most candidates did apply the rule twice, finding first derivative and second derivatives of the numerator and denominator before deducing the limit. The differentiation was very strong on the whole as expected at this level, with few making signs slips or other errors in the derivatives. Missing the 1 in the first derivative of the numerator was the most common such error.

Though most went on to correctly apply l'Hopital's rule and obtain the required limit for the final method mark, the final accuracy was harder to obtain, as many candidates did not show sufficient evidence of substitution into the second derivative of the denominator. Candidates should be advised to make all steps clear and show the substitution into each term.

Question 4

A slight twist on this type of question in the use of a stationary point, but candidates coped well and the question proved relatively straightforward overall, with inaccuracies in part (b) being the main cause of loss of marks.

Part (a) was straightforward for most candidate, who were able to find the value for $\frac{d^2y}{dx^2}$ from the given information and interpret correctly. However, some failed to realise that they needed to substitute $\frac{dy}{dx}=0$ at the stationary point, and therefore could not obtain a value for the second derivative, so moved on to the next part, while a few others made slips in the calculation and obtain a wrong value.

Most who found the correct value recognised that the stationary point was a maximum, though some did think it was a minimum, while a very few referred to concave curves. More common was to not justify the answer, simply stating the type and so lose the accuracy mark.

In part (b) many candidates differentiated correctly and proceeded to achieve full marks. Only a few failed to achieve the second method following an attempt at differentiation. However, there were many who had errors in the differentiation, but which often led to the correct answer (due to the values 0 and 1 for the values of $\frac{dy}{dx}$ and y), with the most common error being the omission of the square in the $\left(\frac{dy}{dx}\right)^2$ term. Consequently, examiners had to check the work closely as some seemingly correct answers actually lost both accuracy marks.

A few candidates tried to differentiate at the stationary point, substituting in $\frac{dy}{dx} = 0$ before attempting the rest of the differentiation, so could make only limited progress.

Those who rearranged to make $\frac{d^2y}{dx^2}$ the subject before differentiation met with mixed fortunes – converting the differential equation to a quotient at the start a tended to result in mismanaging the differentiation.

Substitution of values following differentiation was done well and rearrangement to the correct answer was usually completed correctly though there were a few calculation errors, but again the level of justification was sometimes lacking. As this was a given answer there sufficient working must be shown to convince it was reached genuinely. Several candidates did not attain the last mark as they did not illustrate at least one suitable intermediate line of working.

Part (c) was again generally very well answered with most attaining both marks – the given answers being a big help. The most common error was an incorrect y_0 term, either omitted completely (y_0 =0) or using $y_0 = \frac{\pi}{4}$ or $\frac{\sqrt{2}}{2}$. Occasionally the final mark was lost through the corruption of signs in the final term.

<u>Question 5</u>

A very routine question at this stage of the paper, particularly as part (b) was pure synoptic A-level content – and yet surprisingly poorly attempted by a minority.

Most candidates were familiar with Leibnitz Theorem and correctly differentiated the exponential and the trigonometric terms to swiftly pick up the first four marks of part (a). A very small number made slips at this stage, usually in the powers of the exponential, while some did not differentiate sufficient times, so limited access to the question.

In applying the theorem, most stated coefficients from Pascal's triangle while others used combinatorial notation before simplifying and collecting terms to reach the given answer, which was usually done very well. The majority went on to score full marks on this part, although some omitted to mention $\frac{d^4y}{dx^4}$ at all, and so lost the final mark. Other slips did occur but were rare.

Candidates also performed well on part (b), with many able to quickly carry out the process to find R and α, though there were occasional errors in the latter as is common at A-level in this type of question (the sign errors or fraction the wrong up around being the common errors). Misquote of the final result, either omitting the exponential term or the sin, did lose the final mark for some. Some weaker candidates clearly did not know the $R\sin(x + \alpha)$ topic, and made no sensible progress at all, instead trying to split as $R\sin x + R\sin \alpha$ and it may be questioned if these candidates should be entered for this level of paper, as this routine synoptic topic should be well understood.

Question 6

For those who knew what to do and could find the most efficient way through, this question was again swiftly answered and proved little problem. However, it was more discriminating than the earlier questions and caused problems for many who did not immediately see what to do.
Candidates would be well advised to first of all write out what the given information is saying, as the reason for strife for many was the failure to use the second bullet point from the outset, but to try to use the first bullet point in the eccentricity equation for both curves first, and hence go through a process of setting up more complicated simultaneous equations when eventually bringing the second fact to bear. This was often done via using the x coordinate of the focus being $\sqrt{a^2+b^2}$, rather than finding the coordinate itself.

At the outset most candidates were able to use the equation for eccentricity and obtain the correct eccentricity of the ellipse, and the first mark was therefore generally gained even if no further progress was made. They were then able to find the eccentricity of the hyperbola, but this did not necessarily gain further marks. Those who proceeded to find the coordinate of the focus as a next step usually then made light work of attempting the value of a, usually finding it correctly. Those who did not proceed via this route, but instead set up equations in a and b using the eccentricity and focus equations of the hyperbola (as noted above) had much more difficulty in reaching a value for a, with errors often made along the way, though many did reach the correct answer from this approach.

If a value for a was found, most candidates then were able to attempt b from the relevant equation, although there were sometimes errors with the powers involved to gain the method mark, and this was score even in some cases where an incorrect process to attempt a had been made. But a correct value for a almost invariably led to a correct value for b and full marks ensued.

Question 7

The main problem with this question seemed to be space. Most candidates clearly have not memorised the results for this half angle formulae and spent the first page of the answer booklet deriving them. That left only one page to complete both parts. Alternatively, an excessive number of lines of working simplifying the integral in part (a) pushed the solution quickly on to the second page, and many then tried to cram a lot of work into the margins, or erratically across the page, others used other pages of the answer booklet, while many others used additional answer booklets. The use of additional booklets is preferable to writing answers in other questions on the script. It is unfortunate that only two pages were assigned to this question, and is a lesson taken on board.

The question itself, once results were derived, proved again highly accessible with only the final mark providing a degree of discrimination, largely due to the level of working required to establish a given answer – again.

For part (a) most candidates made the correct substitutions, albeit after some hefty work to derive them sometimes. Where errors were made it was usually as some that did not remember the substitutions made errors when deriving them, with the dθ term causing most problems. Some left the substitution of the dθ until later in the proof, working with mixed variables to begin with, but generally finally managed to get to an integral in t only. Incorrect sin θ and cos θ identities did occasionally arise (e.g. $1 - t^2$ used in the denominator of cos θ, or mixed-up sine and cosine identities), but these were not common.

Simplifying to the quadratic denominator and completion of the square were carried out very well by the majority, and most did achieve the correct expression. The last mark was occasionally lost, either for forgetting the dt in the integral or the integral sign, or because they thought providing the values of a, b and c was sufficient.

For part (b) many candidates could use the standard form of the integral accurately and change the limits correctly to proceed to a correct integral. Indeed, even in cases where incorrect answers to part (a) were found, candidates were apt to select a correct standard form for their answer, though some did not realise the arctanh integral only works for limits in the correct range. This was not, however, penalised in this question, but credit was awarded positively.

Those who had the correct form of answer were generally competent in manipulating the logs. However, most of these showed insufficient working on rationalising the resulting fraction to justify the given answer, and so full marks was not commonly gained in this part. Only the better prepared candidates seem to realise the need to show the full steps of working.

Of those who did not simply use the standard forms, use of partial fractions was common and generally worked well. There were also a range of substitution methods employed, with the simplest ($u = t + 2$) being the most successful. On occasion a needless substitution towards the end of a solution resulted in the logs of negative values, which were not always recovered. Overall, the integration work on display by the majority was very good.

Sight of the given answer may have caused some to overlook the process of rationalising their result, thus losing the final mark, but a better awareness of the need to show all work should be considered.

Question 8

This second coordinate geometry question on the paper again provided some degree of discrimination in part (b), where many candidates failed to identify the values of *a* and *t* at the given points. This led to more complicated equations that needed, and often ground to a halt.

Part (a), though, was accessible to nearly all candidates and full marks was the most common score in it. Candidates differentiated the equation of the parabola successfully, with implicit differentiation the most common approach, followed by parametric differentiation and only a few made *y* the subject first. Substitution of the parametric coordinates followed, with correct rearrangement to the given equation completed in most cases. The few who made little, or no, progress here were generally making little or no progress throughout the paper.

In Part (b), most candidates realised the need to find the point of intersection of the two tangents at *B* and *C*, and tried to do so, but most did not immediately or at all realise that $a = 2$ and $t = k^2$ at the points *B* and *C*, and attempting to solve for equations of tangents in terms of *a* and *t*. This often led to them making little progress in the question with a score profile B0M1M0A0A0 being very common. It was also remarkable how many candidates failed to use the answer to (a) to deduce the tangents but instead resorted to recalculating the gradients for each point and setting up the tangents again from scratch. Usually this was done correctly but overlooks the point of having asked part (a).

Having found the point of intersection, whether in terms of just *k* or in terms of *k*, *a* and *t*, there was usually an attempt to consider the area of the triangle, but due to the complicated or incorrect coordinates of the point of intersection, many candidates failed to find a value for *k* meaning the second method was inaccessible. Sometimes candidates did manage to work out $a = 2$ at this late stage (with copious amounts of work preceding it) to recover the situation, but mostly it was those who realise $a = 2$ from the outset who managed to access beyond the second mark.

The method for the area of the triangle did vary, and although most used the direct approach, attempts at the cross-product formula, or shoelace method, were seen, while some attempted an incorrect dot product formula. Again, these were applied with a mix of *k*, *a* and *t* in the equations, though some erroneous methods managed to eliminate the *a* and *t* conveniently, but no further marks were available in any case.

For the candidates who realised at some point in the solution the values of *a* and *t*, most were successful in eventually finding the value of *k* and correct coordinates, though slips with the triangle area formulae were sometimes made. The correct value of *k*, or correct coordinates, did occasionally arise from incorrect, or suspect working, such as erroneously arriving at $k = \pm 3$ from cube rooting and using the two values to attempt the two coordinates instead of symmetry.

Candidates who drew a clear diagram had a higher successful rate of solving this part of the question, the diagram helped them to visualize the equations, and the most efficient solutions were those who identified the values from the outset and were easily able to identify the equations of the tangents, and quickly deduce the coordinates of *D*.

Question 9

The first two part of this question was well answered while the second part proved far more challenging and discriminating. This is no doubt to do with the fact that part (i) is very familiar vectors work from long standing, while part (ii) contains mathematics novel to the newer specification, which has not been tested in great depth thus far.

In part (i)(a), the strategy for finding the intersection point of the two lines was well know. Candidates equated the two parametrised forms of the two lines and solved to find one, or both, of the parameters. Most clearly used a calculator to solve their simultaneous equations and this was an acceptable method for solution. It was pleasing to see candidates checking their work by verifying that the values of their parameters satisfied the third equation that was unused in their solution.

There were occasional slips in substitution to achieve the position of the point of intersection, but the majority of solutions to this part were correct and completed with little fuss.

Likewise for part (i)(b) most were able to speedily find the normal vector and proceed t the equation of the plane. The usual method of solution to find the equation of the plane was to find the vector product of the direction vectors of the two lines and then to use the **r.n**=**a.n** form of the equation of the plane before writing the equation in a Cartesian form, although sometimes the last mark in this part was lost because a Cartesian form was not given. The cross product was completed very well, with only very few making errors, and so the correct equation usually resulted. Occasionally candidates would use an incorrect vector for **a**, using a direction vector instead of point on the plane, but again these were a minority.

It was rare to see an alternative approach using the Cartesian form of the plane from the outset, and when it was used it was far more prone to error in the solution of the equations.

Part (ii) of the question proved more demanding for candidates with many not having a clear strategy for proceeding and was the part that was more often simply omitted by candidates than any other part of any question on the paper.

It was rare to see the approach using direction cosines from the outset. This unfamiliar topic has yet to be taken on board and those who were aware of it were in the minority. Those who did use the direction cosines generally managed to get as far as $\cos\theta = \frac{1}{2}$ but were not always able to make the step from here to a Cartesian equation for the line. Indeed, many seemed unsure of what the Cartesian equation for a line is, attempting plane equations instead.

Most candidates opted for using the projection on to the coordinate axes approach, using the dot product, but working was often not clearly presented and after setting up correct equations there was again often no progression to a Cartesian equation. Solutions often got the first two marks and then proceeded no further.

There were, nevertheless, some fully correct solutions via both methods, where the candidates showed a good understanding of the equations of lines in 3 dimensions and how to find them. But overall, even among the A* candidates, the latter two marks in this question were ones that were highly likely to prevent full marks on the paper.

Question 10

Although this was a challenging question, many candidates were able to gain some marks for at least one part of the question, particularly part (b), when the more demanding part (a) had not been accessible to them. Solution of a second order differential equation is a very well-versed topic, and a good source of marks for candidates.

In part (a), most of the responses were correct in their approach, accurately differentiating $x = tu$ to the second derivative and substituting these into the given equation and with a valid attempt at expanding brackets. Occasionally candidates showed confusion with regards to which derivative was required, resulting in convoluted or altogether unsuccessful attempts at finding $\frac{\mathrm{d}x}{\mathrm{d}t}$ and $\frac{\mathrm{d}^2x}{\mathrm{d}t^2}$ or even in some cases attempts at $\frac{\mathrm{d}^2t}{\mathrm{d}x^2}$.

When the two B marks were gained, the M mark almost always followed. Most candidates were able to reach the given result, but some fell short due to sign errors, errors in expanding, or sometimes an insufficiently clear line of working where it was not shown how the relevant terms cancel.

In part (b), many fully correct attempts were seen, as well as some that fell just short but with good initial work. The correct general approach of solving the auxiliary equation, finding the associated complementary function, finding the particular integral and then combining these to form a general solution is known by most, but many candidates did not show all the steps clearly. Perhaps as it is so routine, some are forming the habit of doing many steps in one go, and it was not uncommon to only see the complementary function as a part of the general solution.

The most common error at the start of the process was using $y = (At + B)\mathrm{e}^{2t}$ as the complementary function. Others left the e^{0t} in the equation throughout, or at least until the latter stages, but this is not incorrect, just needless, and some used x instead of t in the complementary function, which was usually recovered later though a few persisted with it.

There were seldom errors with the form of the particular integral, and the process of differentiation and substitution to find the constant was carried out well, but some then erroneously use −8 as the particular integral, forgetting the form they had set out.

Most of the candidates went on to use the boundary conditions to form and solve simultaneous equations in their equation and to deduce the overall particular solution of the given differential equation. Many first reversed the substitution to get an equation for x first and needed to use logs in the simultaneous equations only for them to cancel, but the more astute candidates realised they could find the value of u at $x = 0$ first and this simplified the equations. In otherwise correct responses, the main reasons for any loss of marks at this stage were due to errors in dealing with the log terms, or sign errors in the working.

A significant minority of the cohort lost the final mark due to giving the final particular solution for u, rather than x, and thus omitting the required factor of t. But most were able to complete to a correct solution, with full marks being the most common score for this part.

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