Mark Scheme (Results)

Summer 2024

Pearson Edexcel GCE

In A Level Further Mathematics (9FM0)

Paper 03 Further Pure Mathematics 1

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol √ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- ∗ The answer is printed on the paper
- ⌐ The second mark is dependent on gaining the first mark

4. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

5. Where a candidate has made multiple responses <u>and indicates which response they wish to submit</u>, examiners should mark this response.
If there are several attempts at a question <u>which have not been crossed out</u>, examiners should mark the final answer which is the answer that is the <u>most complete</u>.

6. Ignore wrong working or incorrect statements following a correct answer.

7. Mark schemes will firstly show the solution judged to be the most common response expected from candidates. Where appropriate, alternatives answers are provided in the notes. If examiners are not sure if an answer is acceptable, they will check the mark scheme to see if an alternative answer is given for the method used.

Question	Scheme	Marks	AOs
1 (a)	Awrt 2.485	**B1**	1.1b
		(1)	
(b)	$h = 0.5$	**B1**	1.1b
	$\frac{1}{3}\times"0.5"\left[1.946+3.332+2\left(2.485+2.944\right)+4\left(2.225+2.725+3.146\right)\right]$ $=\frac{1}{6}\times 48.52$	**M1**	1.1b
	8.09 cao	**A1**	1.1b
		(3)	
(c)	$0.5\times"8.09"=4.045$ or 4.04 or 4.05 (awrt either)	**B1ft**	2.2a
		(1)	
			(5 marks)

Notes:

(a)
B1: Correct value, seen in table or in the work. Accept awrt.

(b)
B1: Correct step length used, stated or clearly implied in working.
M1: Correct structure for Simpson's rule $\frac{1}{3}"h"\left[\text{ends}+2\text{evens}+4\text{odds}\right]$ using their value of h. If no value of h is stated nor is implied and the $\frac{1}{3}h$ is not clearly seen, award bod for any multiple in front of the bracket that does not imply a clearly incorrect method – but the insides must be correct. Condone a missing closing bracket at the end, but otherwise bracketing must be correct or implied correct by the answer. Condone minor miscopies of the ordinates as long as they are in correct positions.
A1: For 8.09, correct answer only, must be to 3.s.f. Must have scored the M.
Note calculator gives 8.086 …

(c)
B1ft: Deduces the value by finding 0.5 × their answer to (b). Allow for awrt 3.s.f. answers giving tolerance with regards to prior rounding. Accept as a fraction, e.g. $\frac{1213}{300}$ as long as it is half their answer to (b), or rounds correctly to it.

Question	Scheme	Marks	AOs
2	A complete method to form equations in an attempt to find the critical values **Method 1** Uses $x^2-2x=x \Rightarrow x^2-3x=0$ and $x^2-2x=-x \Rightarrow x^2-x=0$ **<u>Method 2</u>** Squares both sides $\left(x^2-2x\right)^2=x^2 \Rightarrow x^4-4x^3+3x^2=0$	**M1**	1.1b
	Solves their equations to find at least all the non-zero the critical values	**dM1**	3.1a
	$x=0, 1, 3$	**A1**	1.1b
	$x=0\,,\ 1 \leqslant x \leqslant 3$	**A1**	2.2a
		(4)	
			(4 marks)

Notes:

M1: A complete method to form equations/an equation that will give **all** the critical values. For method 1, both equations must be attempted. Allow with inequalities in place of equals throughout and do not be concerned if the inequalities are correct or not. This is for the method to find an equation or equations for the CVs.

dM1: Dependent on the previous method mark. Solve their equation(s) to find at least all the non-zero critical values. If using method 1 both equations must be solved, in method 2 the quadratic after factoring out or dividing by x^2 must be solved. Allow for attempts by dividing through by x or x^2.

A1: Correct critical values, including 0 (although this may be later rejected), seen stated or used in the solution. Must have come from correct work.

A1: Deduces the correct range following correct work. Both parts must clearly be identified for the answer (e.g. underlined during working if not on the final line).

Accept alternative notation, e.g. set or interval notation. In words “and” or “or” may be used, but if using mathematical notation, must use union and not intersection.

E.g. $\{0\}\cup\{x:1 \leqslant x \leqslant 3\}$ or $[0]\cup[1,3]$ (if intervals given, must be square brackets not round) are acceptable answers.

Question	Scheme	Marks	AOs
3	$\frac{1}{\sin x}-\frac{1}{x}=\frac{x-\sin x}{x\sin x}$	**M1**	3.1a
	$\frac{\frac{\mathrm{d}}{\mathrm{d}x}(x-\sin x)}{\frac{\mathrm{d}}{\mathrm{d}x}(x\sin x)}=\frac{1\pm\cos x}{\sin x\pm x\cos x}$	**dM1**	1.1b
	$\frac{\frac{\mathrm{d}}{\mathrm{d}x}(x-\sin x)}{\frac{\mathrm{d}}{\mathrm{d}x}(x\sin x)}=\frac{1-\cos x}{\sin x+x\cos x}$	**A1**	1.1b
	$\frac{\frac{\mathrm{d}}{\mathrm{d}x}(1-\cos x)}{\frac{\mathrm{d}}{\mathrm{d}x}(\sin x+x\cos x)}=\frac{\pm\sin x}{\pm\cos x\pm\cos x\pm x\sin x}$	**ddM1**	3.1a
	$\left(\lim_{x\to 0}\left(\frac{1}{\sin x}-\frac{1}{x}\right)=\right)\lim_{x\to 0}\left(\frac{\sin x}{2\cos x-x\sin x}\right)=\frac{\sin(0)}{2\cos(0)-(0)\sin(0)}$	**M1**	1.2
	$=0*$	**A1***	2.1
		(6)	
		(6 marks)	

Notes:

M1: Complete method to write the function as a quotient.

dM1: Attempts differentiation of both numerator and denominator, including use of product rule with the denominator. Either numerator or denominator of the correct form. May be done separately.

A1: Fully correct differentiation of both numerator and denominator (may be separate).

ddM1: Dependent on previous method mark. Recognises the need to differentiate again and carries out the differentiation to complete the method.

M1: Clear demonstration of L'Hospital's Rule being used to attain a limit, e.g. clear statement using limits reaching an expression that can be evaluated. Both numerator and denominator must be used appropriately. The substitution may be implied by 0/2 being reached from a suitable expression (oe if errors) but must have achieved a non-zero denominator. Not dependent, so may be scored if genuine errors in the first derivative lead to an expression that has a non-zero denominator. They may have only differentiated once.

A1*: Fully correct solution. **Must see clear use of a substitution** of $x=0$ into their derivatives. Accept as minimum e.g. $\frac{0}{2-0}$ with each term seen evaluated or equivalent working shown. Needs to be a correct line showing substitution before reaching the printed answer with use of some limit notation. All aspects of the proof should be clear for this mark to be awarded and no errors seen.

NB Proceeding to fourth derivatives before evaluating the limit is a correct approach, and may score the final M once a limit is reached, and final A if all aspects are correct.

Question	Scheme	Marks	AOs
4(a)	$\cos\left(\frac{\pi}{4}\right)\frac{d^2y}{dx^2}+(1)^2(0)+\sin\left(\frac{\pi}{4}\right)=0\Rightarrow\frac{d^2y}{dx^2}=...$	**M1**	3.1a
	$\frac{d^2y}{dx^2}=-1<0$ therefore a (local) maximum	**A1**	2.4
		(2)	
(b)	Achieves $\pm\sin x\frac{d^2y}{dx^2}+\cos x\frac{d^3y}{dx^3}+\alpha y\left(\frac{dy}{dx}\right)^{1\text{ or }2}+y^2\frac{d^2y}{dx^2}\pm\cos x=0$ or $\frac{d^2y}{dx^2}=-\tan x-y^2\sec x\frac{dy}{dx}\rightarrow$ $\frac{d^3y}{dx^3}=\pm\sec^2 x\pm\alpha y\sec x\left(\frac{dy}{dx}\right)^{1\text{ or }2}\pm y^2\sec x\tan x\frac{dy}{dx}\pm y^2\sec x\frac{d^2y}{dx^2}$	**M1**	1.1b
	$-\sin x\frac{d^2y}{dx^2}+\cos x\frac{d^3y}{dx^3}+2y\left(\frac{dy}{dx}\right)^2+y^2\frac{d^2y}{dx^2}+\cos x=0$ or $\frac{d^3y}{dx^3}=-\sec^2 x-2y\sec x\left(\frac{dy}{dx}\right)^2-y^2\sec x\tan x\frac{dy}{dx}-y^2\sec x\frac{d^2y}{dx^2}$	**A1**	1.1b
	$-\sin\left(\frac{\pi}{4}\right)(-1)+\cos\left(\frac{\pi}{4}\right)\frac{d^3y}{dx^3}+2(1)(0)^2+(1)^2(-1)+\cos\left(\frac{\pi}{4}\right)=0\rightarrow\frac{d^3y}{dx^3}=...$ Alt: $\frac{d^3y}{dx^3}=-2-2(1)\sqrt{2}(0)-1^2\sqrt{2}(1)(0)-1^2\sqrt{2}(-1)$	**M1**	1.1b
	$\left(\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}\frac{d^3y}{dx^3}-1+\frac{\sqrt{2}}{2}=0\Rightarrow\frac{\sqrt{2}}{2}\frac{d^3y}{dx^3}=1-\sqrt{2}\right)$ $\frac{d^3y}{dx^3}\left(=\frac{2-2\sqrt{2}}{\sqrt{2}}=\frac{2\sqrt{2}-4}{2}\right)=\sqrt{2}-2$ *	**A1***	2.1
		(4)	
(c)	$(y=)1+\left(x-\frac{\pi}{4}\right)(0)+\frac{\left(x-\frac{\pi}{4}\right)^2}{2!}(\text{their }'-1')+\frac{\left(x-\frac{\pi}{4}\right)^3}{3!}\left(\sqrt{2}-2\right)+...$	**M1**	2.5
	$(y=)1-\frac{\left(x-\frac{\pi}{4}\right)^2}{2}+\frac{\left(x-\frac{\pi}{4}\right)^3\left(\sqrt{2}-2\right)}{6}+...$	**A1**	1.1b
		(2)	
			(8 marks)

Notes:
(a) **M1:** A complete method to find the value for $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$. Substitutes into the differential equation $x=\frac{\pi}{4}, y=1, \frac{\mathrm{d}y}{\mathrm{d}x}=0$ and rearranges to find a value for $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$. **A1:** $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=-1<0$ therefore a maximum. Must achieve the correct value, or at least a correct expression and give reason "<0" (oe) and conclusion and no contradictory statements.
(b) **M1:** Differentiates to the form $\pm\sin x\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+\cos x\frac{\mathrm{d}^3y}{\mathrm{d}x^3}+\alpha y\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^{1\text{ or }2}+y^2\frac{\mathrm{d}^2y}{\mathrm{d}x^2}\pm\cos x=0$. Alternatively, divides through by cos *x* first and differentiates to achieve the form shown in scheme. **A1:** Correct differentiation, any alternative form is acceptable. **M1**: Substitutes in $x=\frac{\pi}{4}, y=1, \frac{\mathrm{d}y}{\mathrm{d}x}=0$, their value of $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}='-1'$ as appropriate for their expression, and rearranges to find a value for $\frac{\mathrm{d}^3y}{\mathrm{d}x^3}$ (if not already done so). **A1*:** Shows that $\frac{\mathrm{d}^3y}{\mathrm{d}x^3}=\sqrt{2}-2$ from correct working with at least one suitable intermediate line – being either an unsimplified $\frac{\mathrm{d}^3y}{\mathrm{d}x^3}=...$ or $\alpha\frac{\mathrm{d}^3y}{\mathrm{d}x^3}=...$, before the final answer. Must have come from a correct derivative expression. Note that in the Alt the correct form comes out much easier, so look for at least one unsimplified line before the final answer.
(c) **M1:** Uses the Taylor's series expansion with the correct values for $y_{\frac{\pi}{4}}=1, \frac{\mathrm{d}y}{\mathrm{d}x}=0$, $\frac{\mathrm{d}^3y}{\mathrm{d}x^3}=\sqrt{2}-2$ (or their $\frac{\mathrm{d}^3y}{\mathrm{d}x^3}$ if they had a different answer to (b)) and their value of $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$. Must include the 1 at the start. **A1:** Correct simplified expansion. The "*y* =" may be missing. Accept with $0\left(x-\frac{\pi}{4}\right)$ as a second term. ISW after a correct simplified expression if further attempts to simplify are made.

Question	Scheme	Marks	AOs
5(a)	$y = e^{3x}\sin x \quad u = e^{3x} \quad v = \sin x$		
	$u' = 3e^{3x},\ u'' = 9e^{3x},\ u''' = 27e^{3x},\ u^{(4)} = 81e^{3x}$	**M1** **A1**	1.1b 1.1b
	$v' = \cos x,\ v'' = -\sin x,\ v''' = -\cos x,\ v^{(4)} = \sin x$	**M1** **A1**	1.1b 1.1b
	Thus $\frac{d^4y}{dx^4} = \left(e^{3x}\times\sin x\right) + \left(4\times 3e^{3x}\times -\cos x\right) + \left(6\times 9e^{3x}\times -\sin x\right)$ $+\left(4\times 27e^{3x}\times\cos x\right) + \left(81e^{3x}\times\sin x\right)$	**M1**	2.1
	$\frac{d^4y}{dx^4} = 28e^{3x}\sin x + 96e^{3x}\cos x$ *	**A1***	1.1b
		(6)	
(b)	$R\left(=\sqrt{28^2+96^2}\right) = 100$	**B1**	1.1b
	$\tan\alpha = \pm\frac{96}{28} \Rightarrow \alpha = \ldots\ (\alpha = 1.287\ldots)$	**M1**	1.1b
	$\left(\frac{d^4y}{dx^4} =\right) 100e^{3x}\sin(x+1.29)$	**A1**	2.1
		(3)	
			(9 marks)

Notes:

(a)

M1: Finds the correct form of the first four derivatives of e^{3x}. Must all be of the form αe^{3x}.

SC: M1 may also be awarded for *at least* four relevant derivatives found if they do not find all four for the e^{3x} or the $\sin x$.

A1: Correct derivatives. Allow if the general form is identified rather than individual ones given. SC allow for at least four correct derivatives if not all are found.

M1: Finds the correct form of the first four derivatives of $\sin x$, condone sign slips only.

A1: Correct derivatives.

M1: Applies Leibnitz's theorem to get the 4th derivative with their expressions. Binomial coefficients must be present and correct numerical expressions (not notational forms).

A1*: Correct simplified 4th derivative from fully correct working, including the $\frac{d^4y}{dx^4} =$ seen at some stage.

<u>Note:</u> **If do not use Leibnitz's theorem then up to the first 4 marks can be awarded for using the product rule to differentiate. Accept correct forms for the derivatives.**

M1: $\frac{dy}{dx} = \pm e^{3x}\cos x + Ae^{3x}\sin x$ and $\frac{d^2y}{dx^2} = Ce^{3x}\cos x + De^{3x}\sin x$

A1: $\frac{dy}{dx} = e^{3x}\cos x + 3e^{3x}\sin x$ and $\frac{d^2y}{dx^2} = 6e^{3x}\cos x + 8e^{3x}\sin x$

dM1: Forms correct next two. **A1**: $\frac{d^3y}{dx^3} = 26e^{3x}\cos x + 18e^{3x}\sin x$ and $\frac{d^4y}{dx^4} = 28e^{3x}\sin x + 96e^{3x}\cos x$

(b) Note this appears on epen as MAA but is being marked as BMA

B1cao: Deduces the correct value of R, simplified, $R = 100$

M1: Attempts the value of α via $\tan\alpha = \pm\frac{B}{A}$ or $\tan\alpha = \pm\frac{A}{B}$ or $\sin\alpha = \pm\frac{B}{R}$ or $\cos\alpha = \pm\frac{A}{R}$

Note that $\alpha = \text{awrt } 1.29$ or $\alpha = \text{awrt } 74°$ imply this mark.

A1: Correct answer, the $\frac{d^4y}{dx^4}$ may be missing but must be given as the correct expression with $\alpha = \text{awrt } 1.29$ not just separate values for R and α

Question	Scheme	Marks	AOs
6	$9=25\left(1-e^2\right)\Rightarrow e=...\left\{\frac{4}{5}\right\}$	**M1**	1.1b
	$ae=5\times\frac{4}{5}=...\{4\}$	**dM1**	1.1b
	A complete method to find the value of a or b e.g. $e=\frac{1}{\text{their }'\frac{4}{5}'}=...\left\{\frac{5}{4}\right\}$ and uses $a=\frac{\text{their }'4'}{\text{their }'\frac{5}{4}'}=...\left\{\frac{16}{5}\right\}$	**M1**	3.1a
	$a=\frac{16}{5}$ or $b=\frac{12}{5}$	**A1**	1.1b
	E.g. Uses $b^2=\left(\text{their }\frac{16}{5}\right)^2\left(\left(\text{their }\frac{5}{4}\right)^2-1\right)$ leading to $b=\ldots$	**M1**	3.1a
	$a=\frac{16}{5}$ and $b=\frac{12}{5}$	**A1**	3.2a
		(6)	
			(6 marks)

Notes:

M1: A complete method to find the eccentricity of E. Uses $b^2=a^2\left(1-e^2\right)$ to find a value for e or e^2
Ignore references to ± for the Ms. Note that identifying a and b correctly is part of the method.
dM1: Dependent on the previous method mark. Finds the x coordinate of the focus of E, 5 multiplied by their value of e (or implied if working with squared forms).
M1: A complete method to find the value of a. Finds the reciprocal of their eccentricity of E and uses their x coordinate of the focus of E divided by their reciprocal of e. Note they may use $e^2=1+\frac{b^2}{a^2}$ and $x_F=\sqrt{a^2+b^2}$ for the hyperbola to form and solve suitable equations.

So for example $a^2+b^2="4"^2$ and $"\frac{5}{4}"^2=1+\frac{b^2}{a^2}$ leading to a value for a or b.

A1: Correct value of a or b. Accept as a decimal.
M1: A complete method to find a value of b or a if b found first. Uses $b^2=a^2\left(e^2-1\right)$ with their values for e and a to find a value for b or vice versa, allowing slips in substitution as long as the correct formula is clear.
A1: Correct values for both a and b. Accept as a decimals.

Question	Scheme	Marks	AOs
7(a)	$\int \frac{1}{2\sin\theta+\cos\theta+2}\,d\theta = \int \frac{1}{2\left(\frac{2t}{1+t^2}\right)+\left(\frac{1-t^2}{1+t^2}\right)+2} \times \frac{2}{1+t^2}\,dt$	**M1**	2.1
	$\int \frac{2}{4t+(1-t^2)+2(1+t^2)}\,dt = \int \frac{2}{t^2+4t+3}\,dt$	**M1**	1.1b
	$\int \frac{2}{(t+2)^2-1}\,dt$	**A1**	2.2a
		(3)	
(b)	$\int \frac{a}{(t+b)^2-c}\,dt = \frac{a}{2\sqrt{c}}\ln\left\|\frac{(t+b)-\sqrt{c}}{(t+b)+\sqrt{c}}\right\|, c>0$ or $\int \frac{a}{(t+b)^2+c}\,dt = \frac{a}{\sqrt{c}}\arctan\left(\frac{t+b}{\sqrt{c}}\right), c>0$ Note correct answer is $\int \frac{2}{(t+2)^2-1}\,dt = \frac{2}{2(1)}\ln\left\|\frac{(t+2)-1}{(t+2)+1}\right\|$	**M1**	3.1a
	Correctly uses the limits $t=1$ and $t=\sqrt{3}$ $\rightarrow \ln\left\|\frac{\sqrt{3}+1}{\sqrt{3}+3}\right\| - \ln\left\|\frac{2}{4}\right\|$ if correct.	**M1**	1.1b
	$=\ln\left(\frac{\sqrt{3}+1}{\sqrt{3}+3} \div \frac{1}{2}\right)$	**M1**	1.1b
	$=\ln\left(2\times\frac{\sqrt{3}+1}{\sqrt{3}+3}\times\frac{\sqrt{3}-3}{\sqrt{3}-3}\right)=\ln\left(\frac{2\sqrt{3}}{3}\right)$ *	**A1***	2.1
		(4)	
		(7 marks)	

Notes:

(a)

M1: Applies the correct substitutions $\sin\theta = \frac{2t}{1+t^2}$, $\cos\theta = \frac{1-t^2}{1+t^2}$ and $d\theta = \frac{2}{1+t^2}\,dt$ The dt may be missing for this and the next M.

M1: Simplifies the integrand to the form $\frac{a}{pt^2+qt+r}$ allowing slips in coefficients but the "$1+t^2$" must be correctly dealt with.

A1: Correct answer in the form required, including the $\int$ dt

(b) **M1:** A correct method for the integration, by use of standard formula. Modulus signs are not necessary. Note that $-\frac{a}{\sqrt{c}}\text{artanh}\left(\frac{t+b}{\sqrt{c}}\right)=-\frac{a}{2\sqrt{c}}\ln\left\|\frac{t+b+\sqrt{c}}{t+b-\sqrt{c}}\right\|$ is also a correct version. If by error they end up with a positive c then accept for the arctan integral shown in scheme. **M1:** Correctly uses the limits $t=1$ and $t=\sqrt{3}$ in their integral to produce an exact expression. Use of the original limits is M0, must have changed to t (unless they reverse the substitution). **M1:** Correctly combines the ln terms with at least one step showing the result of substituting limits before the final answer, and their final answer must follow this step if no further working is shown. May be seen before or after the rationalisation. (M0 if no ln terms.) **A1*cso:** Shows the method to rationalise the denominator and correctly completes to the given answer. Some suitable working must be seen. Note that $\sqrt{3}+3=\sqrt{3}\left(1+\sqrt{3}\right)$ may be used in the denominator, followed by cancelling.			
(b) **Alt**	$\int\frac{2}{(t+2)^2-1}\,dt=\int\frac{2}{(t+1)(t+3)}\,dt$ $=\int\frac{1}{t+1}-\frac{1}{t+3}\,dt=\ln\|t+1\|-\ln\|t+3\|$	**M1**	3.1a
	Correctly uses the limits $t=1$ and $t=\sqrt{3}$ $\ln\left(\sqrt{3}+1\right)-\ln\left(\sqrt{3}+3\right)-\ln 2+\ln 4$	**M1**	1.1b
	$=\ln\left(2\times\frac{\sqrt{3}+1}{\sqrt{3}+3}\right)$	**M1**	2.1
	$=\ln\left(2\times\frac{\sqrt{3}+1}{\sqrt{3}\left(1+\sqrt{3}\right)}\right)=\ln\left(\frac{2\sqrt{3}}{3}\right)$ *	**A1***	1.1b
		(4)	
(7 marks)			
Notes:			
(b) **M1:** A correct method for the integration, applying partial fractions and integrating to ln terms. **M1:** Correctly uses the limits $t=1$ and $t=\sqrt{3}$ (or correct θ limits if substitution is reversed) to produce an exact expression. **M1:** Correctly combines the ln terms with at least one step showing the result of substituting limits before the final answer, and their final answer must follow this step if no further working is shown. May be seen before or after the rationalisation. (M0 if no ln terms.) **A1*cso:** Shows the method to rationalise the denominator and correctly completes to the given answer. Some suitable working must be seen. Note that $\sqrt{3}+3=\sqrt{3}\left(1+\sqrt{3}\right)$ may be used in the denominator, followed by cancelling.			

Question	Scheme	Marks	AOs
8(a)	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{2a}{2at}=\frac{1}{t}$ **or** $2y\frac{\mathrm{d}y}{\mathrm{d}x}=4a\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{4a}{2(2at)}=\frac{1}{t}$ **or** $y=2\sqrt{a}\sqrt{x}\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}x}=2\sqrt{a}\times\frac{1}{2\sqrt{x}}=\frac{\sqrt{a}}{\sqrt{at^2}}=\frac{1}{t}$	**B1**	1.1b
	$y-2at=\frac{1}{t}\left(x-at^2\right)$	**M1**	1.1b
	$yt=x+at^2*$	**A1***	2.1
		(3)	
(b)	Equations of the tangents $yk=x+2k^2$ and/or $-yk=x+2k^2$	**B1**	2.2a
	Solves simultaneously $\left.\begin{array}{r}yk=x+2k^2\\-yk=x+2k^2\end{array}\right\}\Rightarrow x=...\{-2k^2\},\ y=...\{0\}$	**M1**	3.1a
	Finds the area of the triangle, sets equal to 432 and solves to find a value for k $\frac{8k\times\left(2k^2-\text{their '}-2k^2\text{'}\right)}{2}=432$ leading to $k=\ldots$	**M1**	3.1a
	$k=3$	**A1**	1.1b
	$(18,12)$ and $(18,-12)$	**A1**	2.2a
		(5)	
			(8 marks)

Notes:

(a)

B1: Correct derivative in terms of *t* from a correct calculus method. Just stating $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{t}$ with no supporting working is B0.

M1: Uses $y-2at=\text{their }'\frac{1}{t}'\left(x-at^2\right)$, or uses $y='\frac{1}{t}'x+c$ with $\left(at^2,2at\right)$ to find *c*. They may have just stated the $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{t}$ for this mark.

A1*: Achieves the printed equation with no errors, but allow if the $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{t}$ is stated without working (so B0M1A1 is possible).

(b)

B1: Deduces a correct equation for one of the tangents. May find both but accept for one correct. If working in terms of *a* or *a* and *t* they must identify $a=2$ or $at^2=-2k^2$ (oe as relevant) at some stage to gain this mark following a correct tangent in terms of *a* or *a* and *t*.

M1: Finds the intersection of the tangent with the *x*-axis, or solves both tangent equations simultaneously to find at least the *x* coordinate of the point of intersection. May be in terms of *a* and *t* for this mark.

Note the first 2 marks can be implied from stating a correct point of intersection

M1: A complete method to find the value of *k*. E.g. uses $\frac{8k\times\left(2k^2-\text{their }'-2k^2\,'\right)}{2}=432$ and solves to find a value for *k*. Alternative approaches are possible, such as

$$432=\frac{1}{2}\left|\overrightarrow{DC}\times\overrightarrow{DB}\right|=\frac{1}{2}\begin{vmatrix}\mathbf{i}&\mathbf{j}&\mathbf{k}\\4k^2&4k&0\\4k^2&-4k&0\end{vmatrix}=\frac{1}{2}\left|-32k^3\mathbf{k}\right|=16k^3\Rightarrow k=\ldots$$ or

$$432=\frac{1}{2}\begin{vmatrix}-2k&2k&2k&-2k\\0&4k^2&-4k^2&0\end{vmatrix}=\frac{1}{2}\left|\left(-8k^3-8k^3+0-0-8k^3-8k^3\right)\right|=16k^3\Rightarrow k=\ldots$$

A1: $k=3$

A1 Deduces the correct coordinates $(18,12)$ and $(18,-12)$ Do not be concerned with the labelling.

Question	Scheme	Marks	AOs
9(i)(a)	$\left.\begin{array}{r}2+3\lambda=13+\mu\\-3+4\lambda=5-2\mu\\1-\lambda=8+5\mu\end{array}\right\}$ leading to $\lambda=...$ or $\mu=...$ (Note $\lambda=3,\ \mu=-2$)	**M1**	3.1a
	$\begin{pmatrix}2\\-3\\1\end{pmatrix}+3\begin{pmatrix}3\\4\\-1\end{pmatrix}=\begin{pmatrix}11\\9\\-2\end{pmatrix}$ or $\begin{pmatrix}13\\5\\8\end{pmatrix}-2\begin{pmatrix}1\\-2\\5\end{pmatrix}=\begin{pmatrix}11\\9\\-2\end{pmatrix}$	**A1**	1.1b
		(2)	
(i)(b)	$\begin{vmatrix}\mathbf{i}&\mathbf{j}&\mathbf{k}\\3&4&-1\\1&-2&5\end{vmatrix}=\mathbf{i}(20-2)-\mathbf{j}(15+1)+\mathbf{k}(-6-4)$	**M1**	3.1a
	$\pm(18\mathbf{i}-16\mathbf{j}-10\mathbf{k})$	**A1**	1.1b
	e.g. $\mathbf{r}\bullet\begin{pmatrix}18\\-16\\-10\end{pmatrix}=\begin{pmatrix}11\\9\\-2\end{pmatrix}\bullet\begin{pmatrix}18\\-16\\-10\end{pmatrix}=...$	**M1**	1.1b
	$18x-16y-10z=74$ o.e. $k(9x-8y-5z=37)$	**A1**	2.5
		(4)	
(ii)	$\cos^2 60^\circ+\cos^2 45^\circ+\cos^2\theta=1$ leading to $\cos\theta=...$ $\left(\frac{1}{2}\right)^2+\left(\frac{\sqrt{2}}{2}\right)^2+\cos^2\theta=1$	M1	3.1a
	$\cos\theta=(\pm)\frac{1}{2}$	A1	1.1b
	$\frac{x}{\cos 60^\circ}=\frac{y}{\cos 45^\circ}=\frac{z}{\text{their }\cos\theta}$ or $\frac{x}{\left(\frac{1}{2}\right)}=\frac{y}{\left(\frac{\sqrt{2}}{2}\right)}=\frac{z}{\text{their }\cos\theta}$	M1	1.1b
	$2x=\sqrt{2}y=2z$ and $2x=\sqrt{2}y=-2z$ o.e	A1	2.5
		(4)	
			(10 marks)

Notes:

Note: Accept any alternative vector forms of notation throughout.

(i)(a)

M1: Forms and solves two equations to find the value of λ or μ.

A1: Correct point of intersection. Accept as coordinates or vector (oe).

(i)(b)

M1: Finds the cross product of the direction vectors of the lines. If no method shown, two correct components implies the method.

A1: Correct normal vector.

M1: Dependent on having made some attempt at the cross product. Find the equation of a plane using $\mathbf{r}\bullet(\text{their cross product})=(\text{point on the plane})\bullet(\text{their cross product})=...$ Condone minor miscopies of coordinates. **A1:** Correct Cartesian equation of the plane. Accept any multiple of it.
(ii) **M1:** Uses $\cos^2\alpha+\cos^2\beta+\cos^2\gamma=1$ to find the direction cosine to the *z*-axis. Allow with *n* or another letter in place of $\cos\theta$ **A1:** A correct answer for the direction cosine to the *z*-axis. **M1:** Finds a Cartesian equation of one of the lines. **A1:** Two correct Cartesian equations of the line, need not be simplified. Accept for both equation given, don't be concerned if "and" or "or" is used.

Question 9 (i) Alts

Question	Scheme	Marks	AOs
(i)(b) Alt 1	Plane is $ax+by+cz=1$, so $\begin{cases} 2a-3b+c=1 \\ 13a+5b+8c=1 \\ 11a+9b-2c=1 \end{cases}$	**M1** **A1**	3.1a 1.1b
	$\left.\begin{array}{r} 2a-3b+c=1 \\ 13a+5b+8c=1 \\ 11a+9b-2c=1 \end{array}\right\} \Rightarrow \left.\begin{array}{r} 17a+c=4 \\ 49a+29c=8 \end{array}\right\} \Rightarrow a=...,b=...,c=...$	**M1**	1.1b
	$\frac{9}{37}x-\frac{8}{37}y-\frac{5}{37}z=1$ or $9x-8y-5z=37$ o.e.	**A1**	2.5
		(4)	

Notes:

M1: Forms 3 equations in 3 unknowns using three points on the line (or may form four equations in four unknowns)
A1: All correct relevant equations.
M: Full process to solve the equations – which may be by calculator. Accept for any solutions appearing after setting up suitable equations.
A1: Correct Cartesian equation of the plane.

Question	Scheme	Marks	AOs
(i)(b) Alt 2	Let normal vector be $a\mathbf{i}+b\mathbf{j}+c\mathbf{k}$ Then $(a\mathbf{i}+b\mathbf{j}+c\mathbf{k})\cdot(3,4,-1)=0=(a\mathbf{i}+b\mathbf{j}+c\mathbf{k})\cdot(1,-2,5)\Rightarrow$ $\left.\begin{array}{r} 3a+4b-c=0 \\ a-2b+5c=0 \end{array}\right\} \Rightarrow 10b-16c=0 \Rightarrow b=\frac{8}{5}c, a=-\frac{9}{5}c$	**M1**	3.1a
	$\mathbf{n}=\pm k(9\mathbf{i}-8\mathbf{j}-5\mathbf{k})$	**A1**	1.1b
	Finds the Cartesian equation of the plane e.g. $\mathbf{r}\bullet\mathbf{n}=(11\mathbf{i}+9\mathbf{j}-2\mathbf{k})\bullet(9\mathbf{i}-8\mathbf{j}-5\mathbf{k})=...$	**M1**	1.1b
	$9x-8y-5z=37$ o.e.	**A1**	2.5
		(4)	

Notes:

M1: Sets up the normal vector in terms of variables (may set one of them as 1 or another value) and takes the dot product with both directions to form and solve equations to find the normal.
A1: Correct normal vector.
M1: Dependent on having made some attempt at the normal vector. Finds the equation of a plane using $\mathbf{r}\bullet(\text{their normal vector})=(\text{point on the plane})\bullet(\text{their normal vector})=...$ Condone minor miscopies of coordinates.
A1: Correct Cartesian equation of the plane. Accept any multiple of it.

Question 9(ii) Alt

(ii) Alt	Let (unit) direction vector of line be $\mathbf{d}=\begin{pmatrix}a\\b\\c\end{pmatrix}$ then $\mathbf{d}\cdot\begin{pmatrix}1\\0\\0\end{pmatrix}=\lVert\mathbf{d}\rVert\cos 60°=\frac{\lVert\mathbf{d}\rVert}{2}$ and $\mathbf{d}\cdot\begin{pmatrix}0\\1\\0\end{pmatrix}=\lVert\mathbf{d}\rVert\cos 45°=\frac{\lVert\mathbf{d}\rVert}{\sqrt{2}}$	M1	3.1a
	$a=\frac{\lVert\mathbf{d}\rVert}{2}, b=\frac{\lVert\mathbf{d}\rVert}{\sqrt{2}}$	A1	1.1b
	$\mathbf{d}=\lVert\mathbf{d}\rVert\left(\frac{1}{2}\mathbf{i}+\frac{1}{\sqrt{2}}\mathbf{j}+c\mathbf{k}\right)\Rightarrow c=\sqrt{1-\frac{1}{4}-\frac{1}{2}}=...$ $\Rightarrow\mathbf{d}=\lVert\mathbf{d}\rVert\left(\frac{1}{2}\mathbf{i}+\frac{1}{\sqrt{2}}\mathbf{j}\pm"\frac{1}{2}"\mathbf{k}\right)\Rightarrow\frac{x}{1/2}=\frac{y}{1/\sqrt{2}}=\frac{z}{(\pm)1/2}$	M1	1.1b
	$2x=\sqrt{2}y=2z$ and $2x=\sqrt{2}y=-2z$ o.e	A1	2.5
		(4)	

Notes:

(ii)

M1: Sets up the direction vector for the line in unknowns (use of unit vector is fine) and applies dot product with vectors in direction of *x* and *y* axes. Note, may quote the results directly as $\cos 60°=\frac{x}{\lVert\mathbf{d}\rVert}$ etc, for the projections on to the axes.

A1: Correct values for/expression in *a* and *b* (the *x* and *y* components of direction vector) or multiples of them.

M1: Full process to find the third ordinate for the direction vector, and proceeds to form the Cartesian equation of at least one of the lines.

A1: Two correct Cartesian equations of the line, need not be simplified.

Question	Scheme	Marks	AOs
10 (a)	$x = tu \Rightarrow \frac{dx}{dt} = \frac{du}{dt}t + u$ **or** $\frac{du}{dt} = \frac{t\frac{dx}{dt} - x}{t^2}$ (oe)	**B1**	2.2a
	$\frac{dx}{dt} = \frac{du}{dt}t + u \Rightarrow \frac{d^2x}{dt^2} = \frac{d^2u}{dt^2}t + 2\frac{du}{dt}$ **or** e.g. $\frac{d^2u}{dt^2} = -\frac{1}{t^2}\frac{dx}{dt} + \frac{1}{t}\frac{d^2x}{dt^2} - \frac{1}{t^4}\left(t^2\frac{dx}{dt} - 2tx\right)$	**B1**	1.1b
	$t^2\left[\frac{d^2u}{dt^2}t + 2\frac{du}{dt}\right] - 2t(t+1)\left[\frac{du}{dt}t + u\right] + 2(t+1)tu = 8t^3e^t$ $t^3\frac{d^2u}{dt^2} + 2t^2\frac{du}{dt} - 2t(t+1)\frac{du}{dt}t - 2t(t+1)u + 2t(t+1)u = 8t^3e^t$ $t^3\frac{d^2u}{dt^2} + 2t^2\frac{du}{dt} - 2t^3\frac{du}{dt} - 2t^2\frac{du}{dt} - 2t(t+1)u + 2t(t+1)u = 8t^3e^t$	**M1**	1.1b
	$\frac{d^2u}{dt^2} - 2\frac{du}{dt} = 8e^t$ *	**A1***	2.1
		(4)	
(b)	$m^2 - 2m = 0 \Rightarrow m = 0, 2$	**M1**	1.1b
	$u = A + Be^{2t}$	**A1**	1.1b
	$\text{PS} \Rightarrow u = \lambda e^t$	**B1**	2.2a
	$\frac{du}{dt} = \lambda e^t, \frac{d^2u}{dt^2} = \lambda e^t \Rightarrow \lambda e^t - 2\lambda e^t = 8e^t \Rightarrow \lambda = \ldots\{-8\}$ $\Rightarrow u = \text{PS} + \text{CF}$	**M1**	3.4
	$u = "A + Be^{2t}" - 8e^t$ or $x = \left("A + Be^{2t}" - 8e^t\right)t$	**A1ft**	2.2a
	$x = \left(A + Be^{2t} - 8e^t\right)t$ $x = 0, t = \ln 3$ and $t = \ln 5$ $0 = A + Be^{2\ln 3} - 8e^{\ln 3} \Rightarrow 0 = A + 9B - 24$ $0 = A + Be^{2\ln 5} - 8e^{\ln 5} \Rightarrow 0 = A + 25B - 40$	**M1**	3.4
	$\left.\begin{array}{r} A + 9B = 24 \\ A + 25B = 40 \end{array}\right\} \Rightarrow A = \ldots\{15\}, B = \ldots\{1\}$	**dM1**	3.1a
	$x = \left(15 + e^{2t} - 8e^t\right)t$ (oe)	**A1**	1.1b
		(8)	
		(12 marks)	

Notes:
(a) **B1:** Deduces a correct first derivative of $x = tu$ connecting $\frac{\mathrm{d}x}{\mathrm{d}t}$ and $\frac{\mathrm{d}u}{\mathrm{d}t}$. Mos likely the main version shown, but alternatives are possible. **B1:** Correct second derivative of $x = tu$ connecting $\frac{\mathrm{d}^2x}{\mathrm{d}t^2}$ and $\frac{\mathrm{d}^2u}{\mathrm{d}t^2}$. There may be alternatives. **M1**: Substitutes their first and second derivatives into the equation and makes some attempt to expand the brackets (need not reach the final answer). (Note if you see answers substituting into the second equation that you feel are worth merit use review.) **A1*:** Fully correct proof with no errors and omissions. There must be a clear line of working where the relevant terms cancel.
(b) **M1:** Forms and solves the quadratic auxiliary equation $m^2 - 2m = 0$ (may be implied by the correct CF). **A1:** Correct CF. **B1:** Deduces the correct form of the PI. **M1**: Uses the model to find the general solution. Differentiates the correct form of the PI twice (coefficient slips only permitted) and substitutes into the differential equation to find the value of λ, leading to $u = \mathrm{PI} + \mathrm{CF}$ **A1ft:** Deduces the general solution for *u* or for *x*, follow through their CF. **M1:** Uses the model and $x = 0$, $t = \ln 3$ and $t = \ln 5$ to form two simultaneous equations (need not be fully correct, but correct substitution must be seen at least once in each equation, and allow if slips are made rearranging). May undo the substitution first and use *x* and *t*, or may use $x = 0 \Rightarrow u = 0$ and substitute to find the constants in their equation for *u*, but they must form suitable equations using correct conditions. **dM1:** Dependent on the previous method mark. Proceeds to solve to find the values of the constants in their equation. **A1:** Correct particular solution $x = \left(15 + \mathrm{e}^{2t} - 8\mathrm{e}^{t}\right)t$ (may be expanded).

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