Examiners' Report
Principal Examiner Feedback

Summer 2024

Pearson Edexcel GCE

In A Level Further Mathematics (9FM0)

Paper 4A Further Pure Mathematics 2

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Question 1

The first mark was for using Fermat's little theorem, writing $21^{22} \equiv 1(\text{mod}\,23)$ or $21^{23} \equiv 21(\text{mod}\,23)$ or $21^{23} \equiv -2(\text{mod}\,23)$. A few candidates did not use this, they went straight into modulo arithmetic, scoring no marks. Candidate are reminded that if the question asks for a particular method, then must use it to score any marks.

The majority of candidates used $21^{22} \equiv 1(\text{mod}\,23)$ and wrote $80 = 3 \times 22 + 14$ leading to $21^{80} \equiv 21^{14}(\text{mod}\,23)$ scoring the next mark. Candidates who $21^{23} \equiv 21(\text{mod}\,23)$ generally went on to write $80 = 3 \times 23 + 11$ and then incorrectly $21^{80} \equiv 21^{11}(\text{mod}\,23)$ instead of $21^{80} \equiv -8 \times 21^{11}(\text{mod}\,23)$. The majority of candidates did score the first two marks. Candidates could them use their candidates and modulus arithmetic to solve their modulus equation and most did so successfully.

Question 2

Candidates found this question accessible as a standard question finding the closed form of a second order recurrence relation, the majority scoring full marks. The majority of candidates solved the auxiliary equation and correctly wrote the correct form of the complementary function of a repeated root. Candidate were then able to use the values of u_1 and u_2 to form and solve simultaneous equations to find the constants. They then stated the closed form if the form $u_n = ...$

A few candidates had the incorrect auxiliary equation or found the next few terms leading to no marks.

Question 3

Candidate found parts (a) and (b) accessible as standard questions on Euclidean Algorithm to find the gcd and then use back substitution to find the Bezout's identity. Part (c) candidate found more demanding and with many not realising the connection with part (b) and using the multiplicative inverse of 96 or 16.

Part (a) Almost all the candidates score full arks on this question, writing 234 = $p \times 96 + q$ and then completing the algorithm to find the gcd is 6. It was pleasing that candidates stated gcd = 6 of $h = 6$.

Part (b) The majority of candidates knew to start $42 = 3 \times 12 + 6$ and rearrange to get 6 = … and then compered the process to express 6 in terms of 96 and 234. A few candidates incorrect started with the line $12 = 2 \times 6(+0)$ and then rearranged $42 = 3 \times 12 + 6$ to replace 6, this scored no marks.

Part (c) There were quite a few approaches to answer this question. The most common approach was to use the answer to part (b) and use the fact that the multiplicative inverse of 96 or 16 is -17.

Approach 1: divide through by 6 to achieve $16x \equiv 6 \pmod{39}$ and multiply both sides by – 17 to reach $x \equiv -102 \pmod{39}$ (a few candidates incorrectly multiplied by 17). Most candidates then went on to successfully achieve the correct answer $x \equiv 15 \pmod{39}$. With this approach some candidates did not use part (b) and repeated parts (a) and (b) with 16 and 39 to find the multiplicative inverse.

Approach 2: initially multiplied through by – 17 first but then needed to realise that this would give them $6x \equiv -612 \pmod{234}$, quite a few then incorrectly wrote $x \equiv -612 \pmod{234}$. To score the next method mark they needed to divide by 6 to achieve $x \equiv -102 \pmod{39}$ and then solved correctly.

There was a special case for candidate who managed to find one correct value, scoring a mark.

Question 4

Again, the majority of candidates found this question accessible, there was a common error in part (d) when find the matrix **P** many candidates did not find the normalised eigenvectors.

Part (a) There were a few approached to this question, with the majority using $\begin{pmatrix} 4 & 2 & 0 \\ 2 & p & -2 \\ 0 & -2 & 2 \end{pmatrix}\begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix} = \lambda \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix}$ to form and solve an equation for λ.

Other approaches were $\begin{pmatrix} 4 & 2 & 0 \\ 2 & p & -2 \\ 0 & -2 & 2 \end{pmatrix}\begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix} = \begin{pmatrix} 6 \\ -p \\ 6 \end{pmatrix} = \lambda \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix}$ deducing a value for λ and $\begin{pmatrix} 4-\lambda & 2 & 0 \\ 2 & p-\lambda & -2 \\ 0 & -2 & 2-\lambda \end{pmatrix}\begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix} = 0$ deducing a value for λ.

Part (b) the majority of candidates used their answer to λ in part (a) and the middle row to show that $p = 3$. A few candidates restarted and found the determinant and set = 0 with their value for λ.

Part (c) Well done by most candidates finding the determinant, setting = 0 and solving to find the other eigenvalues. Candidate knew the method to find the eigenvectors with a few slips in signs and mixing up 2's and 1's e.g. $\begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix}$ instead of $\begin{pmatrix} -1 \\ 2 \\ 2 \end{pmatrix}$.

Part (d) Candidate who had managed to find eigenvectors generally score B1 for writing down the matrix D. Quite a few candidates did not score marks for finding matrix **P** and they did not find the magnitude of their eigenvectors. Candidates who did used the correct order for their eigenvalues and eigenvectors. This this part allowed follow through on their eigenvalues and eigenvectors.

Question 5

In (i) candidates found this accessible.

Part (a) the majority of candidates knew to substitute $z = x + y\text{i}$ into the equation and applies the modulus to obtain an equation and correctly squared 2. They used algebra correctly to simplify the equation, with some doing extra work in this part to write in the form $x^2 + (y+1)^2 = 4$. Common error was a slip when dividing by 3, forgetting to divide the constant 9.

Part (b) the majority of candidates knew that the required region was the inside of a circle and scored a mark for this. Candidates used their equation $x^2 + (y+1)^2 = 4$ to deduce the correct centre and radius of the circle. If candidate had made a sip in part (a) there was a follow through on their equation. A few candidate incorrectly had the centre coordinate as (– 1, 0) instead of (0, – 1) losing the accuracy mark.

In (ii) this was more demanding

Part (a) some candidates incorrectly said that the transformation was an enlargement scale factor 3. There are two marks of this question and candidates needed to explain what effect the transformation $w = z^3$ had on the modulus and argument. Only a few candidates managed to correct identify that the argument would be multiplied by 3, rotate every point by 2 times the argument. Fewer candidates were able to identify that the modulus would be cubed, some incorrect answers were that the modulus would be multiplied by scale factor of $|z|^3$.

Part (b) was done more successfully that part (a). Even though candidates were not able to describe the transformation correctly they were able to sketch a locus of the correct form and even with the correct argument of $\frac{3\pi}{4}$. Some candidates drew a half line but did not shade the require region so did not score any marks.

Question 6

Part (a) was challenging to many candidates, but it was pleasing to see that candidates used the answer to part (a) to solve part (b)

Part (a) there were many approaches to proving the result with the easiest approach using $I_{n+2} - I_n = \int \frac{\cos(n+2)x - \cos nx}{\sin x}\,\text{d}x$ and then applying the difference of two cosines terms formula, in the A level mathematics section of the formula book.

Candidates who started with $I_{n+2} = \int \frac{\cos(n+2)x}{\sin x}\,dx = \int \frac{\cos(nx)\cos 2x - \sin(nx)\sin 2x}{\sin x}\,dx$ scored the first method mark but then struggled to know where to go from there. Candidates needed to use $\cos 2x = 1 - 2\sin^2 x$ and $\sin 2x = 2\sin x\cos x$ then simplify and compound angle formula to combine terms.

Candidates who stared with $I_n = \int \frac{\cos(n+1-1)x}{\sin x}\,dx = \int \frac{\cos(n+1)x\cos x + \sin(n+1)\sin x}{\sin x}\,dx$ scoring the first method mark but again then struggled to know where to go from there. Candidates needed to use $\cos P\cos Q = \frac{1}{2}\left[\cos(P+Q) + \cos(P-Q)\right]$ which is not explicitly in the formula book, however some candidates were aware of this formula and used it correctly. This approach was less successful that using I_{n+2}.

There were quite a few scripts where no marks were scored in this part.

Part (b) it was pleasing how many candidates who did not manage to successfully prove (a) was it to answer this part. Most candidates who attempted this part correctly found $[I_1] = \int \cot x\,dx = [\ln\sin x]$ and used the correct limits to score two marks. The next mark was for using the recurrence relationship at least once. A common error was incorrectly using $I_5 = 2\frac{\cos 6x}{6} + I_3$ or $I_3 = 2\frac{\cos 4x}{4} + I_1$, this then also lost them the method mark for use of limits to a correct reduction formula. A few candidates incorrectly found the answer to I_7 scoring 3/5 marks.

Question 7

Candidates demonstrated that they understood the group terminology the order of an element/ group and isomorphic groups.

Part (a) the majority of candidates score both marks for finding $\mathbf{B}^3$ and showing that it was equivalent to $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} (\text{mod } 2)$. A few candidates alternatively showed that $\mathbf{B}^4 = \mathbf{B}$.

Candidates draw a conclusion to score the final mark.

Part (b) the majority of candidates were able to find the order of the other elements of the group. The common error was not to state the order of the identity element so scored 2/3 marks.

Part (c) This part was not done very well by most of the candidates.

(i) candidates needed to say that there is no element of G with order 6. Some candidates said that there is no group generator but did not say why.

(ii) There were many reasons that candidates could give including the order of the group, order of elements and compare with G and the symmetries of the regular hexagon. If candidate stated an incorrect order, e.g. symmetries has an element of order 4 this scores B0.

Part (d) the majority of candidate scored full marks demonstrating that the understand isomorphic groups and permutations. Candidates' eithers scored no marks or full marks.

Question 8

This question was split into parts which made all parts accessible to candidates, with proving the result in part (b) the most demanding.

Part (a) Candidates were able to quote and correctly use the surface area formula. Many candidates managed to achieve the correct expression with correct value for *K*. A few candidates had an incorrect value for *K* when they factorised out $\sqrt{\frac{1}{4}}$ ended up multiplying by 2 instead of dividing.

Part (b) this was the most demanding part of the question, using the given substitution to prove the result. The majority of candidates score the first 2 marks for differentiating the substitution and then a full substitution int the equation to obtain an integral in terms of *u* only. Many stopped at this point unsure how to process. Some candidates did recognise that they needed to write as $9\int\sqrt{4+u^2}\,\mathrm{d}u = 9\int\frac{4+u^2}{\sqrt{4+u^2}}\,\mathrm{d}u$, the key step and then split into two integrals $9\int\frac{4+u^2}{\sqrt{4+u^2}}\,\mathrm{d}u = 9\int\frac{2+u^2}{\sqrt{4+u^2}}\,\mathrm{d}u + 9\int\frac{2}{\sqrt{4+u^2}}\,\mathrm{d}u$. They then went onto to achieve the required results. All candidates who attempted this part were able to find the correct limits.

Part (c) It was pleasing that even if candidates could not prove the result in (b) they used it to integrate the result in (a). The majority of candidate achieved the correct result for $18\int\frac{1}{\sqrt{4+u^2}}\,\mathrm{d}u = \beta\,\mathrm{arsinh}\left(\frac{u}{2}\right)$ or $\beta\ln\left(u+\sqrt{u^2+a^2}\right)$ which is in the formula book.

Candidates were less successful with $9\int\frac{2u+u^3}{\sqrt{4u^2+u^4}}\,\mathrm{d}u = \frac{9}{4}\int\frac{8u+4u^3}{\sqrt{4u^2+u^4}}\,\mathrm{d}u = \alpha\sqrt{4u^2+u^4}$ a common incorrect answer was $\ln\left(4u^2+u^4\right)$. The third method mark was dependent on have the correct form of the integrals, using the correct limits and multiplying by their *K*. Quite a few candidates lost this mark as they forgot to multiply be their *K*. For the final mark units for the area were required and the majority of candidates who had a correct answer gave the correct units.

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