

Please check the examination details below before entering your candidate information

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| Candidate surname                                                                                        |  | Other names                                                                         |  |
| Centre Number                                                                                            |  | Candidate Number                                                                    |  |
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**Pearson Edexcel Level 3 GCE**

**Monday 24 June 2024**

Afternoon (Time: 1 hour 30 minutes) **Paper reference** **9FM0/4A**

**Further Mathematics**

**Advanced**

**PAPER 4A: Further Pure Mathematics 2**

**You must have:**  
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

**Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.**

### Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided  
– *there may be more space than you need.*
- You should show sufficient working to make your methods clear.  
Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

### Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 8 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets  
– *use this as a guide as to how much time to spend on each question.*

### Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ►

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1. In this question you must show detailed reasoning.

Use Fermat's Little Theorem to determine the least positive residue of

$$21^{80} \pmod{23}$$

(4)



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Question 1 continued

Lined area for writing the answer to Question 1.

(Total for Question 1 is 4 marks)



2. Determine a closed form for the recurrence system

$$\begin{aligned} u_1 &= 4 & u_2 &= 6 \\ u_{n+2} &= 6u_{n+1} - 9u_n & (n &= 1, 2, 3, \dots) \end{aligned} \quad (5)$$

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Question 2 continued

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Question 2 continued

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Question 2 continued

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(Total for Question 2 is 5 marks)



3.

**In this question you must show all stages of your working.  
Solutions relying on calculator technology are not acceptable.**

- (a) Use the Euclidean Algorithm to determine the highest common factor  $h$  of 234 and 96

(3)

- (b) Hence determine integers  $a$  and  $b$  such that

$$234a + 96b = h$$

(3)

- (c) Solve the congruence equation

$$96x \equiv 36 \pmod{234}$$

(5)



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Question 3 continued

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Question 3 continued

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Question 3 continued

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(Total for Question 3 is 11 marks)





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Question 4 continued

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Question 4 continued

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Question 4 continued

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(Total for Question 4 is 12 marks)





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Question 5 continued

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Question 5 continued

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Question 5 continued

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(Total for Question 5 is 9 marks)



6.

In this question you must show all stages of your working.  
Solutions relying entirely on calculator technology are not acceptable.

$$I_n = \int \frac{\cos(nx)}{\sin x} dx \quad n \geq 1$$

(a) Show that, for  $n \geq 1$

$$I_{n+2} = \frac{2 \cos(n+1)x}{n+1} + I_n \quad (6)$$

(b) Hence determine the exact value of

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{\cos(5x)}{\sin x} dx$$

giving the answer in the form  $a + b \ln c$  where  $a$ ,  $b$  and  $c$  are rational numbers to be found.

(5)



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Question 6 continued

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Question 6 continued

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Question 6 continued

Lined area for writing answers.

(Total for Question 6 is 11 marks)



7. The set of matrices  $G = \{\mathbf{I}, \mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D}, \mathbf{E}\}$  where

$$\mathbf{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad \mathbf{A} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \mathbf{B} = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} \quad \mathbf{C} = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \quad \mathbf{D} = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} \quad \mathbf{E} = \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}$$

with the operation  $\otimes_2$  of matrix multiplication with entries evaluated modulo 2, forms a group.

- (a) Show that  $\mathbf{B}$  is an element of order 3 in  $G$ . (2)
- (b) Determine the orders of the other elements of  $G$ . (3)
- (c) Give a reason why  $G$  is **not** isomorphic to
  - (i) a cyclic group of order 6
  - (ii) the group of symmetries of a regular hexagon. (2)

The group  $H$  of permutations of the numbers 1, 2 and 3 contains the following elements, denoted in two-line notation,

$$\begin{array}{lll} e = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix} & a = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix} & b = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix} \\ c = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix} & d = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix} & f = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix} \end{array}$$

- (d) Determine an isomorphism between the groups  $G$  and  $H$ . (3)

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Question 7 continued

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Question 7 continued

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Question 7 continued

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(Total for Question 7 is 10 marks)



8.



Figure 1

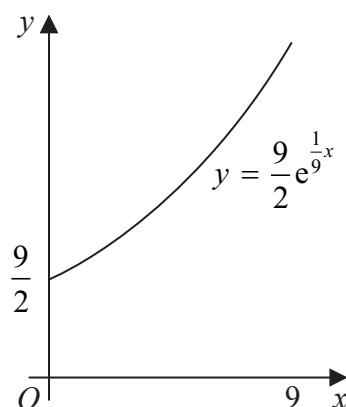


Figure 2

Figure 1 shows a French horn with a detachable bell section.

The shape of the bell section can be modelled by rotating an exponential curve through  $360^\circ$  about the  $x$ -axis, where units are centimetres.

The model uses the curve shown in Figure 2, with equation

$$y = \frac{9}{2}e^{\frac{1}{9}x} \quad 0 \leq x \leq 9$$

- (a) Show that, according to this model, the external surface area of the bell section is given by

$$K \int_0^9 e^{\frac{1}{9}x} \sqrt{4 + e^{\frac{2}{9}x}} \, dx$$

where  $K$  is a real constant to be determined.

(3)

- (b) Use the substitution  $u = e^{\frac{1}{9}x}$  to show that

$$\int_0^9 e^{\frac{1}{9}x} \sqrt{4 + e^{\frac{2}{9}x}} \, dx = 9 \int_a^b \frac{2u + u^3}{\sqrt{4u^2 + u^4}} \, du + 18 \int_a^b \frac{1}{\sqrt{4 + u^2}} \, du$$

where  $a$  and  $b$  are constants to be determined.

(5)

Hence, using algebraic integration,

- (c) determine, according to the model, the external surface area of the bell section of the horn, giving your answer to 3 significant figures.

(5)



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Question 8 continued

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Question 8 continued

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Question 8 continued

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**Question 8 continued**

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**(Total for Question 8 is 13 marks)**

**TOTAL FOR PAPER IS 75 MARKS**

