Pearson Edexcel

Mark Scheme (Results)

Summer 2024

Pearson Edexcel GCE
Advanced Subsidiary Level
Further Mathematics (8FM0)
Paper 21 Further Pure Mathematics 1

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Summer 2024
Question Paper Log number 75671
Publications Code 8FM0_21_2406_MS*

General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS
General Instructions for Marking

1. The total number of marks for the paper is 40.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

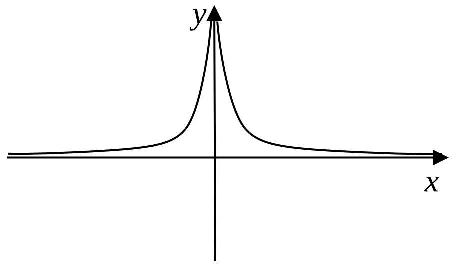
- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- $\ast$ The answer is printed on the paper
- $\sqsubset$ The second mark is dependent on gaining the first mark

4. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

5. Where a candidate has made multiple responses <u>and indicates which response they wish to submit</u>, examiners should mark this response.
If there are several attempts at a question <u>which have not been crossed out</u>, examiners should mark the final answer which is the answer that is the <u>most complete</u>.

6. Ignore wrong working or incorrect statements following a correct answer.

7. Mark schemes will firstly show the solution judged to be the most common response expected from candidates. Where appropriate, alternatives answers are provided in the notes. If examiners are not sure if an answer is acceptable, they will check the mark scheme to see if an alternative answer is given for the method used.

<table>
<tr><th>Question</th><th colspan="2">Scheme</th><th>Marks</th><th>AOs</th></tr>
<tr><td rowspan="3">1(a)</td><td rowspan="2">
</td><td>One branch correct.
See notes.</td><td>M1</td><td>1.1b</td></tr>
<tr><td>See notes</td><td>A1</td><td>1.1b</td></tr>
<tr><td colspan="2"></td><td>(2)</td><td></td></tr>
</table>

Notes:

M1: For one branch with the correct shape in quadrant 1 or quadrant 2 (probably first quadrant) Give tolerance on curves bending away from the axis where it is clear the correct shape is meant provided there is no clear intention to draw a minimum.

Ignore any dashed lines/scale and just look for the shape.

The branch must not clearly intentionally meet or cross either axis.

Condone gaps between the branches and the axes as long as the asymptotic behaviour is intended.

A1: Fully correct sketch. Branches should show asymptotic behaviour to the axes but condone gaps as above for this mark, clear bending away is A0.

Ignore dotted lines at or near either axis if they are clearly intended to indicate they are asymptotes.

(b)	$3-2x^2=\frac{1}{x^2}\Rightarrow 2x^4-3x^2+1=0\Rightarrow x^2=...$ or e.g. $3-2x^2=\frac{1}{x^2}\Rightarrow 3-2x^2-\frac{1}{x^2}=0\Rightarrow \frac{3x^2-2x^4-1}{x^2}=0\Rightarrow x^2=...$	**M1**	1.1b
	$x^2=1,\ \frac{1}{2}$ or e.g. $2x^4-3x^2+1=0\Rightarrow\left(2x^2-1\right)\left(x^2-1\right)=0\Rightarrow x^2=1,\ \frac{1}{2}$ or e.g. $-2x^4+3x^2-1=0\Rightarrow\left(-2x^2+1\right)\left(x^2-1\right)=0\Rightarrow\left(x-1\right)\left(x+1\right)\left(-\sqrt{2}x+1\right)\left(\sqrt{2}x+1\right)=0$ $\Rightarrow x=\pm1,\ \pm\frac{1}{\sqrt{2}}$	**A1**	1.1b
	$x=\pm1,\pm\frac{\sqrt{2}}{2}$	**B1**	1.1b
	$-1<x<-\frac{\sqrt{2}}{2},\qquad \frac{\sqrt{2}}{2}<x<1$	**M1** **A1**	2.1 2.2a
		(5)	

Notes: The question says all working to be shown and solutions not entirely from a calculator.

M1: For an algebraic method to find the critical values so requires:

- e.g. multiplies both sides by x^2 and collects terms to one side or collects terms to one side and puts over a common denominator
- solves the resulting 3 term equation for x^2 (usually $2x^4-3x^2+1=0$). This can be by any method e.g. factors, formula, completing the square or calculator. The usual rules apply so if values are just written down, they must be correct for their equation.

A1: For $x^2=1,\ \frac{1}{2}$ (ignore any reference to $x = 0$) or if using factors may proceed directly to x

B1 (A1 in Epen): For all four correct and exact cv's and no others apart from $x = 0$

M1: Forms regions of the correct form using their **four** critical values.
Must have four **non-zero** cv's a, b, c, d where $a<b<c<d$ and form 2 "inside" inequalities with the cv's in ascending order e.g. $a < x < b,\quad c < x < d$
The directions must be correct but allow strict or non-strict inequalities or a mixture of both.
Accept alternative notation including set notation e.g.

$\left\{x\in\mathbb{R}:-1<x<-\frac{\sqrt{2}}{2}\right\},\left\{x\in\mathbb{R}:\frac{\sqrt{2}}{2}<x<1\right\}$ or e.g. $\left(-1,-\frac{\sqrt{2}}{2}\right),\quad\left(\frac{\sqrt{2}}{2},\ 1\right)$

There must be no other regions.

A1: Correct regions only. Accept with $\frac{1}{\sqrt{2}}$ or e.g. $\sqrt{\frac{1}{2}}$ (must be exact).

Accept equivalent correct notation but if using set notation do not condone $\cap$

Special case:

Candidates who solve the quartic inequality using a calculator e.g.

$3-2x^2>\frac{1}{x^2}\Rightarrow 2x^4-3x^2+1>0\Rightarrow\ -1<x<-\frac{\sqrt{2}}{2},\qquad \frac{\sqrt{2}}{2}<x<1$

Score a maximum of 1 mark – score as M0A0B0M1A0 in EPEN

Question	Scheme	Marks	AOs
2(a)	$\underline{t_0 = 3}$ and step is half a year, so $\underline{h = \frac{1}{2}}$ and $\underline{x_0 = \frac{1}{2}}$	B1	3.3
	$\left(\frac{dx}{dt}\right)_0 = \frac{0.5\times3\times(0.8-0.5)}{0.5^2+5\times3} = ...\left(\frac{9}{305} = 0.0295...\right)$	M1 A1	3.4 1.1b
	So when $t = 3.5$, $x \approx \frac{1}{2} + \frac{1}{2}\times'\frac{9}{305}' = ...$	M1	1.1b
	$= \frac{157}{305} \approx 0.51475....$	A1	3.2a
		(5)	
(b)	Long term proportion is $\frac{4}{5}$	B1	3.4
		(1)	
			(6 marks)

Notes:

(a)

B1: Uses the given information to set up correct parameters for the model, $t_0 = 3, x_0 = \frac{1}{2}$, and $h = \frac{1}{2}$ seen or implied. Look for e.g. $x = \frac{1}{2}$ and t = 3 used in the d.e. and $h = \frac{1}{2}$ used in the approximation formula.

M1: Uses their <u>values</u> for x and t in the given equation to find $\left(\frac{dx}{dt}\right)_0$

Condone one slip when substituting their x and their t into the d.e.

This may be implied by $\left(\frac{dx}{dt}\right)_0 = \text{awrt } 0.03$.

May be seen embedded e.g. $x = \frac{1}{2} + \frac{1}{2}\left(\frac{0.5\times3\times(0.8-0.5)}{0.5^2+5\times3}\right)$

or may be seen in a table e.g.

n	x	t	dy/dx
0	½	3	0.0295…
1			

A1: For $\left(\frac{dx}{dt}\right)_0 = \frac{9}{305}$ or awrt 0.03. May be implied by subsequent work or seen as $\frac{0.45}{15.25}$

M1: Applies the approximation formula with their x, their h and their $\left(\frac{dx}{dt}\right)_0$ to find a <u>value</u> for x.

A1: Correct proportion found, accept as fraction or awrt 0.515 or e.g. 51.5%

(b)

B1: Deduces the correct long term proportion. Allow equivalents e.g. 0.8 or 80%

Question	Scheme	Marks	AOs
3(a)	$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 5 & 4 & -3 \\ a & -6 & 2 \end{vmatrix} = ...\mathbf{i} - ...\mathbf{j} + ...\mathbf{k}$	**M1**	1.1b
	$= -10\mathbf{i} - (10+3a)\mathbf{j} - (30+4a)\mathbf{k}$ or e.g. $\begin{pmatrix} -10 \\ -10-3a \\ -30-4a \end{pmatrix}$	**A1**	1.1b
		(2)	
(b)	$\text{Area} = \frac{1}{2}\left\|2\mathbf{u}\times\mathbf{v}\right\| = 15 \Rightarrow \left\|-10\mathbf{i} - (10+3a)\mathbf{j} - (30+4a)\mathbf{k}\right\| = 15$ Or e.g. $\text{Area} = \frac{1}{2}\left\|2\mathbf{u}\times\mathbf{v}\right\| = \frac{1}{2}\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 10 & 8 & -6 \\ a & -6 & 2 \end{vmatrix} = \frac{1}{2}\left\|\begin{pmatrix} -20 \\ -20-6a \\ -60-8a \end{pmatrix}\right\| = 15$	**M1**	3.1a
	$\Rightarrow 10^2 + (10+3a)^2 + (30+4a)^2 = 15^2$ $\Rightarrow 25a^2 + 300a + 875 = 0 (\Rightarrow a^2 + 12a + 35 = 0)$ or e.g. $20^2 + \left(20+6a\right)^2 + \left(60+8a\right)^2 = 30^2$ $\Rightarrow 400 + 400 + 240a + 36a^2 + 3600 + 960a + 64a^2 = 30^2$ $\Rightarrow 100a^2 + 1200a + 3500 = 0 (\Rightarrow a^2 + 12a + 35 = 0)$	**M1**	1.1b
	$\Rightarrow (a+5)(a+7) = 0 \Rightarrow a = ...$	**ddM1**	1.1b
	$a = -7$ or -5	**A1 cso**	2.2a
		(4)	
		(6 marks)	

Notes:

(a)

M1: Evidence of a correct method for the vector product. May be implied by two out of three correct components if no method shown or by correct work for 1 component.
Attempting $\mathbf{v}\times\mathbf{u}$ scores M0

A1: Correct answer.
Allow equivalent expressions and condone e.g. $-10\mathbf{I} + (-10-3a)\mathbf{J} + (-30-4a)\mathbf{K}$
Award the mark once a correct vector is seen and isw if necessary.
Must be a vector not coordinates.

(b)

M1: Applies a correct method for the area of the triangle and scalar multiple property of vector product to set up a vector equation in *a* using the vector product.

The "1/2" must be seen or implied.

This may be implied by later work e.g. $\frac{1}{2}\sqrt{20^2+\left(20+6a\right)^2+\left(60+8a\right)^2}=15$

M1: Applies the modulus correctly and expands correctly to reach a quadratic in *a*.

Note that $\frac{1}{4}\left(20^2+\left(20+6a\right)^2+\left(60+8a\right)^2\right)=15$ scores M0 (must square both sides)

ddM1: Solves their quadratic by any suitable means including a calculator to obtain at least one real root.

Depends on both previous method marks.

A1cso: Both correct values following correct work e.g. do not condone sign errors in the vector product if they fortuitously lead to the correct answers.

Note there may be more convoluted methods for the area

e.g. using $"\frac{1}{2}ab\sin C"$ **and the scalar product:**

$$\frac{1}{2}\left|\overrightarrow{AB}\right|\left|\overrightarrow{AC}\right|\sin A=15\Rightarrow\frac{1}{2}10\sqrt{2}\sqrt{a^2+40}\sin A=15$$

$$\mathbf{u}.\mathbf{v}=|\mathbf{u}||\mathbf{v}|\cos A\Rightarrow 5a-30=5\sqrt{2}\sqrt{a^2+40}\cos A$$

$$\Rightarrow\sin A=\sqrt{\frac{a^2+12a+44}{2\left(a^2+40\right)}}\Rightarrow 5\sqrt{2}\sqrt{a^2+40}\sqrt{\frac{a^2+12a+44}{2\left(a^2+40\right)}}=15$$

Score M1 for a complete correct method to obtain an equation in *a* only then e.g.

$$5\sqrt{a^2+12a+44}=15\Rightarrow a^2+12a+35=0 \text{ etc.}$$

M2 for simplifying to obtain a quadratic in *a*.

Then as above. Use review if necessary.

Question	Scheme	Marks	AOs
4(a)	$\cos x = \cos^2\frac{x}{2} - \sin^2\frac{x}{2}$ or $2\cos^2\frac{x}{2} - 1$ or $1 - 2\sin^2\frac{x}{2}$ or $\tan x = \dfrac{\tan\frac{x}{2} + \tan\frac{x}{2}}{1 - \tan^2\frac{x}{2}}$	B1	1.2
	$\Rightarrow \cos x = \left(\frac{1}{\sqrt{1+t^2}}\right)^2 - \left(\frac{t}{\sqrt{1+t^2}}\right)^2$ **or** $2\left(\frac{1}{\sqrt{1+t^2}}\right)^2 - 1$ **or** $1 - 2\left(\frac{t}{\sqrt{1+t^2}}\right)^2$ or $\tan x = \frac{2t}{1-t^2} \Rightarrow \cos x = \frac{1-t^2}{\sqrt{\left(1-t^2\right)^2 + 4t^2}}$	M1	1.1b
	$= \frac{1}{1+t^2} - \frac{t^2}{1+t^2} = \frac{1-t^2}{1+t^2}$ * or $= \frac{2}{1+t^2} - 1 = \frac{2-1-t^2}{1+t^2} = \frac{1-t^2}{1+t^2}$ * or $1 - \frac{2t^2}{1+t^2} = \frac{1+t^2-2t^2}{1+t^2} = \frac{1-t^2}{1+t^2}$ * or $\frac{1-t^2}{\sqrt{\left(1-t^2\right)^2 + 4t^2}} = \frac{1-t^2}{\sqrt{t^4 + 2t^2 + 1}} = \frac{1-t^2}{\sqrt{\left(1+t^2\right)^2}} = \frac{1-t^2}{1+t^2}$ *	A1*	2.1
		(3)	

(a) Notes

(a)

B1: Uses any correct appropriate double angle identity for cos *x* or possibly tan *x*.

M1: Substitutes correct expressions for $\cos\frac{x}{2}$ and/or $\sin\frac{x}{2}$ in terms of *t*

A1*: Completes the proof with sufficient working shown with no errors.

Other alternatives are possible and can be marked in a similar way e.g.

$$\cos x = \frac{\sin x}{\tan x} = \frac{2\sin\frac{x}{2}\cos\frac{x}{2}}{\dfrac{2\tan\frac{x}{2}}{1-\tan^2\frac{x}{2}}}$$

Scores **B1** for a correct double angle identity for cos *x*

$$= 2\left(\frac{1}{\sqrt{1+t^2}}\right)\left(\frac{t}{\sqrt{1+t^2}}\right) \times \frac{1-t^2}{2t}$$

M1 for substituting correct expressions for $\cos\frac{x}{2}$, $\sin\frac{x}{2}$ and $\tan\frac{x}{2}$ in terms of t

$$=\frac{2t}{1+t^2}\times\frac{1-t^2}{2t}=\frac{1-t^2}{1+t^2}*$$

A1* for completing the proof with sufficient working shown with no errors.

Allow θ for x for the first 2 marks but must obtain cos x = … for A1*

<u>(a) Special Case – candidates who quote results for tan *x* and/or sin *x* to verify the result:</u>

e.g.

$$t=\tan\frac{x}{2}\Rightarrow \tan x=\frac{2t}{1-t^2},\ \sin x=\frac{2t}{1+t^2}$$

$$\cos x=\frac{\sin x}{\tan x}=\frac{2t}{1+t^2}\times\frac{1-t^2}{2t}=\frac{1-t^2}{1+t^2}$$

or e.g.

$$t=\tan\frac{x}{2}\Rightarrow \sin x=\frac{2t}{1+t^2}$$

$$\cos x=\sqrt{1-\sin^2 x}=\sqrt{1-\frac{4t^2}{\left(1+t^2\right)^2}}=\sqrt{\frac{1+2t^2+t^4-4t^2}{\left(1+t^2\right)^2}}=\sqrt{\frac{\left(1-t^2\right)^2}{\left(1+t^2\right)^2}}=\frac{1-t^2}{1+t^2}$$

Scores SC B1 only

(b)	$3\tan x-10\cos x=10\Rightarrow 3\times\dfrac{2t}{1-t^2}-10\times\dfrac{1-t^2}{1+t^2}=10$	**B1**	1.1a
	$\Rightarrow 6t\left(1+t^2\right)-10\left(1-t^2\right)^2=10\left(1-t^2\right)\left(1+t^2\right)$ $\Rightarrow 6t+6t^3-10+20t^2-10t^4=10-10t^4$ $\Rightarrow 6t^3+20t^2+6t-20=0$ or e.g. $3t^3+10t^2+3t-10=0$	**M1**	2.1
	$\Rightarrow\left(t+2\right)\left(\text{"6"}t^2+...t+...\right)=0$ or e.g. $\Rightarrow\left(t+2\right)\left(\text{"3"}t^2+...t+...\right)=0$	**M1**	1.1b
	$\Rightarrow\left(t+2\right)\left(6t^2+8t-10\right)=0$ or $\left(t+2\right)\left(3t^2+4t-5\right)=0$	**A1**	2.2a
		(4)	
(c)	$t+2=0\Rightarrow x=2\times\arctan\left(-2\right)=...$	**M1**	1.1b
	$x=-126.86...^\circ$ (allow awrt -127°)	**A1**	2.2a
	$3t^2+4t-5=0\Rightarrow t=\dfrac{-4\pm\sqrt{16-4\times3\times-5}}{6}=\dfrac{-2\pm\sqrt{19}}{3}$ $\left(=0.786...,-2.11...\right)$ $\Rightarrow x=2\times\arctan\left(...\right)=...$	**M1**	3.1a
	$x=$ (awrt) $-129^\circ, 76.4^\circ$ $\quad\left(-129.486...,76.355....\right)$ — one / both	**A1** **A1**	1.1b 1.1b
		(5)	

(b) & (c) Notes:

(b)

B1: Applies part (a) and the half angle formula for tan to give a correct **equation** in t.

M1: Attempts to multiply by $1-t^2$ and $1+t^2$, expands and simplifies to a cubic in t.

M1: Attempts to take a factor (t + 2) out of their cubic.
May be done by inspection or long division but must obtain the correct coefficient for t^2 and reach a 3 term quadratic expression.

A1: Correct equation achieved as shown. Accept with any integer multiple of the quadratic.
Must see the equation written down not just values for a, b and c.

(c)

M1: Attempts to solve the t + 2 = 0 equation, look for an attempt at arctan(±2) and an attempt to double. Allow in radians for the M mark.

A1: awrt −127º

M1: Solves their quadratic and attempts arctan and doubles.
The usual rules apply for solving the quadratic and may be implied by their values.

A1: Either awrt −129º or awrt 76.4º (degrees symbol not required) or allow for this mark one of these in radians (awrt −2.25 or awrt 1.33)

A1: Both awrt −129º (or better e.g. −129.5º) and awrt 76.4º (degrees symbol not required)
There must be no other values in range for this mark. Values outside the range can be ignored.

(12 marks)

Question	Scheme	Marks	AOs
5(a)	At P, $x=\frac{p^2}{16}$ and $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{8}{p}$ so $y-p=\frac{8}{p}\left(x-\frac{p^2}{16}\right)$ **or** At P, $x=\frac{p^2}{16}$ and $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{8}{p}$ so $y=\frac{8}{p}x+c$, $p=\frac{8}{p}\times\frac{p^2}{16}+c\Rightarrow c=...$	**M1**	1.1b
	$\Rightarrow py-p^2=8x-\frac{p^2}{2}\Rightarrow 2py=16x+p^2$ *	**A1***	2.1
		(2)	
(b)	$x=-4$ oe (e.g. $x+4=0$)	**B1**	2.2a
		(1)	

Notes:
(a)

M1: Uses equation of line formula with $y_1=p$, $x_1=\frac{p^2}{16}$ and $m=\frac{8}{p}$.

Alternatively uses $y = mx + c$ with $y=p$, $x=\frac{p^2}{16}$ and $m=\frac{8}{p}$ and finds c in terms of p.

A1*: Completes correctly to the given equation, no error seen.

(b)
B1: Deduces the correct equation, accept any form and isw once a correct form is seen.

For part (c), there may be several different attempts so mark the <u>best single attempt</u> i.e. do not "mix and match".

So e.g. if you are awarding B1 for the coordinates of the focus then follow ALT 3 otherwise award M1 for using the correct focus appropriately in the other alternatives.

There may be methods other than those shown. If you are not sure if a particular response deserves credit then use review.

(c) MAIN	Gradient of l is $-\frac{8}{p}$	**B1**	2.2a
	Tangent meets directrix when $2py = 16\times -4 + p^2 \Rightarrow y = \frac{p^2-64}{2p}$	**M1** **A1**	3.1a 1.1b
	Equation of l is $y - \frac{p^2-64}{2p} = "-\frac{8}{p}"(x+4)$ or $y - 0 = "-\frac{8}{p}"(x-4)$	**M1**	3.1a
	Focus on $l \Rightarrow 0 - \frac{p^2-64}{2p} = -\frac{8}{p}(4+4) \Rightarrow p = ...$ or Point on directrix $\Rightarrow \frac{p^2-64}{2p} = "-\frac{8}{p}"(-4-4) \Rightarrow p = ...$	**ddM1**	1.1b
	$\Rightarrow p^2 - 64 = 128 \Rightarrow p = 8\sqrt{3}$	**A1**	1.1b
		(6)	
			(9 marks)

Notes:

(c)

B1: Correct gradient for l seen or implied by working.

M1: Uses $x = -4$ in the tangent to find the y coordinate of the intersection of tangent and directrix.

A1: Correct y coordinate.

M1: Attempts the equation of l using their "changed" gradient and their point of intersection of the tangent with the directrix of the form $(-4, \mathrm{f}(p))$ or using the focus (4, 0) correctly placed in their equation.

ddM1: Uses the focus (4, 0) is on the line or uses their point of intersection of the tangent with the directrix is on the line and solves to find a real non-zero value for p **Depends on both previous method marks.**

A1: Correct value for p, accept if the negative is also included. Accept $\sqrt{192}$

(c) Alt 1	Gradient of l is $-\frac{8}{p}$	**B1**	2.2a
	Reflection of point P has x coordinate $x' = -4 - \left(4 + \frac{p^2}{16}\right) = -8 - \frac{p^2}{16}$	**M1** **A1**	3.1a 1.1b
	Equation of l is $y - p = \text{"}-\frac{8}{p}\text{"}\left(x - \left(-8 - \frac{p^2}{16}\right)\right)$	**M1**	3.1a
	Focus on $l \Rightarrow 0 - p = -\frac{8}{p}\left(4 + 8 + \frac{p^2}{16}\right) \Rightarrow p = ...$	**ddM1**	1.1b
	$\Rightarrow p^2 = 96 + \frac{p^2}{2} \Rightarrow p = 8\sqrt{3}$	**A1**	1.1b
		(6)	

Notes

B1: Correct gradient for l seen or implied by working.
M1: Attempts to find the x coordinate of the reflection of the point P in the directrix.
A1: Correct coordinate.
M1: Attempts the equation of l using their "changed" gradient and coordinate.
ddM1: Uses the focus (4, 0) is on the line and solves to find a real non-zero value for p
Depends on both previous method marks.
A1: Correct value for p, accept if the negative is also included. Accept $\sqrt{192}$

(c) Alt 2	Gradient of l is $-\frac{8}{p}$	**B1**	2.2a
	Tangent meets directrix when $2py = 16 \times -4 + p^2 \Rightarrow y = \frac{p^2 - 64}{2p}$	**M1** **A1**	3.1a 1.1b
	Also y coordinate of intersection satisfies $-\frac{8}{p} = \frac{y}{-8}$ or $-\frac{8}{p} = \frac{p^2 - 64}{-16p}$	**M1**	3.1a
	$\Rightarrow \frac{8}{p} = \frac{p^2 - 64}{16p} \Rightarrow p = ...$	**ddM1**	1.1b
	$\Rightarrow p^2 - 64 = 128 \Rightarrow p = 8\sqrt{3}$	**A1**	1.1b
		(6)	

Notes

B1: Correct gradient for l seen or implied by working.
M1: Uses $x = -4$ in the tangent to find the y coordinate of the intersection of tangent and directrix.
A1: Correct y coordinate.
M1: Uses gradient from intersection of tangent and directrix to focus (4, 0) to form equation in y and p or p only if their y coordinate is already substituted.
ddM1: Substitutes the y coordinate of the intersection of tangent and directrix and solves to find a real non-zero value for p **Depends on both previous method marks.**
A1: Correct value for p, accept if the negative is also included. Accept $\sqrt{192}$

(c) Alt 3	Focus of C is (4, 0)	**B1**	2.2a
	Reflection of focus in directrix is $\left(4-2\left(4--4\right),0\right)=\left(-12,0\right)$	**M1** **A1**	3.1a 1.1b
	So tangent passes through $\left(-12,0\right)\Rightarrow 0=16\times-12+p^2$	**M1**	3.1a
	$\Rightarrow p=...$	**ddM1**	1.1b
	$\Rightarrow p=8\sqrt{3}$	**A1**	1.1b
		(6)	

Notes

B1: Correct focus for C (4, 0) stated or implied by working or e.g. seen on a sketch.

M1: Attempts to find the reflection of the focus of C in the directrix.

A1: Correct reflected focus point (–12, 0)

M1: Realises if l passes through focus then the tangent must pass through the reflection of the focus and substitutes their x from reflection and y = 0 into the tangent equation.

ddM1: Solves to find a real non-zero value for p
Depends on both previous method marks.

A1: Correct value for p, accept if the negative is also included. Accept $\sqrt{192}$

(c) ALT 4	Gradient of l is $-\frac{8}{p}$	**B1**	2.2a
	Equation of l is $y-0=-\frac{8}{p}\left(x-4\right)$	**M1** **A1**	3.1a 1.1b
	Lines intersect when $"-\frac{8}{p}"\left(x-4\right)=\frac{8x}{p}+\frac{p}{2}$	**M1**	3.1a
	Lines intersect when $x=-4\Rightarrow "-\frac{8}{p}"\left(-4-4\right)=\frac{8\left(-4\right)}{p}+\frac{p}{2}\Rightarrow p=...$	**ddM1**	1.1b
	$\Rightarrow p^2-64=128\Rightarrow p=8\sqrt{3}$	**A1**	1.1b
		(6)	
			(9 marks)

Notes:

(c)

B1: Correct gradient for l seen or implied by working.

M1: Uses the focus (4, 0) with their gradient to form an equation for l

A1: Correct equation.

M1: Solves simultaneously with the equation of the tangent to obtain an equation in x and p

ddM1: Substitutes x = –4 and solves to find a real non-zero value for p
Depends on both previous method marks.

A1: Correct value for p, accept if the negative is also included. Accept $\sqrt{192}$

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