Mark Scheme (Results)

Summer 2024

Pearson Edexcel GCE
In AS Further Mathematics (8FM0)
Paper 28 Decision Mathematics 2

Edexcel and BTEC Qualifications

Edexcel and BTEC qualifications are awarded by Pearson, the UK's largest awarding body. We provide a wide range of qualifications including academic, vocational, occupational and specific programmes for employers. For further information visit our qualifications websites at www.edexcel.com or www.btec.co.uk. Alternatively, you can get in touch with us using the details on our contact us page at www.edexcel.com/contactus.

Pearson: helping people progress, everywhere

Pearson aspires to be the world's leading learning company. Our aim is to help everyone progress in their lives through education. We believe in every kind of learning, for all kinds of people, wherever they are in the world. We've been involved in education for over 150 years, and by working across 70 countries, in 100 languages, we have built an international reputation for our commitment to high standards and raising achievement through innovation in education. Find out more about how we can help you and your students at: www.pearson.com/uk

Summer 2024
Question Paper Log Number P75678A
Publications Code 8FM0_28_2406_MS
All the material in this publication is copyright
© Pearson Education Ltd 2024

General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS
General Instructions for Marking

1. The total number of marks for the paper is 40.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\surd$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- $*$ The answer is printed on the paper
- ⌐ The second mark is dependent on gaining the first mark

4. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

5. Where a candidate has made multiple responses and indicates which response they wish to submit, examiners should mark this response.
   If there are several attempts at a question which have not been crossed out, examiners should mark the final answer which is the answer that is the most complete.

6. Ignore wrong working or incorrect statements following a correct answer.

7. Mark schemes will firstly show the solution judged to be the most common response expected from candidates. Where appropriate, alternatives answers are provided in the notes. If examiners are not sure if an answer is acceptable, they will check the mark scheme to see if an alternative answer is given for the method used.

Question	Scheme	Marks	AOs
1 (a)	Initial Flow = 90	B1	1.1b
		(1)	
(b)	The maximum flow into C is 35. Maximum flow in CD is 21 and maximum flow in CG is 19. 35 < 21 + 19 and therefore CD and CG cannot both be saturated	B1	2.4
		(1)	
(c)	(i) 26+18+24+21+19 = 108 (ii) 24+19+17+14+40 = 114	B1 B1	1.1b 1.1b
		(2)	
(d)	SBFJT (+6)	B1	1.1b
		(1)	
(e)	Use of max-flow min-cut theorem Identification of cut through EH EF BF DJ DG and CG Capacity of cut = 96 Therefore it follows that flow is maximal	 M1 A1 A1	 2.1 3.1a 2.2a
		(3)	
		(8 marks)	

Notes:

(a)

B1: CAO

(b)

B1: CAO (as a minimum accept: max inflow to C = 35 and max outflow = 40 and 35 <40)

(c)

(i) B1: CAO

(ii) B1: CAO

(d)

B1: Correct flow augmenting route found from S to T

(e)

M1: Construct argument based on max-flow min-cut theorem (e.g. attempt to find a cut through **saturated** arcs) Cut must be drawn or listed as arcs not values (condone omission of EF for M1 only)

A1: Use appropriate process of finding a minimum cut (both cut and value correct)

A1: Correct deduction that the flow is maximal (dependent on previous A mark) (must use all four words **Max Flow** = **Min Cut**)

<table>
<tr><th>Question</th><th>Scheme</th><th>Marks</th><th>AOs</th></tr>
<tr><td>2 (a)</td><td>Subtract each entry from a constant (e.g. 50) to convert from maximisation problem to minimisation

Add a large value (e.g. 100) to cells BQ, DT and ER so that they cannot be selected</td><td>B1

B1</td><td>1.1b

2.4</td></tr>
<tr><td></td><td></td><td>(2)</td><td></td></tr>
<tr><td>(b)</td><td>e.g.
<table><tr><th></th><th>P</th><th>Q</th><th>R</th><th>S</th><th>T</th></tr><tr><td>A</td><td>18</td><td>10</td><td>15</td><td>9</td><td>13</td></tr><tr><td>B</td><td>12</td><td>100</td><td>10</td><td>23</td><td>17</td></tr><tr><td>C</td><td>9</td><td>22</td><td>13</td><td>14</td><td>15</td></tr><tr><td>D</td><td>15</td><td>17</td><td>12</td><td>14</td><td>100</td></tr><tr><td>E</td><td>10</td><td>12</td><td>100</td><td>11</td><td>16</td></tr></table>Reducing rows
<table><tr><th></th><th>P</th><th>Q</th><th>R</th><th>S</th><th>T</th></tr><tr><td>A</td><td>9</td><td>1</td><td>6</td><td>0</td><td>4</td></tr><tr><td>B</td><td>2</td><td>90</td><td>0</td><td>13</td><td>7</td></tr><tr><td>C</td><td>0</td><td>13</td><td>4</td><td>5</td><td>6</td></tr><tr><td>D</td><td>3</td><td>5</td><td>0</td><td>2</td><td>88</td></tr><tr><td>E</td><td>0</td><td>2</td><td>90</td><td>1</td><td>6</td></tr></table>Reducing columns
<table><tr><th></th><th>P</th><th>Q</th><th>R</th><th>S</th><th>T</th></tr><tr><td>A</td><td>9</td><td>0</td><td>6</td><td>0</td><td>0</td></tr><tr><td>B</td><td>2</td><td>89</td><td>0</td><td>13</td><td>3</td></tr><tr><td>C</td><td>0</td><td>12</td><td>4</td><td>5</td><td>2</td></tr><tr><td>D</td><td>3</td><td>4</td><td>0</td><td>2</td><td>84</td></tr><tr><td>E</td><td>0</td><td>1</td><td>90</td><td>1</td><td>2</td></tr></table>3 lines required (Row A, Columns P, R) so augment by 1
<table><tr><th></th><th>P</th><th>Q</th><th>R</th><th>S</th><th>T</th></tr><tr><td>A</td><td>10</td><td>0</td><td>7</td><td>0</td><td>0</td></tr><tr><td>B</td><td>2</td><td>88</td><td>0</td><td>12</td><td>2</td></tr><tr><td>C</td><td>0</td><td>11</td><td>4</td><td>4</td><td>1</td></tr><tr><td>D</td><td>3</td><td>3</td><td>0</td><td>1</td><td>83</td></tr><tr><td>E</td><td>0</td><td>0</td><td>90</td><td>0</td><td>1</td></tr></table>4 lines required (e.g. Row A, E Column P, R) so augment by 1
<table><tr><th></th><th>P</th><th>Q</th><th>R</th><th>S</th><th>T</th></tr><tr><td>A</td><td>11</td><td>0</td><td>8</td><td>0</td><td>0</td></tr><tr><td>B</td><td>2</td><td>87</td><td>0</td><td>11</td><td>1</td></tr><tr><td>C</td><td>0</td><td>10</td><td>4</td><td>3</td><td>0</td></tr><tr><td>D</td><td>3</td><td>2</td><td>0</td><td>0</td><td>82</td></tr><tr><td>E</td><td>1</td><td>0</td><td>91</td><td>0</td><td>1</td></tr></table></td><td>B1

M1

M1

M1</td><td>1.1b

1.1b

1.1b

1.1b</td></tr>
</table>

Alternatively

4 lines required (e.g. Row A, C, E Column R) so augment by 1

	P	Q	R	S	T
A	10	0	8	0	0
B	1	87	0	11	1
C	0	11	5	4	1
D	2	2	0	0	82
E	0	0	91	0	1

Allocation A – T, B – R, C – P, D – S, E – Q

A1 2.2a

Maximum Score: 192

B1 2.2a

(6)

(8 marks)

Notes:

(a)

B1: Valid statement regarding converting a maximisation problem to a minimisation problem

B1: Explain the need to add a large value to avoid cells being selected – it must be clear which order the changes are applied

(b)

B1: Mark awarded when both steps complete (subtraction and addition of large values) accept one slip in values

M1: Simplifying the initial matrix by reducing rows and then columns (accept one additional slip)

M1: Develop an improved solution – need to see one double covered +e; one uncovered –e; and one single covered unchanged. 3 lines needed to 4 lines needed

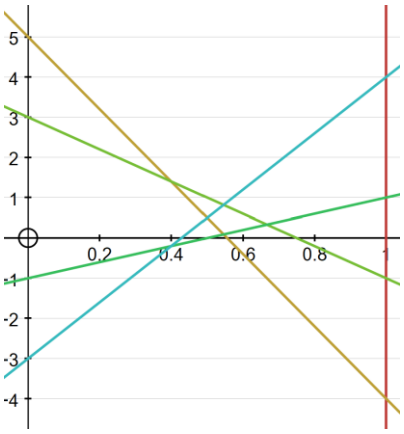
M1: Develop an improved solution – need to see one double covered +e; one uncovered –e; and one single covered unchanged. 4 lines needed to 5 lines needed (so getting to the optimal table)

A1: CSO on final table (so must have scored all previous marks in this part) + deduction of the correct allocation **(note there are different fully correct tables)**

B1: Maximum score stated

If subtracting from 41

	P	Q	R	S	T
A	9	1	6	0	4
B	3	100	1	14	8
C	0	13	4	5	6
D	6	8	3	5	100
E	1	3	100	2	7

<table>
<tr><th>Question</th><th>Scheme</th><th>Marks</th><th>AOs</th></tr>
<tr><td>3 (a)</td><td>(i) Row minima: $-5, -3, -1, -4$ max is -1
Column maxima: 4, 6, 3 min is 3
Row(maximin) ≠ Col(minimax) therefore game is not stable</td><td>M1
A1</td><td>1.1b
2.4</td></tr>
<tr><td></td><td></td><td>(2)</td><td></td></tr>
<tr><td>(b)</td><td>
<table>
<tr><td colspan="2"></td><td colspan="4">Haruki</td></tr>
<tr><td colspan="2"></td><td>Option A</td><td>Option B</td><td>Option C</td><td>Option D</td></tr>
<tr><td rowspan="2">Meera</td><td>Option X</td><td>-4</td><td>-1</td><td>1</td><td>4</td></tr>
<tr><td>Option Z</td><td>5</td><td>3</td><td>-1</td><td>-3</td></tr>
</table>
</td><td>B1</td><td>1.1b</td></tr>
<tr><td></td><td></td><td>(1)</td><td></td></tr>
<tr><td>(c)</td><td>(i)
If H plays Option A, M's gains are $-4p + 5(1 - p) = 5 - 9p$
If H plays Option B, M's gains are $-p + 3(1 - p) = 3 - 4p$
If H plays Option C, M's gains are $p + (-1)(1 - p) = -1 + 2p$
If H plays Option D, M's gains are $4p + (-3)(1 - p) = -3 + 7p$
 
$5 - 9p = -1 + 2p \Rightarrow p = 6/11$
Meera should play Option X with probability $\frac{6}{11}$ and Option Z with probability $\frac{5}{11}$
(ii)
Value to Meera $= -1 + 2 \times \frac{6}{11} = \frac{1}{11}$
Value to Haruki $= \frac{-1}{11}$</td><td>M1
A1

M1
A1

A1

B1 ft</td><td>

1.1b
1.1b

1.1b
1.1b

3.2a

3.4</td></tr>
</table>

	(iii) Haruki never plays B and D	B1	3.2a
		(7)	
(d)	Value of game = 0 When *H* plays Option A, *M*'s gain $5 - 9p = 0$ $p = 5/9$ When *H* plays Option C, *M*'s gain $-5k/9 - 4/9 = 0$ $k = -4/5$	B1 M1 M1 A1	3.1a 1.1b 3.4 2.2a
	Alternatively Considers *H* playing A with probability *q* and C with probability $(1 - q)$ When *M* plays Z *H*'s gain $1 - 6q = 0$ $q = 1/6$ When *M* plays X *H*'s gain $4/6 + 5k/6 = 0$ $k = -4/5$		
		(4)	
			(14 marks)

Notes:

(a)
M1: finding row minimums and column maximums – condone one error
A1: row maximin $\neq$ col minimax so not stable (dependent on correct row mins and col maxs)
(b)
B1: Correct reduced table for Meera (2 rows only)
(c)
M1: setting up four expressions in terms of *p*
A1: all four expressions correct and fully simplified
M1: axes correct, at least one line correctly drawn for their expressions
A1: correct graph (all lines drawn with a ruler, scale clear with correct relative position of lines)
A1: using the graph to obtain the correct probability expressions leading to the correct value of *p* and interpret their value of *p* in the context of the question (dependent on previous A mark)
B1ft: correct value of game to Haruki (ft their *p*)
B1: CAO – B and D
(d):
B1: States value = 0 (may be implied by forming equation in *p* or *q* = 0)
M1: considers Option A and obtains value for *p* (alt considers *H* and obtains value for *q*)
M1: considers Option C and sets up equation in *k* (**dependent on previous M mark**)
A1: CAO (-4/5 o.e.)

Question	Scheme	Marks	AOs
4(a)	$M = 150$	B1	3.4
		(1)	
(b)	(aux equation $m - 1.025 = 0 \Rightarrow$) complementary function is $A(1.025)^n$	B1	1.1b
	Consider a trial solution of the form $u_n = \lambda$ so $\lambda - 1.025\lambda = 1800$ $\Rightarrow \lambda = \ldots$	M1	1.1b
	General solution is $u_n = A(1.025)^n - 72000$	A1	1.1b
	$n = 1$, $u_1 = 6925 \Rightarrow A = \ldots$	M1	3.4
	$u_n = 77000(1.025)^n - 72000$	A1	1.1b
	Alternatively General solution is $u_n = A(1.025)^{n-1} - 72000$ $u_n = 78925(1.025)^{n-1} - 72000$		
		(5)	
(c)	$D = 77000 - 72000 \Rightarrow D = £5000$	B1ft	1.1b
	Alternatively $6925 = 1.025D + 1800 \Rightarrow D = 5000$		
		(1)	
(d)	$77000(1.025)^n - 72000 > 20000$	M1	1.1b
	$(1.025)^n > \frac{92}{77} \Rightarrow n\log(1.025) > \log\left(\frac{92}{77}\right)$	M1	1.1b
	$n > 7.20795\ldots \Rightarrow n = 8$	A1	1.1b
	Alternatively $78925(1.025)^{n-1} - 72000 > 20000$ $(1.025)^{n-1} > \frac{3680}{3157} \Rightarrow (n-1)\log(1.025) > \log\left(\frac{3680}{3157}\right)$ $n - 1 > 6.20795\ldots \Rightarrow n = 8$		
		(3)	
			(10 marks)

Notes
(a) **B1:** CAO **(b)** **B1:** CAO **M1:** substituting their trial solution into the recurrence relation in an attempt to find their λ **A1:** CAO **M1:** using the conditions in the model to calculate A **A1:** CAO (must be $u_n =$ not $u_{n+1} =$) **(c)** **B1ft:** substitutes $n=0$ in their solution to find D **(d)** **M1:** sets their particular solution greater than 20000– their particular solution must be of the correct form $\left(u_n = c(1.025)^n \pm d \text{ or } u_n = c(1.025)^{n-1} \pm d\right)$ **M1:** dependent on previous M mark – correctly re-arranges and then applies the process of taking logs for their particular solution **A1:** CAO

Pearson Education Limited. Registered company number 872828
with its registered office at 80 Strand, London, WC2R 0RL, United Kingdom