Pearson Edexcel

Mark Scheme (Result)

October 2020

Pearson Edexcel GCE In A level Further Mathematics
Paper 9FM0/4D

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate’s response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate’s response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- ∗ The answer is printed on the paper
- ⌐ The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

Question	Scheme	Marks	AOs
1(a)	Subtracting each entry from a constant value ≥ 35 to convert from maximisation problem to minimisation	M1	1.1b
	Add a sufficiently large number (> 15) to cells A3 and B4 e.g. $\begin{bmatrix} 6 & 15 & 100 & 12 \\ 3 & 5 & 7 & 100 \\ 0 & 3 & 1 & 10 \\ 6 & 4 & 8 & 5 \end{bmatrix}$	B1	1.1b
	Reduce rows $\begin{bmatrix} 0 & 9 & 94 & 6 \\ 0 & 2 & 4 & 97 \\ 0 & 3 & 1 & 10 \\ 2 & 0 & 4 & 1 \end{bmatrix}$ and then columns $\begin{bmatrix} 0 & 9 & 93 & 5 \\ 0 & 2 & 3 & 96 \\ 0 & 3 & 0 & 9 \\ 2 & 0 & 3 & 0 \end{bmatrix}$	M1 A1ft	2.1 1.1b
	followed by $\begin{bmatrix} 0 & 7 & 93 & 3 \\ 0 & 0 & 3 & 94 \\ 0 & 1 & 0 & 7 \\ 4 & 0 & 5 & 0 \end{bmatrix}$ or $\begin{bmatrix} 0 & 7 & 91 & 3 \\ 0 & 0 & 1 & 94 \\ 2 & 3 & 0 & 9 \\ 4 & 0 & 3 & 0 \end{bmatrix}$	M1 A1ft	2.1 1.1b
	A – 1, B – 2, C – 3, D – 4	B1ft	1.1b
		(7)	
(b)	£123	B1	1.1b
		(1)	
			(8 marks)

Notes:

(a) M1: convert from maximisation to minimisation (allow at most two errors)
B1: adding a large number (at least 16) to cells A3 and B4
M1: simplifying the initial matrix by reducing rows and then columns
A1ft: cao following on from their earlier subtraction
M1: develop an improved solution – need to see one double covered +e; one uncovered –e; and one single covered unchanged. 3 lines needed to 4 lines needed
A1ft: cao following on from row and column reduction final table
(b) B1ft: correct allocation ft their optimal table (all previous M marks must have been awarded in (a))
B1: cao – solution of original problem

Question	Scheme	Marks	AOs
2(a)	EMV for A is $0.6(350)+0.4(-140)=154$ EMV for B is $0.75(260)+0.25(-190)=147.5$ EMV for C is $0.8(220)+0.2(-230)=130$	M1 A1	3.4 1.1b
	The optimal EMV is £154, which makes option A the best choice using the EMV criterion	A1	2.2a
		(3)	
(b)	$u(350)=0.5831379803...,u(-140)=-0.4190675486...$ $u(260)=0.4779542232...,u(-190)=-0.6080141975...$ $u(220)=0.4230501896...,u(-230)=-0.7771305269...$	M1	3.4
	Calculate all three expected utilities: A is $0.6(0.583...)+0.4(-0.419...)=0.1822557...$ B is $0.75(0.477...)+0.25(-0.608...)=0.2064621...$ C is $0.8(0.423...)+0.2(-0.777...)=0.1830140...$	DM1 A1	1.1b 1.1b
	The optimal expected utility is 0.206 utils, which makes option B the best choice using expected utility as the criterion	A1	2.2a
		(4)	
			(7 marks)

Notes:

(a) M1: Correct method for calculation EMV for either A, B or C
A1: Correct values of EMV for A, B and C
A1: Correct deduction of optimal EMV (dependent on all three correct EMVs)

(b) M1: Uses the correct utility function to replace each pay-off with the corresponding utility
DM1: Calculate all three expected utilities using correct probability values from (a)
A1: At least 2 expected utilities correct (correct to at least 2 decimal places)
A1: Correct deduction of optimal expected utility

<table>
<tr><th>Question</th><th>Scheme</th><th>Marks</th><th>AOs</th></tr>
<tr><td>3 (a)</td><td>
<table>
<tr><td></td><td>P</td><td>Q</td><td>R</td></tr>
<tr><td>A</td><td>$42-\theta$</td><td></td><td>θ</td></tr>
<tr><td>B</td><td>$17+\theta$</td><td>$51-\theta$</td><td></td></tr>
<tr><td>C</td><td></td><td>$21+\theta$</td><td>$4-\theta$</td></tr>
<tr><td>D</td><td></td><td></td><td>(40)</td></tr>
</table>
<table>
<tr><td></td><td>P</td><td>Q</td><td>R</td></tr>
<tr><td>A</td><td>38</td><td></td><td>4</td></tr>
<tr><td>B</td><td>21</td><td>47</td><td></td></tr>
<tr><td>C</td><td></td><td>25</td><td></td></tr>
<tr><td>D</td><td></td><td></td><td>40</td></tr>
</table>
</td><td>M1
A1</td><td>2.1
1.1b</td></tr>
<tr><td></td><td></td><td>(2)</td><td></td></tr>
<tr><td>(b)</td><td>
<table>
<tr><td></td><td></td><td>25</td><td>30</td><td>17</td></tr>
<tr><td></td><td></td><td>P</td><td>Q</td><td>R</td></tr>
<tr><td>0</td><td>A</td><td>X</td><td>–6</td><td>X</td></tr>
<tr><td>–18</td><td>B</td><td>X</td><td>X</td><td>15</td></tr>
<tr><td>–19</td><td>C</td><td>7</td><td>X</td><td>22</td></tr>
<tr><td>–4</td><td>D</td><td>–5</td><td>–11</td><td>X</td></tr>
</table>
</td><td>M1
A1</td><td>1.1b
1.1b</td></tr>
<tr><td></td><td>
<table>
<tr><td></td><td>P</td><td>Q</td><td>R</td></tr>
<tr><td>A</td><td>$38-\theta$</td><td></td><td>$4+\theta$</td></tr>
<tr><td>B</td><td>$21+\theta$</td><td>$47-\theta$</td><td></td></tr>
<tr><td>C</td><td></td><td>(25)</td><td></td></tr>
<tr><td>D</td><td></td><td>θ</td><td>$40-\theta$</td></tr>
</table>
<table>
<tr><td></td><td>P</td><td>Q</td><td>R</td></tr>
<tr><td>A</td><td></td><td></td><td>42</td></tr>
<tr><td>B</td><td>59</td><td>9</td><td></td></tr>
<tr><td>C</td><td></td><td>25</td><td></td></tr>
<tr><td>D</td><td></td><td>38</td><td>2</td></tr>
</table>

Entering cell is DQ and exiting cell is AP
</td><td>M1

A1</td><td>1.1b

2.2a</td></tr>
<tr><td></td><td></td><td>(4)</td><td></td></tr>
<tr><td>(c)</td><td>
<table>
<tr><td></td><td></td><td>14</td><td>19</td><td>17</td></tr>
<tr><td></td><td></td><td>P</td><td>Q</td><td>R</td></tr>
<tr><td>0</td><td>A</td><td>11</td><td>5</td><td>X</td></tr>
<tr><td>–7</td><td>B</td><td>X</td><td>X</td><td>4</td></tr>
<tr><td>–8</td><td>C</td><td>7</td><td>X</td><td>11</td></tr>
<tr><td>–4</td><td>D</td><td>6</td><td>X</td><td>X</td></tr>
</table>

No negative IIs so solution is optimal
</td><td>M1
A1

A1</td><td>2.1
1.1b

2.4</td></tr>
<tr><td></td><td></td><td>(3)</td><td></td></tr>
<tr><td>(d)</td><td>Let x_{ij} be the number of units (of stock) transported from (supply point) i to (sales point) j</td><td>B1</td><td>3.3</td></tr>
<tr><td></td><td>where $i \in \{\mathrm{A,B,C,D}\}$ and $j \in \{\mathrm{P,Q,R}\}$ $(x_{ij} \geq 0)$</td><td>B1</td><td>3.3</td></tr>
<tr><td></td><td>Minimise $25x_{\mathrm{AP}} + 24x_{\mathrm{AQ}} + 17x_{\mathrm{AR}} + 7x_{\mathrm{BP}} + 12x_{\mathrm{BQ}} + 14x_{\mathrm{BR}}$
$+ 13x_{\mathrm{CP}} + 11x_{\mathrm{CQ}} + 20x_{\mathrm{CR}} + 16x_{\mathrm{DP}} + 15x_{\mathrm{DQ}} + 13x_{\mathrm{DR}}$</td><td>B1
B1</td><td>2.5
3.3</td></tr>
</table>

	$\sum x_{Aj} \le 42, \sum x_{Bj} \le 68, \sum x_{Cj} \le 25, \sum x_{Dj} \le 40$ accept $=$	B1	3.3
	$\sum x_{iP} \ge 59, \sum x_{iQ} \ge 72, \sum x_{iR} \ge 44$ accept $=$	B1	3.3
		(6)	
(e)	The Simplex algorithm cannot be used as not all the constraints are in the form $\sum x \le k$ where k is a positive constant	B1	3.5b
		(1)	
(16 marks)			

Notes:

(a) M1: a valid route, only one empty square used, θ's balance
A1: cao
(b) M1: Finding all 7 shadow costs and the 6 improvement indices for the correct 9 entries
A1: Shadow costs and II CAO
M1: A valid route, their most negative II chosen, only one empty square used, θ's balance
A1: CAO – including the deduction of all entering and exiting cells
(c) M1: finding all 7 shadow costs **and** the 6 improvement indices – this mark is dependent on the previous M mark in (b) which will therefore indicate a correct mathematical argument leading from the initial solution to the confirmation or not of the optimality of the current solution
A1: CAO (shadow costs and IIs)
A1: CSO including the correct reasoning that the solution is optimal because there are no negative II
(d) **B1:** Correct definition of x_{ij}
B1: Correctly defining the set of values that i and j can take
B1: 'Minimise' + correct number of terms in objective
B1: Correct objective function
B1: Correct supply constraints (allow 'equals' or written out in full e.g. $x_{AP} + x_{AQ} + x_{AR} \le 42$)
B1: Correct demand constraints (allow 'equals' or written out in full)
(e) B1: Correct justification of why the Simplex algorithm cannot be used to solve transportation LP

Question	Scheme	Marks	AOs
4(a)	CF of $u_n = A(2)^n + B(-1)^n \Rightarrow$ auxiliary equation is $(m-2)(m+1) = 0$	M1	3.1a
	$m^2 - m - 2 = 0 \Rightarrow \alpha = -1, \beta = -2$	A1	2.2a
		(2)	
(b)	particular solution try $u_n = \lambda(-3)^n, u_{n+1} = \lambda(-3)^{n+1}, u_{n+2} = \lambda(-3)^{n+2}$	M1	2.1
	$9\lambda + 3\lambda - 2\lambda = 20 \ (\Rightarrow \lambda = 2)$	M1	1.1b
	$u_n = A(2)^n + B(-1)^n + 2(-3)^n$	A1	1.1b
	$2u_0 = u_1 \Rightarrow 2A + 2B + 4 = 2A - B - 6$	M1	1.1b
	$u_4 = 164 \Rightarrow 16A + B + 162 = 164$	M1	1.1b
	$A = \frac{1}{3}, B = -\frac{10}{3} \Rightarrow u_n = \frac{1}{3}(2)^n - \frac{10}{3}(-1)^n + 2(-3)^n$	A1	2.2a
		(6)	
			(8 marks)

Notes:

(a) M1: Uses given complementary function to find auxiliary equation corresponding to second-order recurrence relation

A1: cao for both α and β

(b) M1: substitute $u_n = \lambda(-3)^n$ into their second-order recurrence relation

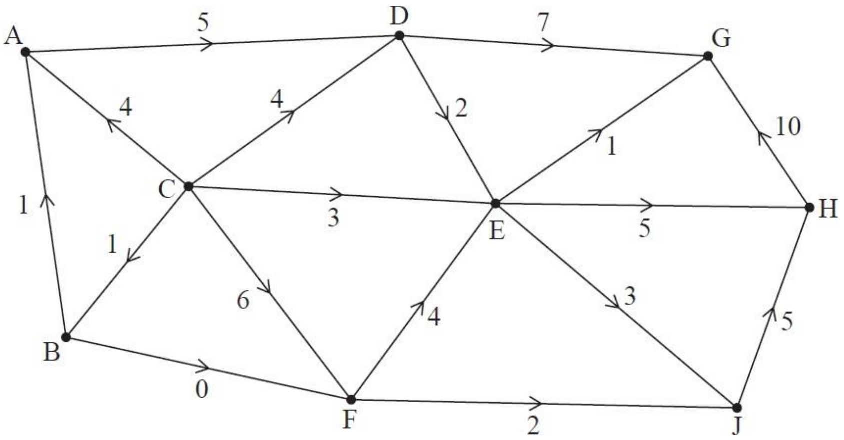
M1: forms linear equation in λ only

A1: correct general solution

M1: use $2u_0 = u_1$ to form an equation in B (and possibly A)

M1: use $u_4 = 164$ to set up a second equation in A and B

A1: cao

Question	Scheme	Marks	AOs
5 (a)	Source node is C	B1	1.1b
		(1)	
(b)	G is the sink node as all the arcs incident to G flow into G	B1	2.4
		(1)	
(c)	Capacity of cut $C_1 = 10 + 2 - 1 + 6 + 8 + 1 - 0 = 26$	B1	1.1b
		(1)	
(d)(i)	Arc JH must be at its upper capacity of 5 as the two arcs that flow into J (EJ and FJ) have a lower capacity of 2 + 3 = 5	B1	2.4
(ii)	Arcs AD and CD must be at the lower capacities (which in total is 9) as the only two arcs (DG and DE) that flow out of D have a total upper capacity of 7 + 2 = 9	B1	2.4
		(2)	
(e)	 	M1 A1	2.2a 1.1b
		(2)	
(f)	Use of max-flow min-cut theorem Identification of cut through DG, DE, CE, CF, CB, BA with a capacity of 18 and value of flow = 18 Therefore it follows that flow is maximal	M1 A1 A1	2.1 3.1a 2.2a
		(3)	
			(10 marks)
Notes:			

(a) **B1:** cao (node C)

(b) **B1:** correct explanation of why G is the sink node

(c) **B1:** cao

(d)(i) B1: correct explanation that JH must be at its upper capacity (must refer to arcs EJ and FJ)

(d)(ii) B1: correct explanation that AD and CD must be at their lower capacities (must refer to arcs DG and DE)

(e) M1: 'flow in = flow out' at all but one vertex – one number only required on each arc (condone blank for arc BF)

A1: a correct valid flow through the network (check that flow in must equal flow out at each vertex)

(f) M1: Construct argument based on max-flow min-cut theorem (e.g. attempt to find a cut through saturated arcs)

A1: Use appropriate process of finding a minimum cut (cut + value correct)

A1: Correct deduction that the flow is maximal

Question	Scheme	Marks	AOs
6 (a)	Option R (or option T) <u>dominates</u> option S Because e.g. 4 > 2 and –3 > –4 and 1 > –2	B1 B1	1.2 2.4
		(2)	
(b)	Row minima: 1, –3, –2 max is 1 Column maxima: 4, 5, 3 min is 3 Row maximin (1) ≠ Column minimax (3) so not stable	M1 A1 A1	1.1b 1.1b 2.4
		(3)	
(c)	V is less than or equal to each of these three expressions since we need to find the maximum value of the worst possible augmented expected pay-off for each value of p	B1	2.3
		(1)	
(d)	It is necessary to use an inequality because it enables the Simplex algorithm to pivot on a row that will increase the value of P	B1	3.5a
		(1)	
(e)	$p_2 = \frac{4}{11}$ Substitute p values to obtain $V \leq \frac{56}{11}, \frac{56}{11}, \frac{58}{11}$ Value of the game to player A = $\frac{56}{11} - 3 = \frac{23}{11}$	B1 M1 A1	1.1b 3.4 2.2a
		(3)	
(f)	$q_1 + 5q_2 + 3q_3 = \frac{23}{11}$ $4q_1 - 3q_2 + q_3 = \frac{23}{11}$ $q_1 + q_2 + q_3 = 1$ Player B should play option X with probability $\frac{8}{11}$, option Y with probability $\frac{3}{11}$ and never play option Z	M1 A1ft A1 A1	3.1a 1.1b 1.1b 3.2a
		(4)	
		(14 marks)	

Notes:

(a) B1: correct statement – must include the word ‘dominate’ (note that T dominates S too)
B1: correct inequalities – must be clear that all inequalities must hold

(b) M1: attempt at row minima and column maxima – condone one error

A1: correct max(row min) and min(col max)

A1: correct reasoning that the game is not stable (accept $1 \neq 3$ + statement)

(c) B1: an understanding that for each value of p we are seeking the minimum possible output

(d) B1: as a minimum accept an answer that implies that an inequality is required so that we can apply the Simplex algorithm

(e) B1: cao

M1: substitute their p values into all three expressions for the upper bound of V

A1: cao for the value of the game to player A

(f) M1: Attempt to set up at least three equations in q_1, q_2, q_3 using the value of the game from (e)

A1ft: Two correct ft "their" V

A1: cao (for exactly three equations correct)

A1: cao in context

Question	Scheme					Marks	AOs
7(a)	Stage	State	Action	Dest.	Value		
	Trainers	0	0	0	0		
		1	1	0	50	B1	3.1a
		2	2	0	90		
		3	3	0	170		
		4	4	0	225		
		5	5	0	295		
	Sandals	0	0	0	0		
		1	1	0	70 + 0 = 70*	M1	3.1a
			0	1	0 + 50 = 50	A1	1.1b
		2	2	0	110 + 0 = 110	A1	1.1b
			1	1	70 + 50 = 120*		
			0	2	0 + 90 = 90		
		3	3	0	165 + 0 = 165		
			2	1	110 + 50 = 160		
			1	2	70 + 90 = 160		
			0	3	0 + 170 = 170*		
		4	4	0	245 + 0 = 245*	M1	1.1b
			3	1	165 + 50 = 215	A1	1.1b
			2	2	110 + 90 = 200	A1	1.1b
			1	3	70 + 170 = 240		
			0	4	0 + 225 = 225		
		5	5	0	300 + 0 = 300*		
			4	1	245 + 50 = 295		
			3	2	165 + 90 = 255		
			2	3	110 + 170 = 280		
			1	4	70 + 225 = 295		
			0	5	0 + 295 = 295		
	High heels	5	5	0	305 + 0 = 305		
			4	1	235 + 70 = 305	M1	1.1b
			3	2	x + 120	A1ft	1.1b
			2	3	115 + 170 = 285		
			1	4	75 + 245 = 320		
			0	5	0 + 300 = 300		
	320 and x +120					A1ft	1.1b
						(10)	
(b)	Trainers: 0	Sandals: 4	High heels: 1			B1	1.1b
	Trainers: 1	Sandals: 1	High heels: 3			B1	2.2a
						(2)	
						(12 marks)	

Notes:
(a) B1: CAO for the first stage (all six rows) – entries in all columns must be correct – candidates may start with state 5 (rather than state 0) which is fine **M1**: Second stage – my states 1, 2 and 3 (so at least 9 rows in the first half of the second stage or at least 20 non-zero rows). Value column must be complete with at least one value correct for each state – ignore entries in all other columns **A1**: Value column for states 1, 2 and 3 correct for the second stage – ignore entries in all other columns and condone additional rows **A1**: CAO for states 0, 1, 2 and 3 of the second stage (no additional rows for these four states) – entries in all columns must be correct **M1**: Second stage – my states 4 and 5 (so at least 11 rows in the second half of the second stage or at least 20 non-zero rows). Value column must be complete with at least one value correct for each state – ignore entries in all other columns **A1**: Value column for states 4 and 5 correct for the second stage – ignore entries in all other columns and condone additional rows **A1**: CAO for states 4 and 5 of the second stage (no additional rows for these two states) - entries in all columns must be correct **M1**: At least 6 rows for the third stage. Value column must be complete with at least 3 values correct – ignore entries in all other columns **A1ft**: CAO for third stage correct (no additional rows for this stage) - entries in all columns must be correct **A1ft**: Must have earned all previous M marks from their completed dynamic programming but ft their result **(b) B1**: One correct allocation (dependent on first three M marks in (a)) **B1:** For both correct (dependent on first three M marks in (a))