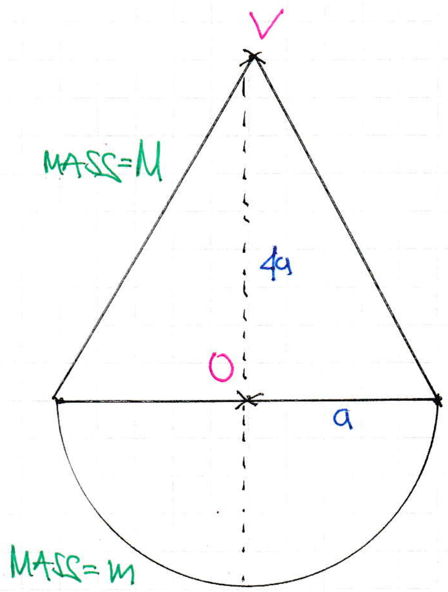
IYGB – FM2 PAPER M – QUESTION 1

- START BY FINDING THE LOCATION OF THE CENTRE OF MASS, ALONG THE AXIS OF SYMMETRY FROM A REFERENCE POINT, SAY O IN THE DIAGRAM

	CONE	HEMISPHERE	TOTAL
MASS RATIO	M	m	$M+m$
DISTANCE OF THE CENTRE OF MASS FROM O	$\frac{1}{4}\times 4a$	$-\frac{3}{8}a$	$\bar{y}$



$$\Rightarrow (M+m)\bar{y} = Ma - \frac{3}{8}ma$$

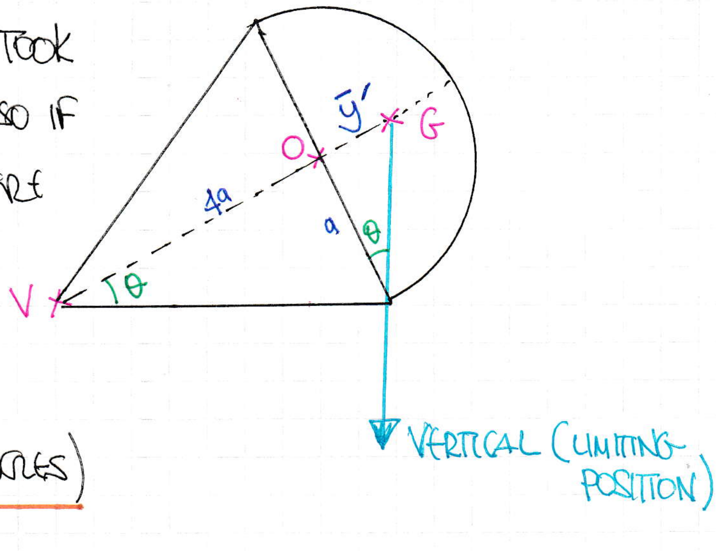
$$\Rightarrow 8(M+m)\bar{y} = (8M-3m)a$$

$$\Rightarrow \bar{y} = \frac{8M-3m}{8(M+m)}a$$

- NOW LOOKING AT THE OBJECT IN EQUILIBRIUM

NOTE IN THE ABOVE CALCULATION WE TOOK $\bar{y}$ TO BE POSITIVE IN THE CONE, SO IF WE TAKE POSITIVE IN THE HEMISPHERE

$$\bar{y}' = \frac{3m-8M}{8(M+m)}a$$



- NOW $|OG| = \frac{1}{4}a$ (SIMILAR TRIANGLES)

$$\Rightarrow \bar{y}' \leqslant \frac{1}{4}a$$

$$\Rightarrow \frac{3m-8M}{8(M+m)}\cancel{a} \leqslant \frac{1}{4}\cancel{a}$$

$$\Rightarrow 3m - 8M \leqslant 2M + 2m$$

$$\Rightarrow m \leqslant 10M$$

AS REQUIRED

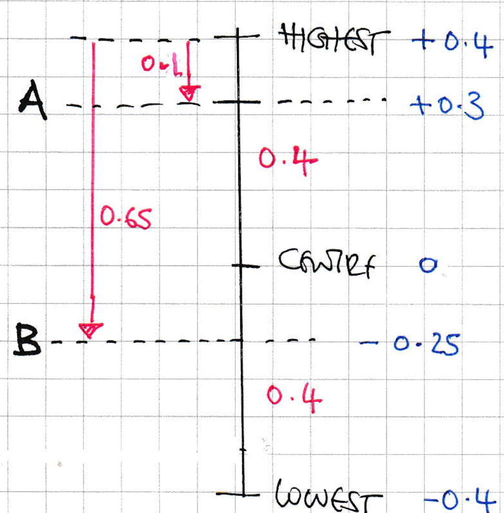
IYGB – FM2 PAPER M – QUESTION 2

a) STARTING WITH A DIAGRAM

- AMPLITUDE $a = 0.4$
- PERIOD $= 2 \times 2.5 = 5$

$$\frac{2\pi}{\omega} = 5$$

$$\omega = \frac{2\pi}{5}$$



USING $v^2 = \omega^2(a^2 - x^2)$

$$v^2 = \left(\frac{2\pi}{5}\right)^2 \left(0.4^2 - 0.3^2\right)$$

$$v^2 = 0.1105\ldots$$

$$|v| \simeq 0.332 \text{ ms}^{-1}$$

b) USING $x = a\cos\omega t$, WITH $t=0$ AT THE HIGHEST POINT

- UP TO "A"

$$0.3 = 0.4\cos\left(\frac{2\pi t}{5}\right)$$

$$0.75 = \cos\left(\frac{2\pi t}{5}\right)$$

$$\frac{2\pi t}{5} \approx 0.7227\ldots$$

$$t_1 \approx 0.57513\ldots$$

- UP TO "B"

$$-0.25 = 0.4\cos\left(\frac{2\pi t}{5}\right)$$

$$-0.625 = \cos\left(\frac{2\pi t}{5}\right)$$

$$\frac{2\pi t}{5} = 2.2459\ldots$$

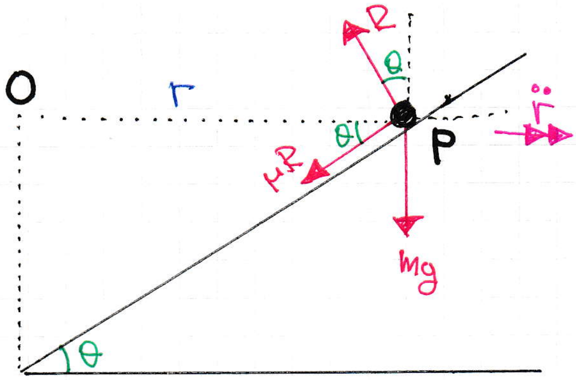
$$t_2 \simeq 1.78725\ldots$$

$\therefore$ REQUIRED TIME $= t_2 - t_1 = 1.78725\ldots - 0.57513\ldots$

$$\simeq 1.21$$

IYGB - FM2 PAPER M - QUESTION 3

- STARTING WITH A CROSS-SECTIONAL DIAGRAM, IN THE LIMITING POSITION, IE THE CAR GOING SO FAST SO IT IS AT THE POINT OF SLIPPING OUT OF THE BEND



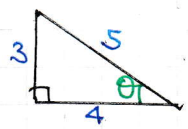
- RESOLVING THE FORCES

$(\uparrow)$: $R\cos\theta = \mu R\sin\theta + mg$ (EQUILIBRIUM)

$(\rightarrow)$: $m\ddot{r} = -R\sin\theta - \mu R\cos\theta$ ($F=ma$)

$\mu = 0.625$

$r = 25$ m



$\tan\theta = \frac{3}{4}$

$\sin\theta = \frac{3}{5}$

$\cos\theta = \frac{4}{5}$

- DIVIDING THE EQUATIONS AFTER REARRANGING, TO ELIMINATE R

$$\Longrightarrow \frac{\cancel{m}\ddot{r}}{\cancel{m}g} = \frac{-\cancel{R}\sin\theta - \mu\cancel{R}\cos\theta}{\cancel{R}\cos\theta - \mu\cancel{R}\sin\theta}$$

$$\Longrightarrow \frac{-\frac{v^2}{r}}{g} = \frac{-\sin\theta - \mu\cos\theta}{\cos\theta - \mu\sin\theta}$$

$$\Longrightarrow \frac{v^2}{rg} = \frac{\sin\theta + \mu\cos\theta}{\cos\theta - \mu\sin\theta}$$

$$\Longrightarrow \frac{v^2}{25\times 9.8} = \frac{\frac{3}{5} + 0.625\times\frac{4}{5}}{\frac{4}{5} - 0.625\times\frac{3}{5}}$$

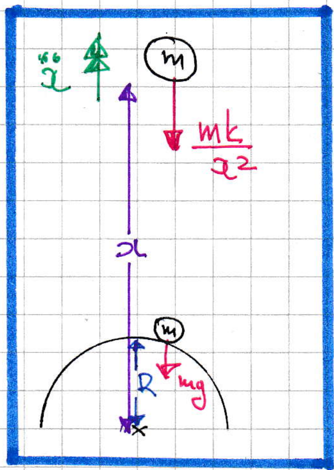
$$\Longrightarrow \frac{v^2}{245} = \frac{11/10}{17/40}$$

$$\Longrightarrow v^2 = \frac{10780}{17}$$

$$\Longrightarrow |v| \approx 25.18 \text{ ms}^{-1}$$

IYGB - FM2 PAPER M - QUESTION 4

STARTING WITH A DIAGRAM



ON EARTH'S SURFACE, $x = R$

$$\frac{mk}{R^2} = mg$$

$$k = gR^2$$

NEXT THE EQUATION OF MOTION

$$\Rightarrow \cancel{m}\ddot{x} = -\frac{\cancel{m}k}{x^2}$$

$$\Rightarrow \ddot{x} = -\frac{gR^2}{x^2}$$

$$\Rightarrow v\frac{dv}{dx} = -\frac{gR^2}{x^2}$$

$$\Rightarrow v\,dv = -\frac{gR^2}{x^2}$$

INTEGRATE SUBJECT TO CONDITION $x = R$, $v = U$

$$\Rightarrow \int_{v=U}^{v=V} v\,dv = \int_{x=R}^{x=2R} -\frac{gR^2}{x^2}\,dx$$

$$\Rightarrow \left[\tfrac{1}{2}v^2\right]_{v=U}^{v=V} = \left[\frac{gR^2}{x}\right]_{x=R}^{x=2R}$$

$$\Rightarrow \tfrac{1}{2}V^2 - \tfrac{1}{2}U^2 = \frac{gR^{\cancel{2}}}{2\cancel{R}} - \frac{gR^{\cancel{2}}}{\cancel{R}}$$

$$\Rightarrow \tfrac{1}{2}V^2 - \tfrac{1}{2}U^2 = -\tfrac{1}{2}gR$$

$$\Rightarrow V^2 - U^2 = -gR$$

$$\Rightarrow V^2 = U^2 - gR$$

IYGB – FM2 PAPER M – QUESTION 4

NOW CONSIDER THE KINETIC ENERGIES

INITIAL KINETIC ENERGY $= \frac{1}{2}mU^2$

"FINAL" KINETIC ENERGY $= \frac{1}{2}mV^2 = \frac{1}{2}m(U^2 - gR)$

$$\Rightarrow \frac{1}{2} \times \frac{1}{2}\cancel{m}U^2 = \frac{1}{2}\cancel{m}(U^2 - gR)$$

$$\Rightarrow U^2 = 2(U^2 - gR)$$

$$\Rightarrow U^2 = 2U^2 - 2gR$$

$$\Rightarrow U^2 = 2gR$$

$$\Rightarrow gR = \frac{1}{2}U^2$$

$$\Rightarrow gR^2 = \frac{1}{2}RU^2$$

$$\Rightarrow k = \frac{1}{2}RU^2$$

IYGB - FM2 PAPER M - QUESTION 5

$$\ddot{x} = \frac{0.5}{v+3} \qquad t=0,\ x=0,\ \dot{x}=v=0$$

START FROM OBTAINING A RELATIONSHIP BETWEEN v & t

$$\Rightarrow \frac{dv}{dt} = \frac{0.5}{v+3}$$

$$\Rightarrow (v+3)\,dv = \tfrac{1}{2}\,dt$$

$$\Rightarrow \int_{v=0}^{v} v+3 \; dv = \int_{t=0}^{t} \tfrac{1}{2}\,dt$$

$$\Rightarrow \left[\tfrac{1}{2}v^2+3v\right]_0^v = \left[\tfrac{1}{2}t\right]_0^t$$

$$\Rightarrow \tfrac{1}{2}v^2+3v = \tfrac{1}{2}t$$

$$\Rightarrow v^2+6v = t$$

COMPLETING THE SQUARE TO COMPLETE THE REARRANGEMENT

$$\Rightarrow v^2+6v+9 = t+9$$

$$\Rightarrow (v+3)^2 = t+9$$

$$\Rightarrow v+3 = \pm\sqrt{t+9}$$

$$\Rightarrow v = -3+\sqrt{t+9} \qquad v \geqslant 0 \text{ FOR } t \geqslant 0$$

$$\Rightarrow \frac{dx}{dt} = -3+(t+9)^{\frac{1}{2}}$$

$$\Rightarrow \int_{x=0}^{x} 1\,dx = \int_{t=0}^{7} -3+(t+9)^{\frac{1}{2}}$$

IYGB – FM2 PAPER M – QUESTION 5

$$\Longrightarrow \left[x\right]_0^x = \left[-3t + \tfrac{2}{3}(t+9)^{\frac{3}{2}}\right]_0^7$$

$$\Longrightarrow x = \left(-21 + \tfrac{2}{3}\times 64\right) - \left(0 + \tfrac{2}{3}\times 27\right)$$

$$\Longrightarrow x = -21 + \frac{128}{3} - 18$$

$$\Longrightarrow x = \frac{128}{3} - 39 = \frac{128}{3} - \frac{117}{3} = \frac{11}{3}$$

ALTERNATIVE

$$\boxed{\text{AFTER USING } \frac{dv}{dt} = \frac{0.5}{v+3} \text{ \& OBTAINING } v^2+6v = t}$$

$$\Longrightarrow v\frac{dv}{dx} = \frac{0.5}{v+3}$$

$$\Longrightarrow (v^2+3v)\,dv = 0.5\,dx$$

$$\Longrightarrow \int_{v=0}^{v} v^2+3v \; dv = \int_{x=0}^{x} 0.5\,dx$$

$$\Longrightarrow \left[\tfrac{1}{3}v^3 + \tfrac{3}{2}v^2\right]_0^v = \left[\tfrac{1}{2}x\right]_0^x$$

$$\Longrightarrow \tfrac{1}{3}v^3 + \tfrac{3}{2}v^2 = \tfrac{1}{2}x$$

$$\Longrightarrow x = \tfrac{2}{3}v^3 + 3v^2$$

NOW USING $t=7$

$$\Longrightarrow v^2+6v = 7$$

$$\Longrightarrow v^2+6v-7 = 0$$

$$\Longrightarrow (v-1)(v+7) = 0$$

$$\Longrightarrow v = \begin{cases} 1 \\ \cancel{-7} \end{cases}$$

HENCE WE FINALLY HAVE

$$x = \tfrac{2}{3}v^3 + 3v^2$$

$$x = \tfrac{2}{3} + 3$$

$$x = \frac{11}{3}$$

AS BEFORE.

IYGB - FM2 PAPER M - QUESTION 6

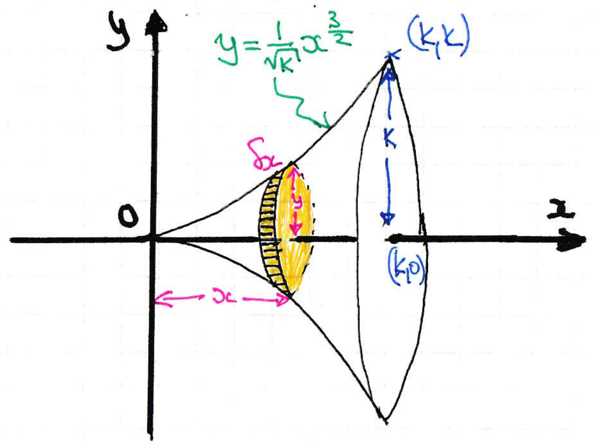
- START BY FINDING THE VOLUME OF REVOLUTION

$$V = \pi \int_{x_1}^{x_2} (y(x))^2 \, dx = \pi \int_0^k \left(\frac{1}{\sqrt{k}} x^{\frac{3}{2}}\right)^2 dx$$

$$V = \pi \int_0^k \frac{x^3}{k} \, dx = \frac{\pi}{4k} \left[x^4\right]_0^k$$

$$V = \frac{\pi}{4k}\left[k^4 - 0\right] = \frac{1}{4}\pi k^3$$

- NEXT LOOKING AT THE DIAGRAM BELOW



- THE MASS OF THE INFINITESIMAL DISC OF THICKNESS δx IS

$$\delta m = \rho \pi y^2 \delta x \qquad (\rho = \text{DENSITY})$$

- THE "MOMENT" OF THE INFINITESIMAL DISC ABOUT THE y AXIS IS GIVEN BY

$$(\rho \pi y^2 \delta x) x = \rho \pi x y^2 \delta x$$

- SUMMING UP, TAKING LIMITS AND CARRYING OUT THE RESULTING INTEGRATIONS

$$\Longrightarrow \; M\bar{x} = \int_{x=0}^{x=k} \rho \pi x y^2 \, dx$$

$$\Longrightarrow \; \left(\frac{1}{4}\pi k^3 \rho\right)\bar{x} = \rho\pi \int_{x=0}^{x=k} x \left(\frac{1}{\sqrt{k}} x^{\frac{3}{2}}\right)^2 dx$$

$$\Longrightarrow \; \frac{1}{4}k^3 \bar{x} = \int_{x=0}^{x=k} \frac{1}{k} x^4 \, dx$$

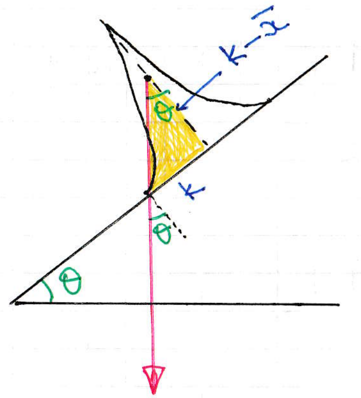
IYGB - FM2 PAPER M - QUESTION 6

$$\Rightarrow \tfrac{1}{4}k^3\bar{x} = \frac{1}{5k}\left[x^5\right]_0^k$$

$$\Rightarrow \tfrac{1}{4}k^3\bar{x} = \tfrac{1}{5}k^4$$

$$\Rightarrow \bar{x} = \tfrac{4}{5}k$$

- FINALLY DRAWING THE SOLID ON THE INCLINED PLANE



$$\tan\theta = \frac{k}{k-\bar{x}}$$

$$\tan\theta = \frac{k}{k-\frac{4}{5}k}$$

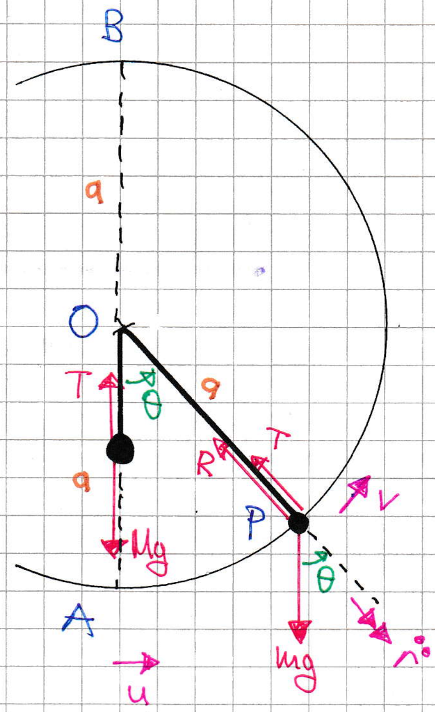
$$\tan\theta = \frac{1}{1-\frac{4}{5}}$$

$$\tan\theta = \frac{1}{\frac{1}{5}}$$

$$\tan\theta = 5$$

IYGB - FM2 PAPER M - QUESTION 7

STARTING WITH A DETAILED DIAGRAM



BY ENERGIES TAKING THE LEVEL OF "O" AS THE ZERO POTENTIAL LEVEL

$$\Rightarrow KE_A + PE_A = KE_\theta + PE_\theta$$

$$\Rightarrow \tfrac{1}{2}mu^2 - mga = \tfrac{1}{2}mv^2 - mg(a\cos\theta)$$

$$\Rightarrow u^2 - 2ag = v^2 - 2ag\cos\theta$$

$$\Rightarrow v^2 = u^2 - 2ag + 2ag\cos\theta$$

AT THE TOP, $V = 12ag$ WITH $\theta = \pi$

$$\Rightarrow 12ag = u^2 - 2ag + 2ag\cos\pi$$

$$\Rightarrow 16ag = u^2$$

$$\Rightarrow u = 4\sqrt{ag}$$

NEXT WE OBTAIN THE RADIAL EQUATION OF MOTION

$$\Rightarrow m\ddot{r} = mg\cos\theta - T - R$$

$$\Rightarrow m\left(-\frac{v^2}{a}\right) = mg\cos\theta - Mg - R$$

$$\Rightarrow R = \frac{m}{a}v^2 + mg\cos\theta - Mg$$

$$\Rightarrow R = \frac{m}{a}\left[u^2 - 2ag + 2ag\cos\theta\right] + mg\cos\theta - Mg$$

IYGB – FM2 PAPER M – QUESTION 7

$$\Rightarrow R = \frac{m}{a}\left(16ag - 2ag + 2ag\cos\theta\right) + mg\cos\theta - Mg$$

$$\Rightarrow R = 14mg + 3mg\cos\theta - Mg$$

NOW WE HAVE $R = 0$

$$\Rightarrow 0 = 14mg + 3mg\cos\theta - Mg$$

$$\Rightarrow 0 = 14m + 3m\cos\theta - M$$

$$\Rightarrow M - 14m = 3m\cos\theta$$

$$\Rightarrow \cos\theta = \frac{M - 14m}{3m}$$

BUT $-1 \leqslant \cos\theta \leqslant 1$

$$\Rightarrow -1 \leqslant \frac{M - 14m}{3m} \leqslant 1$$

$$\Rightarrow -3m \leqslant M - 14m \leqslant 3m$$

$$\Rightarrow 11m \leqslant M \leqslant 17m$$