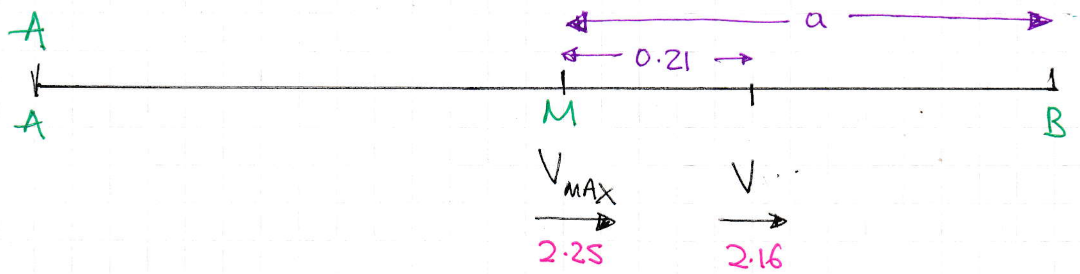
IYGB - FM2 PAPER N - QUESTION 1

- START WITH A STANDARD DIAGRAM FOR S.H.M KINEMATICS



- $V_{MAX} = a\omega$

$2.25 = a\omega$

- WHEN $x = 0.21$, $V = 2.16$

$\Rightarrow V^2 = \omega^2(a^2 - x^2)$

$\Rightarrow \quad = \omega^2 a^2 - \omega^2 x^2$

$\Rightarrow 2.16^2 = 2.25^2 - \omega^2 \times 0.21^2$

$\Rightarrow 4.6656 = 5.0625 - 0.0441\omega^2$

$\Rightarrow 0.0441\omega^2 = 0.3969$

$\Rightarrow \omega^2 = 9$

$\Rightarrow \omega = +3$

- HENCE WE HAVE

$\Rightarrow a\omega = 2.25$

$\Rightarrow a \times 3 = 2.25$

$\Rightarrow a = 0.75$

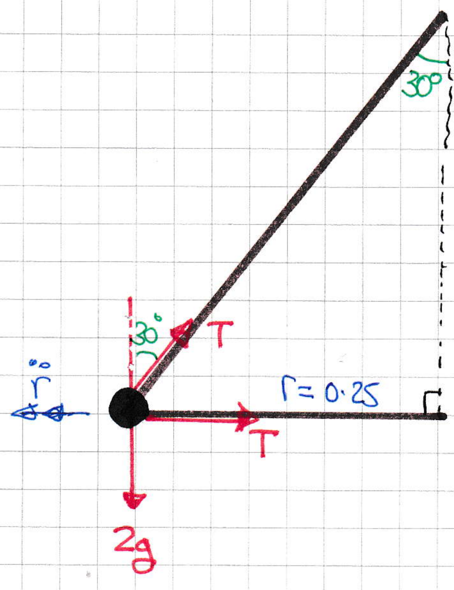
$\Rightarrow |AB| = 1.5 \text{ m}$

IYGB - FM2 PAPER N - QUESTION 2

RESOLVING FORCES VERTICALLY & HORIZONTALLY

$(\uparrow)$: $T\cos 30 = 2g$ (EQUILIBRIUM)

$(\rightarrow)$ $m\ddot{r} = -T - T\sin 30$ ("F = ma")



SIMPLIFYING THE EQUATIONS

$$\left.\begin{aligned} \frac{\sqrt{3}}{2}T &= 2g \\ \left(-\frac{v^2}{r}\right) &= -T - \frac{1}{2}T \end{aligned}\right\} \Longrightarrow$$

$$\left.\begin{aligned} T &= \frac{4g}{\sqrt{3}} \\ \frac{2v^2}{0.25} &= \frac{3}{2}T \end{aligned}\right\} \Longrightarrow$$

$$\Longrightarrow 8v^2 = \frac{3}{2}\left(\frac{4g}{\sqrt{3}}\right)$$

$$\Longrightarrow v^2 = \frac{\sqrt{3}}{2}g$$

$$\Longrightarrow v \approx 2.91 \text{ ms}^{-1}$$

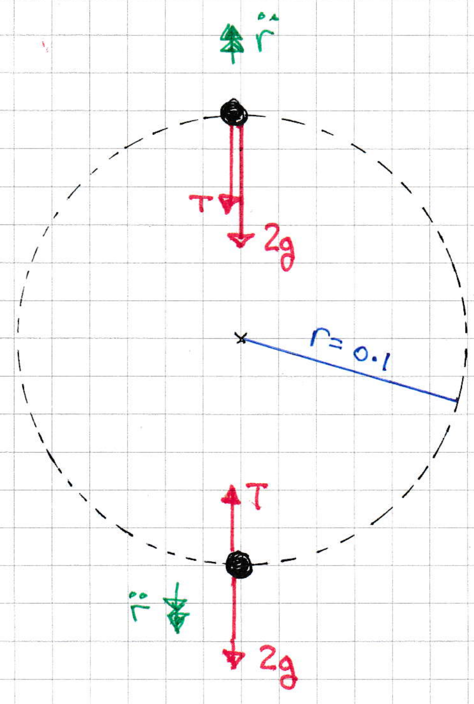
IYGB - FM2 PAPER N - QUESTION 3

START BY CONVERTING THE ANGULAR SPEED INTO SI UNITS

$$\omega = 750 \text{ REVS / MINUTE}$$

$$\omega = \frac{750 \times 2\pi}{60} = 25\pi \text{ rad s}^{-1}$$

NOW LOOKING AT THE EQUATION OF RADIAL MOTION AT THE HIGHEST AND LOWEST POINT OF THE PATH



AT THE TOP

$$\Rightarrow m\ddot{r} = -T - 2g$$

$$\Rightarrow 2(-\omega^2 r) = -T - 2g$$

$$\Rightarrow T = 2\omega^2 r - 2g$$

$$\Rightarrow T = 2(25\pi)^2(0.1) - 2g$$

$$\Rightarrow T \approx 1214 \text{ N}$$

(4 sf)

AT THE BOTTOM

$$\Rightarrow m\ddot{r} = 2g - T$$

$$\Rightarrow 2(-\omega^2 r) = 2g - T$$

$$\Rightarrow T = 2g + 2\omega^2 r$$

$$\Rightarrow T = 2g + 2(25\pi)^2(0.1)$$

$$\Rightarrow T \approx 1253 \text{ N}$$

4 sf

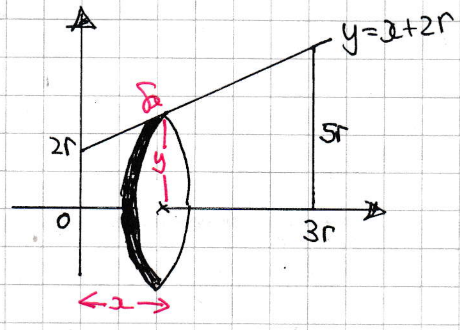
IYGB - FM2 PAPER N - QUESTION 4

a) SET UP THE VOLUME OF REVOLUTION

$$V = \pi \int_{x_1}^{x_2} y(x)\, dx = \pi \int_0^{3r} (x+2r)^2\, dx = \pi \times \frac{1}{3}\left[(x+2r)^3\right]_0^{3r}$$

$$= \frac{\pi}{3}\left[125r^3 - 8r^3\right] = 39\pi r^3$$

NEXT LOOKING AT THE DIAGRAM



- ρ = MASS PER UNIT VOLUME (DENSITY)
- MASS OF INFINITESIMAL DISC IS $\pi y^2 \delta x \rho$
- TAKING MOMENTS ABOUT THE y AXIS, SUMMING UP & TAKING LIMITS GIVES

$$\Rightarrow M\bar{x} = \int_{x=0}^{3r} \pi \rho y^2 x\, dx$$

$$\Rightarrow (39r^3\pi\rho)\bar{x} = \pi\rho \int_0^{3r} (x+2r)^2 x\, dx$$

$$\Rightarrow 39r^3\bar{x} = \int_0^{3r} x^3 + 4x^2r + 4xr^2\, dx$$

$$\Rightarrow 39r^3\bar{x} = \left[\frac{1}{4}x^4 + \frac{4}{3}rx^3 + 2x^2r^2\right]_0^{3r}$$

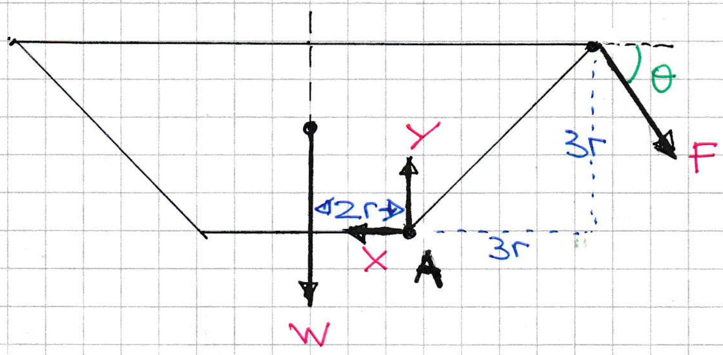
$$\Rightarrow 39r^3\bar{x} = \left(\frac{81}{4}r^4 + 36r^4 + 18r^4\right) - (0)$$

$$\Rightarrow 39r^3\bar{x} = \frac{297}{4}r^4$$

$$\therefore \bar{x} = \frac{99}{52}r$$

IYGB – FM2 PAPER N – QUESTION 4

b) LOOKING AT THE DIAGRAM



TAKING MOMENTS ABOUT A

$$W \times 2r = F\sin\theta \times 3r + F\cos\theta \times 3r$$

$$2W = 3F(\sin\theta + \cos\theta)$$

$$F = \frac{2W}{3(\sin\theta + \cos\theta)}$$

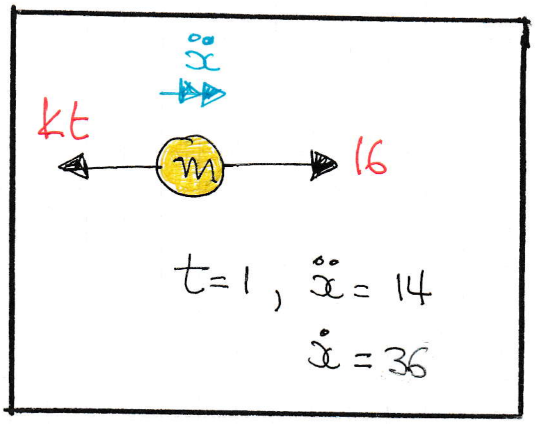
BY STANDARD "R-TRANSFORMATIONS", $\sin\theta + \cos\theta \equiv \sqrt{2}\sin(\theta + 45°)$

$$\Longrightarrow F = \frac{2W}{3\sqrt{2}\sin(\theta + 45°)}$$

LEAST VALUE OCCURS WHEN DENOMINATOR IS MAX, WHICH OCCURS WHEN $\theta = 45$, SO $\sin(\theta + 45) = 1$

$$\therefore F_{MIN} = \frac{2}{3\sqrt{2}}W = \frac{1}{3}\sqrt{2}W \quad \text{WHEN } \theta = \frac{\pi}{4} = 45°$$

IYGB – FM2 PAPER N – QUESTION 5



- WE HAVE "$F=ma$"

$$\Longrightarrow 16-kt=\frac{1}{2}\ddot{x}$$

- APPLY A CONDITION & FIND k

$$t=1,\ \ddot{x}=14$$

$$16-(k\times 1)=\frac{1}{2}(14)$$

$$16-k=7$$

$$k=9$$

$$\therefore\ \frac{1}{2}\ddot{x}=16-9t$$

$$\boxed{\ddot{x}=32-18t}$$

- SOLVE THE O.D.E, BY SEPARATION OF VARIABLES AND SUBJECT TO THE CONDITIONS GIVEN

$$\Longrightarrow \frac{dv}{dt}=32-18t$$

$$\Longrightarrow \int_{v=36}^{v=28} 1\,dv=\int_{t=1}^{t} 32-18t\,dt$$

$$\Longrightarrow \Big[v\Big]_{36}^{28}=\Big[32t-9t^2\Big]_1^t$$

$$\Longrightarrow 28-36=(32t-9t^2)-(32-9)$$

$$\Longrightarrow -8=32t-9t^2-23$$

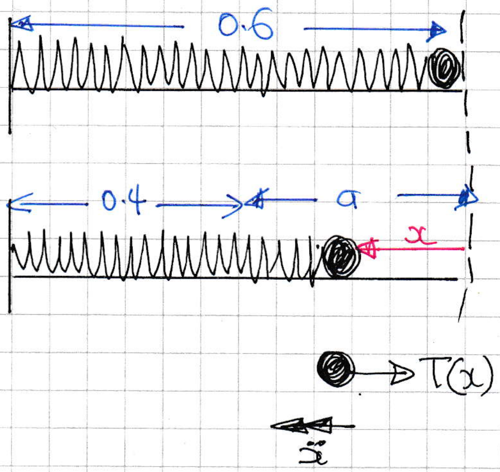
$$\Longrightarrow 9t^2-32t+15=0$$

$$\Longrightarrow (t-3)(9t-5)=0$$

$$\therefore\ t=\begin{cases}3\\ 5/9\end{cases}$$

IYGB — FM2 PAPER N — QUESTION 6

a) LOOKING AT THE DIAGRAM(S)



$\lambda = 168$
$l = 0.6$
$m = 0.7$

EQUATION OF MOTION IS

$$m\ddot{x} = -T(x)$$
$$m\ddot{x} = -\frac{\lambda}{l}x$$
$$\ddot{x} = -\frac{\lambda}{ml}x$$
$$\ddot{x} = -\frac{168}{0.7 \times 0.6}x$$
$$\ddot{x} = -400x$$

∴ S.H.M ABOUT EQUILIBRIUM POSITION WITH $\omega^2 = 400$

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{20} = \frac{\pi}{10}$$

$$T \simeq 0.314\,s$$

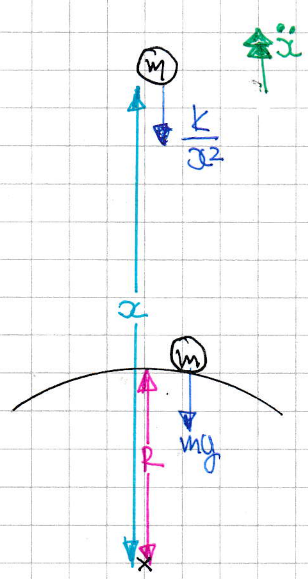
b) FINALLY THE AMPLITUDE OF THE MOTION FROM THE DIAGRAM

$$a = 0.6 - 0.4 = 0.2$$

$$|v|_{MAX} = a\omega = 0.2 \times 20 = 4\ ms^{-1}$$

IYGB - FM2 PAPER N - QUESTION 7

START WITH A DIAGRAM & CONSIDER THE PARTICLE ON THE SURFACE OF THE EARTH, IN ORDER TO GET AN EXPRESSION FOR THE PROPORTIONALITY CONSTANT



ON EARTH'S SURFACE $x = R$

$$mg = \frac{k}{R^2}$$
$$k = mgR^2$$

NOW LOOKING AT THE EQUATION OF MOTION IN GENERAL

$$\Rightarrow m\ddot{x} = -\frac{k}{x^2}$$

$$\Rightarrow \cancel{m} v\frac{dv}{dx} = -\frac{\cancel{m}gR^2}{x^2}$$

$$\Rightarrow v\,dv = -\frac{gR^2}{x^2}\,dx$$

INTEGRATE SUBJECT TO THE CONDITION $x = R$, $v = \sqrt{\frac{3}{2}gR}$

$$\Rightarrow \int_{v=\sqrt{\frac{3}{2}gR}}^{v=V} v\,dv = \int_{x=R}^{x=3R} -\frac{gR^2}{x^2}\,dx$$

$$\Rightarrow \left[\frac{1}{2}v^2\right]_{v=\sqrt{\frac{3}{2}gR}}^{v=V} = \left[\frac{gR^2}{x}\right]_{x=R}^{x=3R}$$

$$\Rightarrow \frac{1}{2}V^2 - \frac{1}{2}\left(\frac{3}{2}gR\right) = gR^2\left[\frac{1}{3R} - \frac{1}{R}\right]$$

IYGB - FM2 PAPER N - QUESTION 7

$$\Longrightarrow \frac{1}{2}V^2 - \frac{3}{4}gR = gR^2\left(-\frac{2}{3R}\right)$$

$$\Longrightarrow \frac{1}{2}V^2 - \frac{3}{4}gR = -\frac{2}{3}gR$$

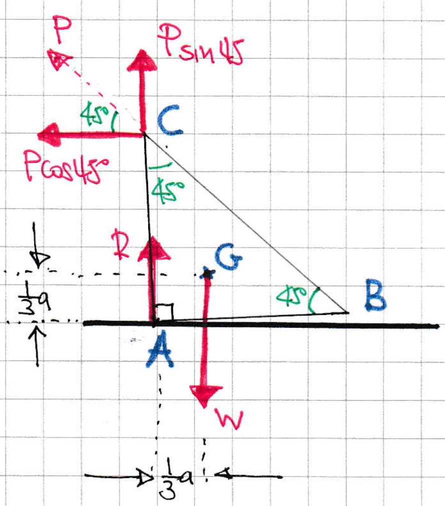
$$\Longrightarrow \frac{1}{2}V^2 = \frac{1}{12}gR$$

$$\Longrightarrow V^2 = \frac{1}{6}gR$$

$$\Longrightarrow |V| = \sqrt{\frac{1}{6}gR}$$

IYGB - FM2 PAPER N - QUESTION 8

START WITH A DIAGRAM FOR "TOPPLING"

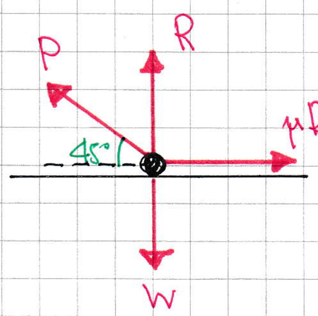


- LET $|AB| = |AC| = a$
- THEN THE LOCATION OF THE CENTRE OF MASS OF THE LAMINA WILL BE $\frac{1}{3}a$ FROM THE RIGHT ANGLE, ALONG AB AND ALONG AC
- RESOLVE P INTO COMPONENTS
- $\overset{\curvearrowright}{A}$: $(P\cos 45°) \times \cancel{a} = W \times \frac{1}{3}\cancel{a}$

$$\frac{1}{\sqrt{2}}P = \frac{1}{3}W$$

$$P = \frac{\sqrt{2}}{3}W$$

FOR SLIDING PURPOSES THE LAMINA CAN BE REDUCED TO A PARTICLE



$(\uparrow)$: $R + P\sin 45° = W$

$(\rightarrow)$: $P\cos 45° = \mu R$

BY SUBSTITUTION

$\Rightarrow P\cos 45° = \mu(W - P\sin 45°)$

$\Rightarrow P\cos 45° = \mu W - \mu P\sin 45°$

$\Rightarrow P\cos 45° + \mu P\sin 45° = \mu W$

$\Rightarrow P(\cos 45° + \mu\sin 45°) = \mu W$

$\Rightarrow P = \dfrac{\mu W}{\cos 45° + \mu\sin 45°}$

IYGB - FM2 PAPER N - QUESTION 8

$$\Longrightarrow P = \frac{\mu W}{\frac{1}{\sqrt{2}} + \mu \times \frac{1}{\sqrt{2}}}$$

MULTIPLY TOP & BOTTOM OF THE FRACTION BY $\sqrt{2}$

$$\Longrightarrow P = \frac{\mu W \sqrt{2}}{1+\mu}$$

FINALLY THE LAMINA SLIDES BEFORE IT TOPPLES

$$\Longrightarrow P_{\text{SLIDE}} < P_{\text{TOPPLE}}$$

$$\Longrightarrow \frac{\mu W \sqrt{2}}{1+\mu} < \frac{\sqrt{2}}{3} W$$

$$\Longrightarrow \frac{\mu}{1+\mu} < \frac{1}{3}$$

$$\Longrightarrow 3\mu < 1 + \mu \qquad (\mu + 1 > 0)$$

$$\Longrightarrow 2\mu < 1$$

$$\Longrightarrow \mu < \frac{1}{2}$$

$$\therefore \quad 0 < \mu < \frac{1}{2}$$

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