IYGB GCE

Mathematics FP1

Advanced Level

Practice Paper L

Difficulty Rating: 3.6267/1.6854

Time: 1 hour 30 minutes

Candidates may use any calculator allowed by the regulations of this examination.

Information for Candidates

This practice paper follows closely the Pearson Edexcel Syllabus, suitable for first assessment Summer 2018.

The standard booklet "Mathematical Formulae and Statistical Tables" may be used.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 9 questions in this question paper.
The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
Non exact answers should be given to an appropriate degree of accuracy.
The examiner may refuse to mark any parts of questions if deemed not to be legible.

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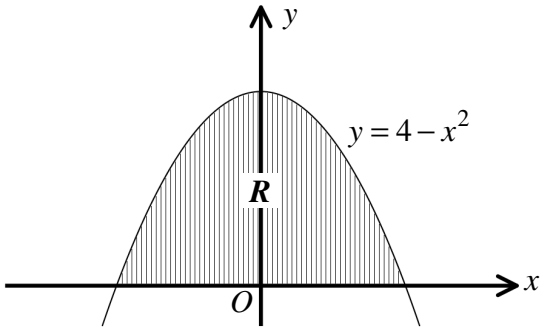
Question 1



The figure above shows the graph of the curve with equation

$$y = 4 - x^2 .$$

The shaded region R, is bounded by the curve and the x axis.

The region R is rotated through 2π radians about the x axis to form a solid of revolution.

Show that the volume of the solid is $\dfrac{256\pi}{15}$. **(4)**

Question 2

Use standard results on summations to find the value of the following sum.

$$\sum_{k=2}^{16} \left[(k-1)(k+2) \right] . \qquad \textbf{(7)}$$

Question 3

The 3×3 matrix $\mathbf{A}$ is defined by

$$\mathbf{A}=\begin{pmatrix}0 & 0 & -1\\ 0 & 1 & 0\\ 1 & 0 & 0\end{pmatrix}.$$

a) Describe geometrically the transformation given by $\mathbf{A}$. **(2)**

The 3×3 matrix $\mathbf{B}$ represents a rotation of $180°$ about the line $x=z$, $y=0$.

b) Determine the elements of $\mathbf{B}$. **(3)**

The 3×3 matrix $\mathbf{C}$ is represents the transformation defined by $\mathbf{B}$, followed by the transformation defined by $\mathbf{A}$.

c) Describe geometrically the transformation represented by $\mathbf{C}$. **(5)**

Question 4

Solve the following quadratic equation

$$z^2-z+8+2(z+1)\mathrm{i}=0,\quad z\in\mathbb{C}.$$

Give the answers in the form $a+b\mathrm{i}$, $a\in\mathbb{R}$, $b\in\mathbb{R}$. **(10)**

Question 5

The 2×2 matrix $\mathbf{A}=\begin{pmatrix}2 & 1\\ -6 & 3\end{pmatrix}$ is given.

Under the transformation represented by $\mathbf{A}$, a straight line passing through the origin is reflected about the y axis.

Determine the possible equations of this line. **(6)**

Question 6

Sketch on a standard Argand diagram the locus of the points $z=\sqrt{2}(1+\mathrm{i})$, $w=\sqrt{3}-\mathrm{i}$ and $z+w$, and use geometry to prove that

$$\tan\left(\frac{\pi}{24}\right)=\sqrt{6}-\sqrt{3}+\sqrt{2}-2.$$

You **must justify all the steps in this proof.** **(9)**

Question 7

A cubic equation is given below as

$$ax^3+bx^2+cx+d=0,$$

where a, b, c and d are non zero constants.

Given that the product of two of the three roots of above cubic equation is 1, show that

$$a^2-d^2=ac-bd\ .$$ **(6)**

Question 8

Prove by mathematical induction that if n is a positive integer then

$$\sum_{r=1}^{n}\frac{3r+2}{r(r+1)(r+2)}=\frac{n(2n+3)}{(n+1)(n+2)}.$$

You may not use other methods of proof in this question. **(10)**

Question 9

The straight lines l_1 and l_2 have respective vector equations

$$\mathbf{r}_1 = 5\mathbf{i}+3\mathbf{j}+\mathbf{k}+\lambda(\mathbf{i}+\mathbf{j}+\mathbf{k}) \quad \text{and} \quad \mathbf{r}_2 = -3\mathbf{i}+4\mathbf{j}+8\mathbf{k}+\mu(2\mathbf{i}-\mathbf{j}-3\mathbf{k}),$$

where λ and μ are scalar parameters.

a) Show that l_1 and l_2 intersect at some point P, further finding its coordinates. **(5)**

b) Calculate the acute angle between l_1 and l_2. **(3)**

The point $A(7,5,3)$ lies on l_1 and the point B lies on l_2, such that the straight line AB is perpendicular to l_2.

c) Determine the area of the triangle ABP. **(5)**