IYGB GCE

Mathematics FP1

Advanced Level

Practice Paper W

Difficulty Rating: 4.2267/2.2556

Time: 1 hour 30 minutes

Candidates may use any calculator allowed by the regulations of this examination.

Information for Candidates

This practice paper follows closely the Pearson Edexcel Syllabus, suitable for first assessment Summer 2018.

The standard booklet "Mathematical Formulae and Statistical Tables" may be used.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 10 questions in this question paper.
The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
Non exact answers should be given to an appropriate degree of accuracy.
The examiner may refuse to mark any parts of questions if deemed not to be legible.

Question 1

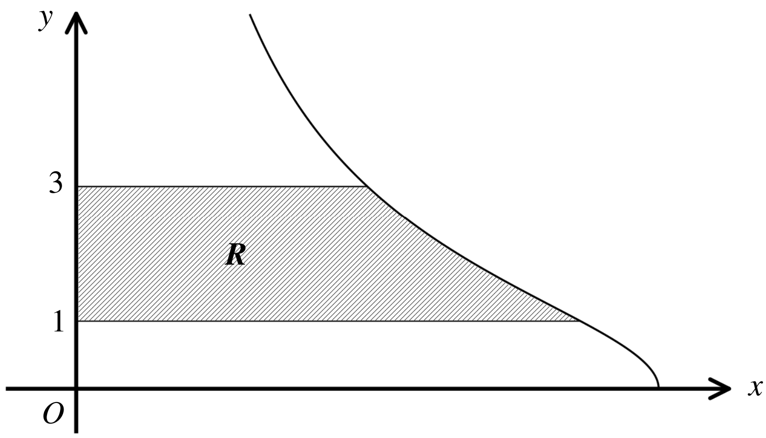
The quadratic equation

$$z^2 - 2z + 1 - 2\mathrm{i} = 0, \; c \in \mathbb{R},$$

has a solution $z = -\mathrm{i}$.

Find the other solution. **(3)**

Question 2



The figure above shows the curve with parametric equations

$$x = 2\cos^2\theta, \quad y = \sqrt{3}\tan\theta, \quad 0 \le \theta < \frac{\pi}{2}.$$

The finite region R shown shaded in the figure, bounded by the curve, the y axis, and the straight lines with equations $y = 1$ and $y = 3$.

Use integration in parametric to show that the volume of the solid formed when R is fully revolved about the y axis is $\frac{\pi^2}{\sqrt{3}}$. **(5)**

Question 3

It is given that

$$\mathbf{w} = \mathbf{u} + \mathbf{v},$$

where $\mathbf{w} = 2\mathbf{i} + 8\mathbf{j} - \mathbf{k}$.

Given further that $\mathbf{u}$ is in the direction $\mathbf{i} + \mathbf{j} + \mathbf{k}$, and the vectors $\mathbf{u}$ and $\mathbf{v}$ are perpendicular to one another, determine $\mathbf{u}$ and $\mathbf{v}$ in component form. **(5)**

Question 4

Prove by induction that for all even natural numbers n

$$\frac{d^n}{dx^n}(\sin 3x) = (-1)^{\frac{n}{2}} \times 3^n \times \sin 3x. \qquad \mathbf{(6)}$$

Question 5

The skew straight lines L_1 and L_2 have vector equations

$$\mathbf{r}_1 = (-13\mathbf{j} + \mathbf{k}) + \lambda(-3\mathbf{i} + 4\mathbf{j} - 7\mathbf{k}),$$

$$\mathbf{r}_2 = (5\mathbf{i} + 25\mathbf{j}) + \mu(2\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}),$$

where λ and μ are scalar parameters.

a) **Without** using the vector product (cross product), find a vector which mutually perpendicular to L_1 and L_2. **(6)**

The point A lies on L_1 and the point B lies on L_2.

b) Given that the distance AB is least, determine the coordinates of A and B. **(8)**

Question 6

The 2×2 matrix $\mathbf{A}$ is given below.

$$\mathbf{A}=\begin{pmatrix}3 & 2\\ 7 & 5\end{pmatrix}.$$

a) Find scalar constants, k and h, so that

$$\mathbf{A}^2+k\mathbf{I}=h\mathbf{A}.$$ **(4)**

b) Use part **(a)** to determine $\mathbf{A}^{-1}$, the inverse of $\mathbf{A}$.

No credit will be given for finding $\mathbf{A}^{-1}$ by a direct method. **(4)**

Question 7

The complex number z satisfies the relationship

$$z+\frac{1}{z}=-1,\ z\neq 0.$$

Show clearly that ….

a) … $z^3=1$. **(4)**

b) … $z^8+z^4=-1$. **(4)**

Question 8

By using an algebraic method, find the value of

$$99^2-97^2+95^2-93^2+...+3^2-1^2$$ **(6)**

Question 9

A plane Π is defined parametrically by

$$\mathbf{r} = \mathbf{i} + \mathbf{k} + \lambda(\mathbf{i} + \mathbf{j} - 2\mathbf{k}) + \mu(\mathbf{i} - \mathbf{j} + \mathbf{k}),$$

where λ and μ are a scalar parameters.

Determine a Cartesian equation for the transformation of Π under the matrix

$$\begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 2 & 1 & 0 \end{pmatrix}.$$ **(10)**

Question 10

The roots of the cubic equation

$$x^3 - 4x^2 - 3x - 2 = 0$$

are denoted in the usual notation by α, β and γ.

Show that the cubic equation whose roots are

$$\alpha + \beta, \quad \beta + \gamma \text{ and } \quad \gamma + \alpha,$$

is given by

$$x^3 - 8x^2 + 13x + 14 = 0$$ **(10)**
