IYGB GCE

Mathematics FP2

Advanced Level

Practice Paper J

Difficulty Rating: 3.5333/1.6216

Time: 1 hour 30 minutes

Candidates may use any calculator allowed by the regulations of this examination.

Information for Candidates

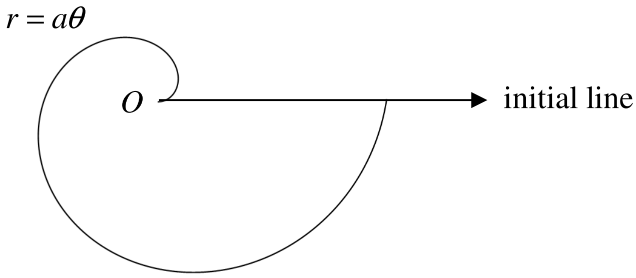
This practice paper follows closely the Pearson Edexcel Syllabus, suitable for first assessment Summer 2018.

The standard booklet "Mathematical Formulae and Statistical Tables" may be used.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 9 questions in this question paper.
The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
Non exact answers should be given to an appropriate degree of accuracy.
The examiner may refuse to mark any parts of questions if deemed not to be legible.

Question 1



The figure above shows a spiral curve with polar equation

$$r = a\theta, \ 0 \le \theta \le 2\pi,$$

where a is a positive constant.

Find the area of the finite region bounded by the spiral and the initial line. **(3)**

Question 2

$$f(x) \equiv \frac{4x}{1-x^4}.$$

a) Express $f(x)$ into partial fractions. **(5)**

b) Hence find, as a single natural logarithm, the value of

$$\int_0^{\frac{1}{2}} f(x)\ dx.$$ **(5)**

Question 3

$$f(r) = \frac{1}{\sqrt{r+2}+\sqrt{r}}, \ r \geq 0 .$$

a) Rationalize the denominator of $f(r)$. **(1)**

b) Find an expression for

$$\sum_{r=1}^{n} f(r) .$$ **(6)**

c) Show clearly that

$$\sum_{r=1}^{48} f(r) = 3 + 2\sqrt{2}$$ **(2)**

Question 4

The polar curve C has equation

$$r = 2(\cos\theta - \sin\theta), \ 0 \leq \theta < 2\pi .$$

Find a Cartesian equation for C and show it represents a circle, indicating its radius and the Cartesian coordinates of its centre. **(6)**

Question 5

Find the Maclaurin expansion of $\ln(2-e^x)$, up and including the term in x^3. **(8)**

Question 6

$$y = \operatorname{artanh} x, \ -1 < x < 1$$

a) By using the definitions of hyperbolic functions in terms of exponentials prove that

$$\operatorname{artanh} x = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right). \qquad \textbf{(4)}$$

b) Hence solve the equation

$$x = \tanh\left(\ln\sqrt{6x}\right). \qquad \textbf{(6)}$$

Question 7

A curve C, with equation $y = f(x)$, passes through the points with coordinates $(1,1)$ and $(2,k)$, where k is a constant.

Given further that the equation of C satisfies the differential equation

$$x^2\frac{dy}{dx} + xy(x+3) = 1,$$

determine the exact value of k. **(10)**

Question 8

The function f is defined as

$$f(x) \equiv \tanh^2 x, \ x \in \mathbb{R}, \ 0 \le x \le \ln 3.$$

Determine the mean value of f, in its entire domain. **(5)**

Question 9

It is given that

$$\cot 4\theta = \frac{\cot^4\theta - 6\cot^2\theta + 1}{4\cot^3\theta - 4\cot\theta}.$$

a) Use de Moivre's theorem to prove the validity of the above trigonometric identity. **(4)**

b) Deduce that $x = \cot^2\left(\frac{\pi}{8}\right)$ is one of the two solutions of the equation

$$x^2 - 6x + 1 = 0.$$ **(6)**

c) Show further that

$$\operatorname{cosec}^2\left(\frac{\pi}{8}\right) + \operatorname{cosec}^2\left(\frac{3\pi}{8}\right) = 8.$$ **(4)**