IYGB - FP2 PAPER L - QUESTION 1

a) STARTING FROM THE R.H.S

$$\cosh A\cosh B - \sinh A\sinh B = \tfrac{1}{2}(e^{A}+e^{-A})\times\tfrac{1}{2}(e^{B}+e^{-B}) - \tfrac{1}{2}(e^{A}-e^{-A})\times\tfrac{1}{2}(e^{B}-e^{-B})$$

$$= \tfrac{1}{4}(e^{A+B}+e^{A-B}+e^{-A+B}+e^{-A-B}) - \tfrac{1}{4}(e^{A+B}-e^{A-B}-e^{-A+B}+e^{-A-B})$$

$$= \tfrac{1}{4}\left[\cancel{e^{A+B}}+e^{A-B}+e^{-A+B}+\cancel{e^{-A-B}}-\cancel{e^{A+B}}+e^{A-B}+e^{-A+B}-\cancel{e^{-A-B}}\right]$$

$$= \tfrac{1}{2}\left[e^{A-B}+e^{-A+B}\right]$$

$$= \tfrac{1}{2}\left[e^{A-B}+e^{-(A-B)}\right]$$

$$= \cosh(A-B)$$

AS REQUIRED

b) USING PART (a)

$$\Rightarrow \cosh(x-\ln 3) = \sinh x$$

$$\Rightarrow \cosh x\cosh(\ln 3) - \sinh x\sinh(\ln 3) = \sinh x$$

$$\Rightarrow \cosh x\left[\tfrac{1}{2}e^{\ln 3}+\tfrac{1}{2}e^{-\ln 3}\right] - \sinh x\left[\tfrac{1}{2}e^{\ln 3}-\tfrac{1}{2}e^{-\ln 3}\right] = \sinh x$$

$$\Rightarrow \cosh x\left[\tfrac{3}{2}+\tfrac{1}{6}\right] - \sinh x\left[\tfrac{3}{2}-\tfrac{1}{6}\right] = \sinh x$$

$$\Rightarrow \tfrac{5}{3}\cosh x - \tfrac{4}{3}\sinh x = \sinh x$$

$$\Rightarrow 5\cosh x - 4\sinh x = 3\sinh x$$

DIVIDE BY $\cosh x$ TO CREATE $\tanh x$

$$\Rightarrow 5 - 4\tanh x = 3\tanh x$$

$$\Rightarrow \tanh x = \frac{5}{7}$$

$$\Rightarrow x = \operatorname{artanh}\left(\frac{5}{7}\right)$$

$$\operatorname{artanh} x \equiv \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right)$$

$$\Rightarrow x = \frac{1}{2}\ln\left(\frac{1+\frac{5}{7}}{1-\frac{5}{7}}\right)$$

$$\Rightarrow x = \frac{1}{2}\ln\left(\frac{7+5}{7-5}\right)$$

$$\therefore x = \tfrac{1}{2}\ln 6$$

IYGB - FP2 PAPER L - QUESTION 2

a) $f(x) = \ln(1+\cos 2x)$

$$f'(x) = \frac{1}{1+\cos 2x} \times (-2\sin 2x)$$

$$f'(x) = -\frac{2\sin 2x}{1+\cos 2x}$$

b) MANIPULATE ABOVE FIRST

$$f'(x) = -\frac{2(2\sin x\cos x)}{1+(2\cos^2 x-1)} = \frac{-4\sin x\cos x}{2\cos^2 x} = -2\tan x$$

NOW WE HAVE

$$\Rightarrow f''(x) = -2\sec^2 x = -2(1+\tan^2 x)$$

$$\Rightarrow f''(x) = -2-2\tan^2 x$$

$$\Rightarrow 2f''(x) = -4-4\tan^2 x$$

$$\Rightarrow 2f''(x) = -4-(-2\tan x)^2$$

$$\Rightarrow 2f''(x) = -4-(f'(x))^2$$

$$\Rightarrow f''(x) = -2-\frac{1}{2}(f'(x))^2$$

AS REQUIRED

c) USING PART (b)

$$f'''(x) = 0-(f'(x))\times f''(x) = -f'(x)f''(x)$$

PRODUCT RULE

$$f''''(x) = -f''(x)f''(x) - f'(x)f'''(x)$$

IYGB – FP2 PAPER L – QUESTION 2

EVALUATE AT $x=0$

$$f(0) = \ln(1+\cos 0) = \ln 2$$

$$f'(0) = -2\tan 0 = 0$$

$$f''(0) = -2 - \frac{1}{2}\left(f'(0)\right)^2 = -2$$

$$f'''(0) = -f'(0)f''(0) = 0$$

$$f''''(0) = -f''(0)f''(0) - f'(0)f'''(0) = -(-2)(-2) - 0 = -4$$

FINALLY WE HAVE

$$f(x) = f(0) + xf'(0) + \frac{x^2}{2!}f''(0) + \frac{x^3}{3!}f'''(0) + \frac{x^4}{4!}f''''(0) + O(x^5)$$

$$\ln(1+\cos 2x) = \ln 2 + 0 + \frac{1}{2}x^2(-2) + 0 + \frac{x^4}{24}(-4) + O(x^5)$$

$$\ln(1+\cos 2x) = \ln 2 - x^2 - \frac{1}{6}x^4 + O(x^5)$$

AS REQUIRED

IYGB - FP2 PAPER L - QUESTION 3

a) $$u_r - u_{r-1} = \frac{1}{6}r(r+1)(4r+11) - \frac{1}{6}(r-1)r[4(r-1)+11]$$

$$= \frac{1}{6}r(r+1)(4r+11) - \frac{1}{6}r(r-1)(4r+7)$$

$$= \frac{1}{6}r\left[(r+1)(4r+11) - (r-1)(4r+7)\right]$$

$$= \frac{1}{6}r\left[\cancel{4r^2}+15r+11-\cancel{4r^2}-3r+7\right]$$

$$= \frac{1}{6}r(12r+18)$$

$$= r(2r+3)$$

b) PROCEED AS FOLLOWS

$$(1\times5)+(2\times7)+(3\times9)+(4\times11)+\dots+(100\times203)$$

$$\Longrightarrow \boxed{u_r - u_{r-1} \equiv r(2r+3)}$$

- $r=1$ $\quad \cancel{u_1} - u_0 = 1\times5$
- $r=2$ $\quad \cancel{u_2} - \cancel{u_1} = 2\times7$
- $r=3$ $\quad \cancel{u_3} - \cancel{u_2} = 3\times9$
- $r=4$ $\quad \cancel{u_4} - \cancel{u_3} = 4\times11$

$\vdots$

- $r=100$ $\quad u_{100} - \cancel{u_{99}} = 100\times203$

$$\Longrightarrow u_{100} - u_0 = (1\times5)+(2\times7)+(3\times9)+\dots+(100\times203)$$

$$\Longrightarrow \frac{1}{6}\times100\times101\times411 - 0 = \sum_{n=1}^{100} n(2n+3)$$

$$\Longrightarrow \sum_{n=1}^{100} n(2n+3) = 691850$$

IYGB - FP2 PAPER L - QUESTION 4

a) PARALLEL TO THE INITIAL LINE IMPLIES $\frac{dy}{dx}=0$

$$\Rightarrow \frac{dy/d\theta}{dx/d\theta}=0$$

$$\Rightarrow dy/d\theta = 0$$

$$\Rightarrow \frac{d}{d\theta}(y)=0$$

$$\Rightarrow \frac{d}{d\theta}(r\sin\theta)=0$$

$$\Rightarrow \frac{d}{d\theta}(r^2\sin^2\theta)=0$$

$$\Rightarrow \frac{d}{d\theta}(2\cos2\theta\sin^2\theta)=0$$

$$\Rightarrow -4\sin2\theta\sin^2\theta + 4\cos2\theta\sin\theta\cos\theta=0$$

$\sin2\theta \equiv 2\sin\theta\cos\theta$

$$\Rightarrow -8\sin^3\theta\cos\theta + 4\cos2\theta\sin\theta\cos\theta=0$$

$$\Rightarrow \cancel{4\sin\theta\cos\theta}(\cos2\theta - 2\sin^2\theta)=0$$

$\sin\theta \neq 0$, $\cos\theta \neq 0$ AT THE REQUIRED POINT

$$\Rightarrow \cos2\theta - 2\left(\tfrac{1}{2} - \tfrac{1}{2}\cos2\theta\right)=0$$

$$\Rightarrow \cos2\theta - 1 + \cos2\theta = 0$$

$$\Rightarrow 2\cos2\theta = 1$$

$$\Rightarrow \cos2\theta = \frac{1}{2}$$

$$\Rightarrow 2\theta = \frac{\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}, \frac{11\pi}{3}, \dots$$

$$\Rightarrow \theta = \frac{\pi}{6} \qquad 0 \leq \theta < \frac{\pi}{4}$$

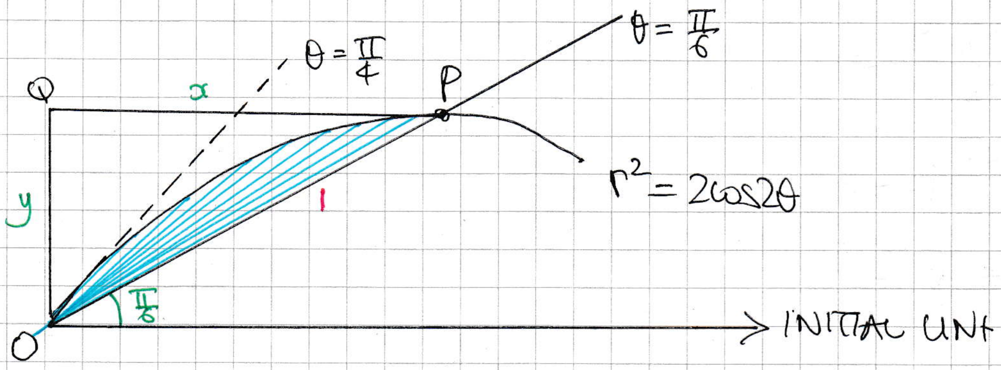
$$r^2 = 2\cos\left(\frac{\pi}{3}\right) = 2\times\frac{1}{2} = 1$$

$r=1 \qquad r \geq 0$

$\therefore P\left(1, \frac{\pi}{6}\right)$

IYGB - FP2 PAPER L - QUESTION 4

LOOKING AT THE DIAGRAM

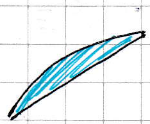


$$x = |OP|\cos\frac{\pi}{6} = 1 \times \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}$$

$$y = |OP|\sin\frac{\pi}{6} = 1 \times \frac{1}{2} = \frac{1}{2}$$

$$O\hat{Q}P \text{ AREA} = \frac{1}{2} \times \frac{1}{2} \times \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{8}$$

NOW THE "SHADED BLUE AREA" OF POLAR SECTOR IS GIVEN BY



$$= \frac{1}{2}\int_{\theta_1}^{\theta_2} r^2 \, d\theta = \frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} 2\cos 2\theta \, d\theta$$

$$= \left[\frac{1}{2}\sin 2\theta\right]_{\frac{\pi}{6}}^{\frac{\pi}{4}} = \frac{1}{2}\left(\sin\frac{\pi}{2} - \sin\frac{\pi}{3}\right)$$

$$= \frac{1}{2}\left(1 - \frac{\sqrt{3}}{2}\right) = \frac{1}{2} - \frac{\sqrt{3}}{4}$$

FINALLY THE REQUIRED AREA IS GIVEN BY

$$\frac{\sqrt{3}}{8} - \left(\frac{1}{2} - \frac{\sqrt{3}}{4}\right) = \frac{\sqrt{3}}{8} - \frac{1}{2} + \frac{\sqrt{3}}{4}$$

$$= \frac{3}{8}\sqrt{3} - \frac{1}{2}$$

$$= \frac{1}{8}\left(3\sqrt{3} - 4\right)$$

AS REQUIRED

IYGB – FP2 PAPER L – QUESTION 5

USING THE SUGGESTION GIVEN

$$\Longrightarrow (2x - 4y^2)\frac{dy}{dx} + y = 0$$

LET $x \longmapsto Y$ & $y \longmapsto X$

$$\Longrightarrow (2Y - 4X^2)\frac{dX}{dY} + X = 0$$

$$\Longrightarrow \frac{dX}{dY} = -\frac{X}{2Y - 4X^2}$$

$$\Longrightarrow \frac{dY}{dX} = \frac{4X^2 - 2Y}{X}$$

$$\Longrightarrow \frac{dY}{dX} = 4X - \frac{2Y}{X}$$

$$\Longrightarrow \frac{dY}{dX} + \frac{2}{X}Y = 4X$$

INTEGRATING FACTOR

$$e^{\int \frac{2}{X}\,dX} = e^{2\ln X} = e^{\ln X^2} = X^2$$

MULTIPLYING THROUGH BY THE INTEGRATING FACTOR TO MAKE THE LEFT SIDE EXACT

$$\Longrightarrow \frac{d}{dX}(YX^2) = 4X^3$$

$$\Longrightarrow YX^2 = \int 4X^3\,dX$$

$$\Longrightarrow YX^2 = X^4 + C$$

$$\Longrightarrow xy^2 = y^4 + C$$

IYGB - FP2 PAPER L - QUESTION 6

a) DIFFERENTIATE THE SUM OF THE TWO PRODUCTS

$$f(x) = (2x^2-1)\arcsin x + x(1-x^2)^{\frac{1}{2}}$$

$$f'(x) = 4x\arcsin x + (2x^2-1)\times\frac{1}{(1-x^2)^{\frac{1}{2}}} + 1\times(1-x^2)^{\frac{1}{2}} + x\times\frac{1}{2}(1-x^2)^{-\frac{1}{2}}(-2x)$$

$$= 4x\arcsin x + \frac{2x^2-1}{(1-x^2)^{\frac{1}{2}}} + (1-x^2)^{\frac{1}{2}} - \frac{x^2}{(1-x^2)^{\frac{1}{2}}}$$

$$= 4x\arcsin x + \frac{2x^2-1}{(1-x^2)^{\frac{1}{2}}} + \frac{(1-x^2)^1}{(1-x^2)^{\frac{1}{2}}} - \frac{x^2}{(1-x^2)^{\frac{1}{2}}}$$

$$= 4x\arcsin x + \cancel{\frac{2x^2-1+1-x^2-x^2}{(1-x^2)^{\frac{1}{2}}}}$$

$$= 4x\arcsin x$$

b) USING PART (a)

$$\int_0^{\sqrt{2}/2} x\arcsin x\,dx = \frac{1}{4}\int_0^{\frac{\sqrt{2}}{2}} 4x\arcsin x\,dx$$

$$= \frac{1}{4}\left[(2x^2-1)\arcsin x + x\sqrt{1-x^2}\right]_0^{\sqrt{2}/2}$$

$$= \frac{1}{4}\left[\left(0+\frac{1}{2}\right) - (0-0)\right]$$

$$= \frac{1}{8}$$

IYGB - FP2 PAPER L - QUESTION 7

a) LET $\cos\theta + i\sin\theta = C + iS$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1

$$\Rightarrow (\cos\theta + i\sin\theta)^5 = (C+iS)^5$$

$$\Rightarrow \cos 5\theta + i\sin 5\theta = C^5 + 5iC^4S - 10C^3S^2 - 10iC^2S^3 + 5CS^4 + iS^5$$

EQUATING REAL & IMAGINARY AND WRITE AS A TAN

$$\tan 5\theta = \frac{\sin 5\theta}{\cos 5\theta} = \frac{5C^4S - 10C^2S^3 + S^5}{C^5 - 10C^3S^2 + 5CS^4}$$

$$\tan 5\theta = \frac{\frac{5C^4S}{C^5} - \frac{10C^2S^3}{C^5} + \frac{S^5}{C^5}}{\frac{C^5}{C^5} - \frac{10C^3S^2}{C^5} + \frac{5CS^4}{C^5}}$$

$$\therefore \tan 5\theta = \frac{5\tan\theta - 10\tan^3\theta + \tan^5\theta}{1 - 10\tan^2\theta + \tan^4\theta}$$

b) LET $\tan 5\theta = 0$, WITH SOLUTIONS $\theta = 0, \frac{\pi}{5}, \frac{2\pi}{5}, \frac{3\pi}{5}, \frac{4\pi}{5}$

$$\Rightarrow 5\tan\theta - 10\tan^3\theta + \tan^5\theta = 0$$

$$\Rightarrow \tan\theta\left[\tan^4\theta - 10\tan^2\theta + 5\right] = 0$$

EITHER $\tan\theta = 0$

$\cancel{\theta = 0}$

CLEARLY NOT A SOLUTION OF THE QUARTIC

OR $\tan^4\theta - 10\tan^2\theta + 5 = 0$

$t^4 - 10t^2 + 5 = 0$

WITH SOLUTIONS

$$t = \tan\frac{\pi}{5}, \tan\frac{2\pi}{5}, \tan\frac{3\pi}{5}, \tan\frac{4\pi}{5}$$

IYGB - FP2 PAPER L - QUESTION 7

SOLVING THE QUARTIC AS A QUADRATIC NOTING $\tan\frac{\pi}{5} \neq \tan\frac{2\pi}{5}$

$$\tan^4\theta - 10\tan^2\theta + 5 = 0$$
$$(\tan^2\theta - 5)^2 - 20 = 0$$
$$(\tan^2\theta - 5)^2 = 20$$
$$\tan^2\theta - 5 = \pm 2\sqrt{5}$$
$$\tan^2\theta = 5 \pm 2\sqrt{5}$$

$\therefore \tan^2\frac{\pi}{5} \begin{cases} 5+2\sqrt{5} \\ 5-2\sqrt{5} \end{cases}$ $\quad \tan^2\frac{2\pi}{5} \begin{cases} 5+2\sqrt{5} \\ 5-2\sqrt{5} \end{cases}$

BUT $\tan\frac{\pi}{5} < \tan\frac{\pi}{4} = 1$

$$\tan^2\frac{\pi}{5} < \tan^2\frac{\pi}{4} = 1$$
$$\therefore \tan^2\frac{\pi}{5} < 1$$
$$\therefore \tan^2\frac{\pi}{5} = 5 - 2\sqrt{5}$$

SIMILARLY

$$\tan\frac{2\pi}{5} > \tan\frac{\pi}{4} = 1$$
$$\tan^2\frac{2\pi}{5} > \tan^2\frac{\pi}{4} = 1$$
$$\tan^2\frac{2\pi}{5} > 1$$
$$\therefore \tan^2\frac{2\pi}{5} = 5 + 2\sqrt{5}$$

IYGB - FP2 PAPER L - QUESTION 7

FINALLY THE RESULT FOLLOWS

$$\tan^2\frac{\pi}{5}\tan^2\frac{2\pi}{5} = (5-2\sqrt{5})(5+2\sqrt{5}) = 25-20 =$$

$$\tan\frac{\pi}{5}\tan\frac{2\pi}{5} = +\sqrt{5}$$ (AS BOTH ARE POSITIVE)

$\frac{\pi}{5} = 36°$ $\frac{2\pi}{5} = 72°$

VARIATION USING POLYNOMIAL ROOTS RELATIONSHIPS

$$\tan^4\theta - 10\tan^2\theta + 5 = 0$$

$$T^2 - 10T^2 + 5 = 0 \qquad T = \tan^2\theta$$

$\tan^2\frac{\pi}{5}$ & $\tan^2\frac{2\pi}{5}$ ARE TWO DISTINCT ROOTS OF THIS

$$\tan^2\frac{\pi}{5}\tan^2\frac{2\pi}{5} = \text{"}\frac{c}{a}\text{"} = \frac{5}{1}$$

$$\tan^2\frac{\pi}{5}\tan^2\frac{2\pi}{5} = 5$$

$$\tan\frac{\pi}{5}\tan\frac{2\pi}{5} = +\sqrt{5}$$

$\frac{\pi}{5} = 36°$

$\frac{2\pi}{5} = 72°$

BOTH POSITIVE