# IYGB - FP2 PAPER P - QUESTION 1

a) BY DIRECT DIFFERENTIATION

- $f(x) = \ln(1+\sin x)$ $\qquad f(0) = \ln 1 = 0$

- $f'(x) = \dfrac{\cos x}{1+\sin x}$ $\qquad f'(0) = 1$

- $f''(x) = \dfrac{(1+\sin x)(-\sin x) - \cos x \cos x}{(1+\sin x)^2}$

$$= \frac{-\sin x - \sin^2 x - \cos^2 x}{(1+\sin x)^2} = -\frac{1+\sin x}{(1+\sin x)^2}$$

$$= -\frac{1}{1+\sin x} = -(1+\sin x)^{-1} \qquad f''(0) = -1$$

- $f'''(x) = (1+\sin x)^{-2}\cos x = \dfrac{\cos x}{(1+\sin x)^2}$ $\qquad f'''(0) = 1$

BY McLAURIN'S THEOREM

$$f(x) = f(0) + x f'(0) + \frac{x^2}{2!}f''(0) + \frac{x^3}{3!}f'''(0) + O(x^4)$$

$$\ln(1+\sin x) = 0 + 1x - \frac{1}{2}x^2 + \frac{1}{6}x^3 + O(x^4)$$

$$\ln(1+\sin x) = x - \frac{1}{2}x^2 + \frac{1}{6}x^3 + O(x^4)$$

ALTERNATIVE USING STANDARD EXPANSIONS

$$\ln(1+y) = y - \frac{1}{2}y^2 + \frac{1}{3}y^3 + O(y^4)$$

$$\sin x = x - \frac{1}{6}x^3 + O(x^5)$$

$$\therefore \ln(1+y) = \ln(1+\sin x) = \left(x - \tfrac{1}{6}x^3\right) - \tfrac{1}{2}\left(x - \tfrac{1}{6}x^3\right)^2 + \tfrac{1}{3}\left(x - \tfrac{1}{6}x^3\right)^3 + O(x^4)$$

$$= x - \tfrac{1}{6}x^3 - \tfrac{1}{2}(x^2 + \dots) + \tfrac{1}{3}(x^3 + \dots) + O(x^4)$$

$$= x - \tfrac{1}{6}x^3 - \tfrac{1}{2}x^2 + \tfrac{1}{3}x^3 + O(x^4)$$

$$= x - \tfrac{1}{2}x^2 + \tfrac{1}{6}x^3 + O(x^4)$$

AS ABOVE

# IYGB - FP2 PAPER P - QUESTION 1

b) AS $x$ IS SMALL IN THE LIMITS

$$\int_0^{\frac{1}{4}} \ln(1+\sin x)\,dx \approx \int_0^{\frac{1}{4}} x - \frac{1}{2}x^2 + \frac{1}{6}x^3 \, dx$$

$$\approx \left[ \frac{1}{2}x^2 - \frac{1}{6}x^3 + \frac{1}{24}x^4 \right]_0^{\frac{1}{4}}$$

$$\approx \left( \frac{1}{32} - \frac{1}{384} + \frac{1}{6144} \right) - (0)$$

$$\approx \frac{59}{2048}$$

$$\approx 0.028809$$

AS REQUIRED

# IYGB - FP2 PAPER P - QUESTION 2

PROCEED BY PARTIAL FRACTIONS (COVER UP OR FULL METHOD)

$$\frac{3n-2}{n(n+1)(n+2)} = \frac{\frac{-2}{1\times 2}}{n} + \frac{\frac{-5}{-1\times 1}}{n+1} + \frac{\frac{-8}{-2(-1)}}{n+2}$$

$$= \frac{-1}{n} + \frac{5}{n+1} - \frac{4}{n+2}$$

SETTING UP THE METHOD OF DIFFERENCES BASED ON THE ABOVE RESULT

$$\boxed{\frac{3r-2}{r(r+1)(r+2)} \equiv -\frac{1}{r} + \frac{5}{r+1} - \frac{4}{r+2}}$$

- IF $r=1$ $\quad \frac{1}{1\times 2\times 3} = -\frac{1}{1} + \frac{5}{2} - \cancel{\frac{4}{3}}$
- IF $r=2$ $\quad \frac{4}{2\times 3\times 4} = -\frac{1}{2} + \cancel{\frac{5}{3}} - \cancel{\frac{4}{4}}$
- IF $r=3$ $\quad \frac{7}{3\times 4\times 5} = -\cancel{\frac{1}{3}} + \cancel{\frac{5}{4}} - \cancel{\frac{4}{5}}$
- IF $r=4$ $\quad \frac{10}{4\times 5\times 6} = -\cancel{\frac{1}{4}} + \cancel{\frac{5}{5}} - \cancel{\frac{4}{6}}$

$\vdots \qquad \vdots \qquad \vdots \qquad \vdots \qquad \vdots$

- IF $r=n-1$ $\quad \frac{3(n-1)-2}{(n-1)n(n+1)} = -\cancel{\frac{1}{n-1}} + \cancel{\frac{5}{n}} - \frac{4}{n+1}$
- IF $r=n$ $\quad \frac{3n-2}{n(n+1)(n+2)} = -\cancel{\frac{1}{n}} + \frac{5}{n+1} - \frac{4}{n+2}$

$$\sum_{r=1}^{n} \frac{3r-2}{r(r+1)(r+2)} = \left(-1 + \frac{5}{2} - \frac{1}{2}\right) + \left(\frac{1}{n+1} - \frac{4}{n+2}\right)$$

$$= 1 + \frac{1}{n+1} - \frac{4}{n+2}$$

IYGB – FP2 PAPER P – QUESTION 2

$$= 1 + \frac{1}{n+1} - \frac{4}{n+2}$$

$$= \frac{(n+1)(n+2) + (n+2) - 4(n+1)}{(n+1)(n+2)}$$

$$= \frac{n^2 + \cancel{3n} \cancel{+2} \cancel{+n} \cancel{+2} \cancel{-4n} \cancel{-4}}{(n+1)(n+2)}$$

$$= \frac{n^2}{(n+1)(n+2)}$$

# IYGB – FP2 PAPER P – QUESTION 3

a) DETERMINE THE CARTESIAN LOCUS

$A(-1,0) \quad B(1,0) \quad P(x,y)$

- $|AP| = \sqrt{(x+1)^2 + y^2}$
- $|BP| = \sqrt{(x-1)^2 + y^2}$

$$\Longrightarrow |AP||BP| = 1$$

$$\Longrightarrow |AP|^2|BP|^2 = 1$$

$$\Longrightarrow \left[(x+1)^2+y^2\right]\left[(x-1)^2+y^2\right] = 1$$

$$\Longrightarrow \left\{\begin{matrix} y^2(x+1)^2 + (x+1)^2(x-1)^2 \\ y^2(x-1)^2 + y^4 \end{matrix}\right\} = 1$$

$$\Longrightarrow y^2\left[(x+1)^2+(x-1)^2\right] + y^4 + (x+1)^2(x-1)^2 = 1$$

$$\Longrightarrow y^2\left[x^2+2x+1+x^2-2x+1\right] + y^4 + (x^2-1)^2 = 1$$

$$\Longrightarrow y^2\left[2x^2+2\right] + y^4 + x^4 - 2x^2 + 1 = 1$$

REGROUPING

$$\Longrightarrow 2x^2y^2 + 2y^2 + y^4 + x^4 - 2x^2 = 0$$

$$\Longrightarrow \left(y^4 + 2x^2y^2 + x^4\right) + 2(y^2 - x^2) = 0$$

$$\Longrightarrow (x^2+y^2)^2 + 2(y^2-x^2) = 0$$

TRANSFORM INTO POLARS

$$\Longrightarrow (r^2)^2 + 2\left(r^2\sin^2\theta - r^2\cos^2\theta\right) = 0$$

$$\Longrightarrow r^4 + 2r^2(\sin^2\theta - \cos^2\theta) = 0$$

$$\Longrightarrow r^2 + 2(\sin^2\theta - \cos^2\theta) = 0$$

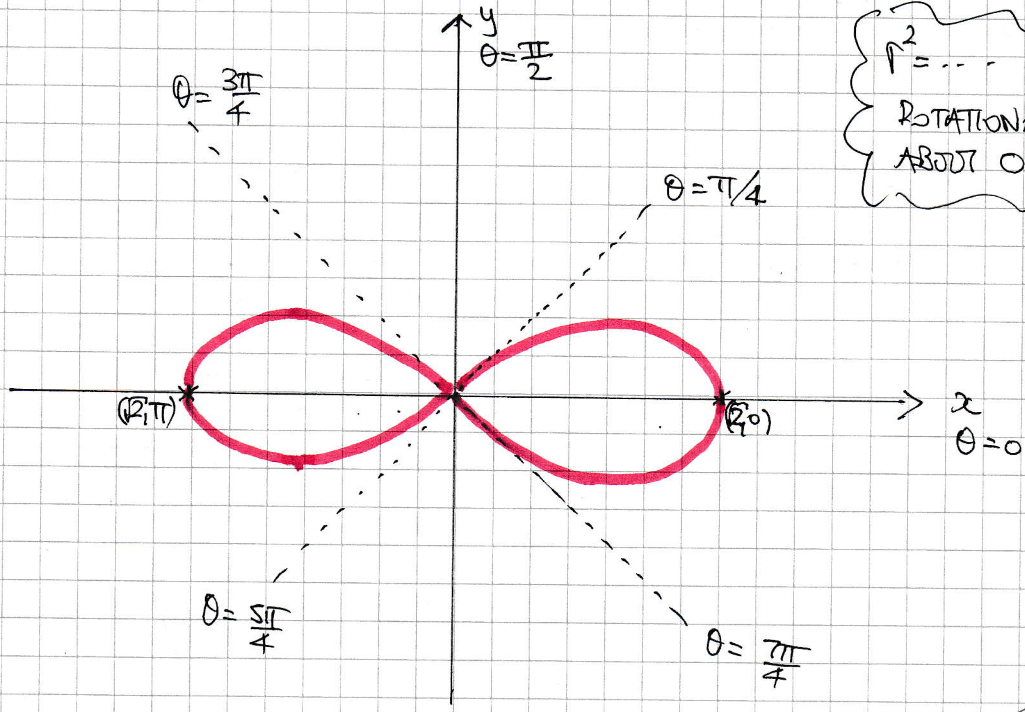
$$\Longrightarrow r^2 = 2(\cos^2\theta - \sin^2\theta)$$

$$\Longrightarrow r^2 = 2\cos 2\theta$$

AS REQUIRED

IYGB - FP2 PAPER P - QUESTION 3

b)




$r^2 = \ldots$

ROTATIONAL SYMMETRY ABOUT O

# IYGB – FP2 PAPER P – QUESTION 4

$$\boxed{f(x) = \frac{ax+b}{e^{x}} = (ax+b)e^{-x}}$$

PROCEED BY INDEFINITE INTEGRATION BY PARTS

$$\int \frac{ax+b}{e^{x}}\,dx = -(ax+b)e^{-x} - \int -ae^{-x}\,dx$$

| $ax+b$ | $a$ |
|---|---|
| $-e^{-x}$ | $e^{-x}$ |

$$= -(ax+b)e^{-x} + \int ae^{-x}\,dx$$

$$= -(ax+b)e^{-x} - ae^{-x} + C$$

$$= -e^{-x}\left[ax+b+a\right] + C$$

NEXT THE MEAN VALUE OF THE FUNCTION IN $(\ln 2, \ln 4)$

$$\Rightarrow \text{MEAN VALUE} = \frac{1}{b-a}\int_a^b f(x)\,dx$$

$$\Rightarrow \frac{1}{4\ln 2} = \frac{1}{\ln 4 - \ln 2}\int_{\ln 2}^{\ln 4} \frac{ax+b}{e^{x}}\,dx$$

$$\Rightarrow \frac{1}{4\cancel{\ln 2}} = \frac{1}{\cancel{\ln 2}}\left[-e^{-x}\left[ax+b+a\right]\right]_{\ln 2}^{\ln 4}$$

$$\Rightarrow \frac{1}{4} = \left[+e^{-x}(ax+a+b)\right]_{\ln 4}^{\ln 2}$$

$$\Rightarrow \frac{1}{4} = e^{-\ln 2}(a\ln 2 + a + b) - e^{-\ln 4}(a\ln 4 + a + b)$$

$$\Rightarrow \frac{1}{4} = \frac{1}{2}(a\ln 2 + a + b) - \frac{1}{4}(2a\ln 2 + a + b)$$

$$\Rightarrow 1 = \cancel{2(a\ln 2} + a + b) - (\cancel{2a\ln 2} + a + b)$$

# IYGB - FP2 PAPER P - QUESTION 4

$\Longrightarrow 1 = 2a + 2b - a - b$

$\Longrightarrow \underline{a + b = 1}$

NEXT THE IMPROPER INTEGRAL

$$\Longrightarrow \int_1^{\infty} \frac{ax+b}{e^x}\,dx = \lim_{k \to \infty}\left[\left[-e^{-x}(ax+b+a)\right]_1^k\right]$$

$$\Longrightarrow \frac{3}{e} = \lim_{k \to \infty}\left[\left[+e^{-x}(ax+b+a)\right]_k^1\right]$$

$$\Longrightarrow \frac{3}{e} = \lim_{k \to \infty}\left[e^{-1}(2a+b) - e^{-k}(ak+b+a)\right]$$

$$\Longrightarrow \frac{3}{e} = \lim_{k \to \infty}\left[\frac{2a+b}{e} - \cancel{\frac{ak}{e^k}} - \cancel{\frac{a+b}{e^k}}\right]$$

TENDS TO ZERO, AS $e^k$ GROWS FASTER THAN THE ALGEBRAIC NUMERATOR GROWS, AS $k \to \infty$

TENDS TO ZERO AS $k \to \infty$

$$\Longrightarrow \frac{3}{e} = \frac{2a+b}{e}$$

$\Longrightarrow \underline{2a + b = 3}$

SOLVING THE EQUATIONS BY INSPECTION

$$\left.\begin{array}{l} a+b=1 \\ 2a+b=3 \end{array}\right\} \Longrightarrow \underline{a=2}$$

&

$\underline{b=-1}$

# IYGB – FP2 PAPER P – QUESTION 5

a) $y = \arcsin(4x-3) + (4x-3)\left[-8(2x^2-3x+1)\right]^{\frac{1}{2}}$

$\Rightarrow \frac{dy}{dx} = \frac{1}{\sqrt{1-(4x-3)^2}} \times 4 + 4\left[-8(2x^2-3x+1)\right]^{\frac{1}{2}} + (4x-3)\times\frac{1}{2}\left[-8(2x^2-3x+1)\right]^{-\frac{1}{2}}(-32x+24)$

$\Rightarrow \frac{dy}{dx} = \frac{4}{\sqrt{1-(4x-3)^2}} + 4\left[8(-2x^2+3x-1)\right]^{\frac{1}{2}} - (4x-3)(16x-12)\left[8(-2x^2+3x-1)\right]^{-\frac{1}{2}}$

$\Rightarrow \frac{dy}{dx} = \frac{4}{\sqrt{1-(16x^2-24x+9)}} + 4\left[8(-2x^2+3x-1)\right]^{\frac{1}{2}} - 4(4x-3)\left[8(-2x^2+3x-1)\right]^{-\frac{1}{2}}$

$\Rightarrow \frac{dy}{dx} = \frac{4}{\sqrt{-16x^2+24x-8}} + 4\left[8(-2x^2+3x-1)\right]^{\frac{1}{2}} - 4(4x-3)^2\left[8(-2x^2+3x-1)\right]^{-\frac{1}{2}}$

$\Rightarrow \frac{dy}{dx} = 4\left[8(-2x^2+3x-1)\right]^{-\frac{1}{2}} + 4\left[8(-2x^2+3x-1)\right]^{\frac{1}{2}} - 4(4x-3)^2\left[8(-2x^2+3x-1)\right]^{-\frac{1}{2}}$

$\Rightarrow \frac{dy}{dx} = 4\left[8(-2x^2+3x-1)\right]^{-\frac{1}{2}}\left[1 + \left[8(-2x^2+3x-1)\right]^{1} - (4x-3)^2\right]$

$\Rightarrow \frac{dy}{dx} = 4\left[8(-2x^2+3x-1)\right]^{-\frac{1}{2}}\left[1 - 16x^2 + 24x - 8 - (16x^2-24x+9)\right]$

$\Rightarrow \frac{dy}{dx} = 4\left[8(-2x^2+3x-1)\right]^{-\frac{1}{2}}\left[-32x^2+48x-16\right]$

$\Rightarrow \frac{dy}{dx} = 4\left[8(-2x^2+3x-1)\right]^{-\frac{1}{2}}\left[16(-2x^2+3x-1)\right]$

$\Rightarrow \frac{dy}{dx} = 4 \times 8^{-\frac{1}{2}} \times 16 \times (-2x^2+3x-1)^{-\frac{1}{2}}(-2x^2+3x-1)$

# IYGB - FP2 PAPER P - QUESTION 5

$$\Rightarrow \frac{dy}{dx} = 4 \times \frac{1}{\sqrt{8}} \times 16 \times \left(-2x^2+3x-1\right)^{\frac{1}{2}}$$

$$\Rightarrow \frac{dy}{dx} = \frac{64\sqrt{8}}{8}\left(-2x^2+3x-1\right)^{\frac{1}{2}}$$

$$\Rightarrow \frac{dy}{dx} = 16\sqrt{2}\left(-2x^2+3x-1\right)^{\frac{1}{2}}$$

// $k = 16\sqrt{2}$

b) USING PART (a)

$$\int_{\frac{1}{2}}^{1} \sqrt{-2x^2+3x-1} = \frac{1}{16\sqrt{2}} \int_{\frac{1}{2}}^{1} 4\sqrt{2}\left(-2x^2+3x-1\right)^{\frac{1}{2}}\,dx$$

THUS WE HAVE

$$= \frac{1}{16\sqrt{2}}\left[\arcsin(4x-3) + (4x-3)\left[-8(2x^2-3x+1)\right]^{\frac{1}{2}}\right]_{\frac{1}{2}}^{1}$$

$$= \frac{1}{16\sqrt{2}}\left\{\left[\arcsin 1 + \cancel{\sqrt{-8(2-3+1)}}\right] - \left[\arcsin(-1) - \cancel{\sqrt{-8(\tfrac{1}{2}-\tfrac{3}{2}+1)}}\right]\right\}$$

$$= \frac{1}{16\sqrt{2}}\left\{\frac{\pi}{2} + \frac{\pi}{2}\right\}$$

$$= \frac{\pi}{16\sqrt{2}}$$

# IYGB – FP2 PAPER P – QUESTION 6

a) PROCEED AS FOLLOWS

LET $\text{artanh}\,x = \alpha$ , $|x| < 1$

$$\Rightarrow x = \tanh\alpha$$

$$\Rightarrow x = \frac{e^{2\alpha}-1}{e^{2\alpha}+1}$$

$$\Rightarrow xe^{2\alpha} + x = e^{2\alpha} - 1$$

$$\Rightarrow 1 + x = e^{2\alpha} - xe^{2\alpha}$$

$$\Rightarrow 1 + x = e^{2\alpha}(1-x)$$

$$\Rightarrow e^{2\alpha} = \frac{1+x}{1-x}$$

$$\Rightarrow 2\alpha = \ln\left(\frac{1+x}{1-x}\right)$$

$$\Rightarrow \alpha = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right)$$

$$\Rightarrow \text{artanh}\,x = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right)$$

// AS REQUIRED

b) STARTING FROM THE TRIGONOMETRIC IDENTITY

$$\Rightarrow 1 + \tan^2\theta \equiv \sec^2\theta$$

$$\Rightarrow 1 + \frac{\sin^2\theta}{\cos^2\theta} \equiv \frac{1}{\cos^2\theta}$$

LET $\theta = ix$ & NOTE $\cos ix \equiv \cosh x$ & $\sin ix = i\sinh x$

$$\Rightarrow 1 + \frac{\sin^2(ix)}{\cos^2(ix)} = \frac{1}{\cos^2(ix)}$$

$$\Rightarrow 1 + \frac{i^2\sinh^2 x}{\cosh^2 x} = \frac{1}{\cosh^2 x}$$

$$\Rightarrow 1 - \frac{\sinh^2 x}{\cosh^2 x} = \frac{1}{\cosh^2 x}$$

# IYGB – FP2 PAPER P – QUESTION 6

$\Longrightarrow \quad 1 - \tanh^2 x = \operatorname{sech}^2 x$ // AS REQUIRED

c) USING PART (b)

$\Longrightarrow \quad 6\operatorname{sech}^2 x - \tanh x = 4$

$\Longrightarrow \quad 6(1 - \tanh^2 x) - \tanh x = 4$

$\Longrightarrow \quad 6 - 6\tanh^2 x - \tanh x = 4$

$\Longrightarrow \quad 0 = 6\tanh^2 x + \tanh x - 2$

$\Longrightarrow \quad (3\tanh x + 2)(2\tanh x - 1) = 0$

$\Longrightarrow \quad \tanh x = \begin{cases} \frac{1}{2} \\ -\frac{2}{3} \end{cases}$

$\Longrightarrow \quad x = \begin{cases} \operatorname{artanh}\left(\frac{1}{2}\right) \\ \operatorname{artanh}\left(-\frac{2}{3}\right) = -\operatorname{artanh}\left(\frac{2}{3}\right) \end{cases}$

USING PART (a)

$\Longrightarrow \quad x = \begin{cases} \frac{1}{2}\ln\left(\frac{1+\frac{1}{2}}{1-\frac{1}{2}}\right) = \frac{1}{2}\ln\left(\frac{2+1}{2-1}\right) = \frac{1}{2}\ln 3 \\ -\frac{1}{2}\ln\left(\frac{1+\frac{2}{3}}{1-\frac{2}{3}}\right) = -\frac{1}{2}\ln\left(\frac{3+2}{3-2}\right) = -\frac{1}{2}\ln 5 \end{cases}$

$\Longrightarrow \quad x = \begin{cases} \frac{1}{2}\ln 3 \\ -\frac{1}{2}\ln 5 \end{cases}$

$\therefore \quad k = 3 \quad \text{OR} \quad k = 5$ //

# IYGB - FP2 PAPER P - QUESTION 7

LET $z = (1 + i\tan\theta)^3$

$$\Rightarrow (1+i\tan\theta)^3 = \left(1 + \frac{i\sin\theta}{\cos\theta}\right)^3$$

$$\Rightarrow (1+i\tan\theta)^3 = \left(\frac{\cos\theta + i\sin\theta}{\cos\theta}\right)^3$$

CUBING BOTH SIDES

$$\Rightarrow 1 + 3i\tan\theta + 3i^2\tan^2\theta + i^3\tan^3\theta = \frac{(\cos\theta + i\sin\theta)^3}{\cos^3\theta}$$

DE MOIVRE THEOREM

$$\Rightarrow 1 + 3i\tan\theta - 3\tan^2\theta - i\tan^3\theta = \frac{\cos3\theta + i\sin3\theta}{\cos^3\theta}$$

EQUATING REAL PARTS

$$1 - 3\tan^2\theta = \frac{\cos3\theta}{\cos^3\theta}$$

// AS REQUIRED

# IYGB – FP2 PAPER P – QUESTION 8

DIFFERENTIATE EITHER OF THE TWO EQUATIONS WITH RESPECT TO $t$, REARRANGE AND SUBSTITUTE INTO THE OTHER

$$\frac{d}{dt}\left(\frac{dx}{dt}+y\right) = \frac{d}{dt}\left(e^{-t}\right)$$

$$\frac{d^2x}{dt^2} + \frac{dy}{dt} = -e^{-t}$$

$$\frac{d^2x}{dt^2} + \left[x+e^{t}\right] = -e^{-t}$$

$$\frac{d^2x}{dt^2} + x = -e^{t} - e^{-t}$$

AUXILIARY EQUATION IS $\lambda^2+1=0$, WHICH YIELDS $\lambda = \pm i$

COMPLEMENTARY FUNCTION IS $x = A\cos t + B\sin t$

FOR PARTICULAR INTEGRAL, LET $x = Pe^{t} + Qe^{-t}$

$$\frac{dx}{dt} = Pe^{t} - Qe^{-t}$$

$$\frac{d^2x}{dt^2} = Pe^{t} + Qe^{-t}$$

SUB INTO THE O.D.E

$$\left(Pe^{t}+Qe^{-t}\right) + \left(Pe^{t}+Qe^{-t}\right) \equiv -e^{t} - e^{-t}$$

$$2Pe^{t} + 2Qe^{-t} \equiv -e^{t} - e^{-t}$$

$$\therefore P = Q = -\tfrac{1}{2}$$

$\therefore$ GENERAL SOLUTION IS

$$x = A\cos t + B\sin t - \tfrac{1}{2}e^{t} - \tfrac{1}{2}e^{-t}$$

$$x = A\cos t + B\sin t - \cosh t$$

# IYGB - FP2 PAPER P - QUESTION 8

APPLY CONDITIONS $x=0$ $t=0$ YIELDS

$$0 = A - 1$$
$$A = 1$$

$$\therefore x = \cos t + B\sin t - \cosh t$$

DIFFERENTIATE WITH RESPECT TO $t$

$$\frac{dx}{dt} = -\sin t + B\cos t - \sinh t$$

$$-y + e^{-t} = -\sin t + B\cos t - \sinh t$$

APPLY CONDITION $t=0$, $y=0$

$$0 + 1 = 0 + B - 0$$
$$B = 1$$

$$\therefore x = \cos t + \sin t - \cosh t$$

FINALLY REARRANGING

$$y = e^{-t} - \frac{dx}{dt} \implies y = e^{-t} - \frac{d}{dt}\left[\cos t + \sin t - \cosh t\right]$$

$$\implies y = e^{-t} - \left[-\sin t + \cos t - \sinh t\right]$$

$$\implies y = e^{-t} + \sin t - \cos t + \sinh t$$

$$\implies y = e^{-t} + \sin t - \cos t + \tfrac{1}{2}e^{t} - \tfrac{1}{2}e^{-t}$$

$$\implies y = \sin t - \cos t + \tfrac{1}{2}e^{t} + \tfrac{1}{2}e^{-t}$$

$$\implies y = \sin t - \cos t + \cosh t$$

# IYGB - FP2 PAPER P - QUESTION 8

APPLY CONDITIONS $x=0$ $t=0$ YIELDS

$$0 = A - 1$$
$$A = 1$$

$$\therefore x = \cos t + B\sin t - \cosh t$$

DIFFERENTIATE WITH RESPECT TO $t$

$$\frac{dx}{dt} = -\sin t + B\cos t - \sinh t$$

$$-y + e^{-t} = -\sin t + B\cos t - \sinh t$$

APPLY CONDITION $t=0$, $y=0$

$$0 + 1 = 0 + B - 0$$
$$B = 1$$

$$\therefore \underline{x = \cos t + \sin t - \cosh t}$$

FINALLY REARRANGING

$$y = e^{-t} - \frac{dx}{dt} \Rightarrow y = e^{-t} - \frac{d}{dt}\left[\cos t + \sin t - \cosh t\right]$$

$$\Rightarrow y = e^{-t} - \left[-\sin t + \cos t - \sinh t\right]$$

$$\Rightarrow y = e^{-t} + \sin t - \cos t + \sinh t$$

$$\Rightarrow y = e^{-t} + \sin t - \cos t + \tfrac{1}{2}e^{t} - \tfrac{1}{2}e^{-t}$$

$$\Rightarrow y = \sin t - \cos t + \tfrac{1}{2}e^{t} + \tfrac{1}{2}e^{-t}$$

$$\Rightarrow \underline{y = \sin t - \cos t + \cosh t}$$