I.Y.G.B. www.madasmaths.com

IYGB GCE

Mathematics FP3

Advanced Level

Practice Paper R

Difficulty Rating: 3.200/1.4388

Time: 1 hour 30 minutes

Candidates may use any calculator allowed by the regulations of this examination.

Information for Candidates

This practice paper follows closely the Pearson Edexcel Syllabus, suitable for first assessment Summer 2018.

The standard booklet "Mathematical Formulae and Statistical Tables" may be used.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. (**2**).
There are 9 questions in this question paper.
The total mark for this paper is 75.

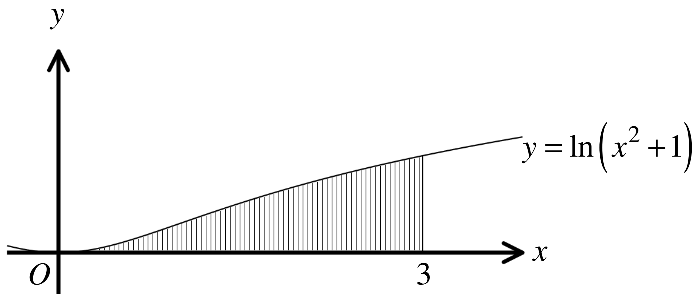
Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
Non exact answers should be given to an appropriate degree of accuracy.
The examiner may refuse to mark any parts of questions if deemed not to be legible.

I.Y.G.B. www.madasmaths.com

I.Y.G.B. www.madasmaths.com

Question 1



The figure above shows part of the curve with equation

$$y = \ln\left(x^2+4\right).$$

The region R, shown shaded in the figure, is bounded by the curve, the x axis and the straight line with equation $x = 3$.

a) Use Simpson's rule with 7 equally spaced ordinates to estimate the area of R.
[*The answer must be supported with detailed calculations*.] **(3)**

b) Deduce an estimate for the value of

$$\int_0^3 \ln\left(\tfrac{1}{4}x^2+1\right)\,dx.$$ **(3)**

Question 2

Find the value of the following limit

$$\lim_{x\to\infty}\left[x\left(2^{\frac{1}{x}}-1\right)\right].$$ **(6)**

Question 3

Solve the following rational inequality.

$$\frac{(x+1)^2(x-2)}{(x-1)^2(x+2)} < 0. \qquad (4)$$

Question 4

Relative to a fixed origin O, the points $A(-2,3,5)$, $B(1,-3,1)$ and $C(4,-6,-7)$ lie on the plane Π.

a) Find a Cartesian equation for Π. **(6)**

The perpendicular from the point $P(26,2,7)$ meets the Π at the point Q.

b) Determine the coordinates of Q. **(4)**

Question 5

The function with equation $y = f(x)$ is differentiable n times, $n \in \mathbb{N}$, and satisfies the following relationship.

$$\left(1+x^2\right)\frac{d^2y}{dx^2} + x\frac{dy}{dx} - 4y = 0.$$

Use the Leibniz rule to show that at $x = 0$

$$\frac{d^{n+2}y}{dx^{n+2}} = \left(4-n^2\right)\frac{d^ny}{dx^n}. \qquad (5)$$

I.Y.G.B. www.madasmaths.com

I.Y.G.B www.madasmaths.com

Question 6

The curve with equation $y = f(x)$, passes through the point $(1,0)$ and satisfies the following differential equation.

$$\frac{dy}{dx} = x + \ln x, \ x > 0.$$

Use the approximation

$$\left(\frac{dy}{dx}\right)_0 \approx \frac{y_1 - y_0}{h}, \quad h = 0.1,$$

to find the value of y at $x = 1.1$, and use this answer with the approximation

$$\left(\frac{dy}{dx}\right)_0 \approx \frac{y_1 - y_{-1}}{2h}, \ h = 0.1,$$

to find, correct to 3 decimal places, the value of y at $x = 1.2$, $x = 1.3$ and $x = 1.4$. **(10)**

Question 7

A curve has an equation $y = f(x)$ that satisfies the differential equation

$$e^{-x}\frac{d^2y}{dx^2} = 2y\frac{dy}{dx} + y^2 + 1$$

with $y = 1$, $\frac{dy}{dx} = 2$ at $x = 0$.

a) Show clearly that

$$e^{-x}\frac{d^3y}{dx^3} = \left(2y + e^{-x}\right)\frac{d^2y}{dx^2} + 2\frac{dy}{dx}\left(y + \frac{dy}{dx}\right).$$ **(7)**

b) Find a series solution for $f(x)$, up and including the term in x^3. **(3)**

Question 8

Use the substitution $y = xv$, where $v = v(x)$, to solve the following differential equation

$$2\frac{dy}{dx} = 1 + \frac{y^2}{x^2}, \quad y(\mathrm{e}) = -\mathrm{e}. \qquad (12)$$

Question 9

The general point $P(a\sec\theta, b\tan\theta)$ lies on the hyperbola with Cartesian equation

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1,$$

where a and b are positive constants.

a) Show that an equation of the normal at P is given by

$$by + ax\sin\theta = \left(a^2 + b^2\right)\tan\theta. \qquad (5)$$

The normal to the hyperbola meets the x axis at the point X.

The eccentricity of the hyperbola is $\frac{3}{2}$ and its foci are denoted by S and S', where S has a positive x coordinate.

b) Given that $|OX| = 3|OS|$, find the possible values of θ for $0 \le \theta \le 2\pi$. **(7)**

I.Y.G.B. www.madasmaths.com