IYGB GCE

Mathematics FP4

Advanced Level

Practice Paper O

Difficulty Rating: 3.6200/1.6807

Time: 1 hour 30 minutes

Candidates may use any calculator allowed by the regulations of this examination.

Information for Candidates

This practice paper follows closely the Pearson Edexcel Syllabus, suitable for first assessment Summer 2018.

The standard booklet "Mathematical Formulae and Statistical Tables" may be used.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 7 questions in this question paper.
The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
Non exact answers should be given to an appropriate degree of accuracy.
The examiner may refuse to mark any parts of questions if deemed not to be legible.

I.Y.G.B www.madasmaths.com

Question 1

A group G consists of the elements $\{a, b, ab, e\}$ under the operation of multiplication.

It is further given that $ab = ba$ and $a^2 = b^2 = e$, where e is the identity element.

Construct a table for G, fully justifying the entries where appropriate. **(6)**

Question 2

The numbers 1, 2, 3 and 4 are to be used to make a four digit password.

Calculate the number of the four digit passwords that can be created if …

a) ... any repetitions are allowed. **(2)**

b) ... no repetitions are allowed. **(2)**

c) ... a digit can be repeated at most twice. **(6)**

Question 3

Suppose that when a positive integer is divided by 6 the remainder is 4, and when the same positive integer is divided by 12 the remainder is 8.

a) Determine whether such positive integer exists. **(5)**

Suppose next that when a positive integer is divided by 6, the quotient is q and the remainder is 1.

When the square of the same positive integer is divided by q, the quotient is 984 and the remainder is 1.

b) Determine whether the second positive integer described in the question exists. **(6)**

Question 4

By forming and using a suitable reduction formula show that

$$\int_0^1 x^5 \, \mathrm{e}^{-x^2} \, dx = \frac{2\mathrm{e}-5}{2\mathrm{e}}.$$

No credit will be given if no reduction formula is not used in this question **(11)**

Question 5

A 3×3 matrix $\mathbf{A}$ has characteristic equation

$$2\lambda^3 - 7\lambda^2 + \lambda + 10 = 0.$$

a) Show that $\lambda = 2$ is an eigenvalue of $\mathbf{A}$ and find the other two eigenvalues. **(3)**

b) Show further that

$$2\mathbf{A}^4 + 71\mathbf{A}^2 = 27\mathbf{A}^3 + 100\mathbf{I}. \quad \textbf{(5)}$$

An eigenvector corresponding to $\lambda = 2$ is $\mathbf{u}$.

It is further given that $\mathbf{u} = \begin{pmatrix} 2 \\ -4 \\ -5 \end{pmatrix}$, $\mathbf{v} = \begin{pmatrix} 0.4 \\ -0.8 \\ -1 \end{pmatrix}$ and $\mathbf{x} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$.

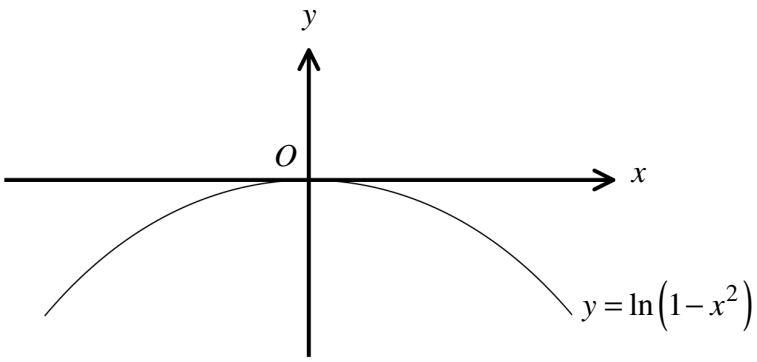
c) Evaluate each of the following expressions.

i. $\mathbf{Au}$. **(2)**

ii. $\mathbf{A}^2\mathbf{v}$. **(2)**

d) Solve the equation $\mathbf{Ax} = \mathbf{v}$. **(4)**

Question 6



The figure above shows the graph of the curve with equation

$$y = \ln\left(1-x^2\right), \quad \frac{1}{2} \le x \le \frac{1}{2}.$$

a) Show that the length s of the curve is given by

$$s = \int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{1+x^2}{1-x^2}\, dx. \qquad (\mathbf{5})$$

b) Hence find the exact length of the curve. $(\mathbf{9})$

Question 7

The complex function $w = f(z)$ satisfies

$$w = \frac{1}{z},\ z \in \mathbb{C},\ z \neq 0 .$$

This function maps the point $P(x, y)$ in the z plane onto the point $Q(u, v)$ in the w plane.

It is further given that P traces the curve with equation

$$\left| z + \frac{1}{2}\mathrm{i} \right| = \frac{1}{2}.$$

Find, in Cartesian form, the equation of the locus of Q. **(7)**
