IYGB GCE

Mathematics FP4

Advanced Level

Practice Paper P

Difficulty Rating: 3.3133/1.4888

Time: 1 hour 30 minutes

Candidates may use any calculator allowed by the regulations of this examination.

Information for Candidates

This practice paper follows closely the Pearson Edexcel Syllabus, suitable for first assessment Summer 2018.

The standard booklet "Mathematical Formulae and Statistical Tables" may be used.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 8 questions in this question paper.
The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
Non exact answers should be given to an appropriate degree of accuracy.
The examiner may refuse to mark any parts of questions if deemed not to be legible.

Question 1

The table below shows the group G, representing the symmetries of an **equilateral triangle**, under the composition of their transformations

	a	b	c	d	e	f
a	e	f	d	c	a	b
b	d	e	f	a	b	c
c	f	d	e	b	c	a
d	b	c	a	f	d	e
e	a	b	c	d	e	f
f	c	a	b	e	f	d

a) State the identity element and classify the rest of the elements as reflections or rotations.

Full justifications must be given. **(5)**

b) Find a subgroup of order 3. **(2)**

The multiplication table of another group H is given below.

	u	v	w	x	y	z
u	v	w	u	y	z	x
v	w	u	v	z	x	y
w	u	v	w	x	y	z
x	y	z	x	w	u	v
y	z	x	y	v	w	u
z	x	y	z	u	v	w

c) Find all the self inverse elements. **(2)**

d) Determine the order of the element u **(2)**

e) Show that G and H are isomorphic, stating a possible isomorphism of their elements. **(4)**

Question 2

A sequence of integers is defined inductively by the relation

$$a_{n+1} = 3a_n + 4, \quad a_1 = 3, \quad n = 1, 2, 3, \ldots$$

Prove by induction that its n^{th} term is given by

$$a_n = 5 \times 3^{n-1} - 2, \quad n = 1, 2, 3, \ldots \qquad (5)$$

Question 3

Use Euclid's algorithm to find the Highest common factor of 3059 and 7728. **(4)**

Question 4

The 3×3 matrix $\mathbf{A}$ is given below.

$$\mathbf{A} = \begin{pmatrix} -1 & k & 0 \\ k & 0 & 2 \\ 0 & 2 & 1 \end{pmatrix}.$$

a) If one of the eigenvalues of $\mathbf{A}$ is 3, find the possible values of k. **(4)**

b) Determine the other two eigenvalues of $\mathbf{A}$, given that $k > 0$. **(4)**

c) Find an eigenvector corresponding to the eigenvalue 3. **(4)**

Question 5

The point P represents the number $z = x + \mathrm{i}y$ in an Argand diagram and further satisfies the equation

$$\arg(z-7) - \arg(z-1) = \frac{\pi}{4}, \quad z \neq -\mathrm{i}.$$

Determine in exact form the maximum value of $|z|$. **(6)**

Question 6

$$I_n = \int \operatorname{cosec}^n x \, dx, \quad n \in \mathbb{N}.$$

a) Show clearly that

$$I_n = \frac{n-2}{n-1} I_{n-2} - \frac{1}{n-1} (\cot x)(\operatorname{cosec} x)^{n-2}, \quad n \geq 2.$$ **(5)**

b) Use part **(a)** to evaluate

$$I_n = \int_{\frac{1}{4}\pi}^{\frac{1}{2}\pi} \operatorname{cosec}^6 x \, dx$$ **(5)**

Question 7

Alex, Beth and Cain are 3 students in a class which consists of a total of 8 students.

a) Determine the number of selections of 4 students which contain both Alex and Beth but not Cain. **(2)**

Next all 8 students are standing next to each for a group photo.

b) Determine the number of arrangements in which ...

i. ... Alex is standing at one end and Beth and Cain are standing next to each other. **(4)**

ii. ... Alex and Beth are standing next to each other and Cain is standing next to them. **(4)**

Question 8

A transformation T from the z plane to the w plane is defined by

$$w = \frac{z - \mathrm{i}}{z+1}, \ z \in \mathbb{C}, \ z \neq -1.$$

T transforms the circle with equation $|z| = 1$ in the z plane, into the straight line L in the w plane.

a) Find a Cartesian equation for L. **(5)**

T transforms the y axis in the z plane, into the curve C in the w plane.

b) Find a Cartesian equation for C. **(5)**

The region R in the z plane, satisfies $|z| \leq 1$ such that $-\frac{\pi}{2} \leq \arg z \leq \frac{\pi}{2}$.

c) Shade the image of R under T in the w plane. **(3)**
