# IYGB GCE

## Mathematics FP4

## Advanced Level

### Practice Paper Q

Difficulty Rating: 3.1733/1.4151

### Time: 1 hour 30 minutes

**Candidates may use any calculator allowed by the regulations of this examination.**

**Information for Candidates**

This practice paper follows closely the Pearson Edexcel Syllabus, suitable for first assessment Summer 2018.

The standard booklet "Mathematical Formulae and Statistical Tables" may be used.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 8 questions in this question paper.
The total mark for this paper is 75.

**Advice to Candidates**

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
Non exact answers should be given to an appropriate degree of accuracy.
The examiner may refuse to mark any parts of questions if deemed not to be legible.

**Question 1**

The $2\times2$ matrix $\mathbf{A}=\begin{pmatrix}7 & 6\\ 6 & 2\end{pmatrix}$ is given.

Use the Caley- Hamilton theorem to show that

$$\mathbf{A}^4=\lambda\mathbf{A}+\mu\mathbf{I},$$

where $\mathbf{I}$ is $2\times2$ identity matrix. **(6)**

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**Question 2**

$$I_n=\int_0^1 x^n\,\mathrm{e}^{-\frac{1}{2}x^2}\,dx,\quad n\in\mathbb{N}.$$

Show clearly that …

**a)** … $I_n=(n-1)I_{n-2}-\mathrm{e}^{-\frac{1}{2}}$, $n\geq 2$. **(5)**

**b)** … $\displaystyle\int_0^1 x^5\,\mathrm{e}^{-\frac{1}{2}x^2}\,dx=8-13\mathrm{e}^{-\frac{1}{2}}$. **(5)**

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**Question 3**

The 10 letters of the word

$$B\ A\ C\ A\ B\ A\ C\ A\ B\ A$$

are written on 10 separate pieces of card.

These cards are selected at random and arranged in a line next to each other.

Determine the number of arrangements which start and finish with the same letter. **(8)**

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## Question 4

Three Cayley tables are shown below.

| $G$ | 2 | 4 | 6 | 8 |
|---|---|---|---|---|
| 2 | 4 | 8 | 2 | 6 |
| 4 | 8 | 6 | 4 | 2 |
| 6 | 2 | 4 | 6 | 8 |
| 8 | 6 | 2 | 8 | 4 |

| $H$ | 1 | $-1$ | i | $-$i |
|---|---|---|---|---|
| 1 | 1 | $-1$ | i | $-$i |
| $-1$ | $-1$ | 1 | $-$i | i |
| i | i | $-$i | $-1$ | 1 |
| $-$i | $-$i | i | 1 | $-1$ |

| $J$ | 1 | 5 | 7 | 11 |
|---|---|---|---|---|
| 1 | 1 | 5 | 7 | 11 |
| 5 | 5 | 1 | 11 | 2 |
| 7 | 7 | 11 | 1 | 5 |
| 11 | 11 | 7 | 5 | 1 |

It is given that $G = \{2,4,6,8\}$ under multiplication modulo $10$, $H = \{1,-1,\text{i},-\text{i}\}$ under complex multiplication and $G = \{1,5,7,11\}$ under multiplication modulo $12$.

Determine with full justification …

**a)** … whether $G$ is a group. **(5)**

**b)** … whether $G$ is isomorphic to $H$, assuming that $H$ is a group. **(2)**

**c)** … whether $H$ is cyclic, assuming that $H$ is a group. **(2)**

**d)** … whether $J$ is isomorphic to $H$, assuming that both form groups. **(2)**

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## Question 5

A sequence of numbers $u_1$, $u_2$, $u_3$, $u_4$, $u_5$ … , $u_n$, … is generated by the recurrence relation

$$u_{n+1} = u_n + 3n - 2, \quad u_1 = -1.$$

Determine an expression for the $n^{\text{th}}$ term of this sequence.

Give the answer in the form $u_n = \frac{1}{2}(an+b)(n+c)$, where $a$, $b$ and $c$ are integers. **(12)**

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**Question 6**

When $165$ is divided by some integer the quotient is $7$ and the remainder is $R$.

Determine the possible values of $R$. **(7)**

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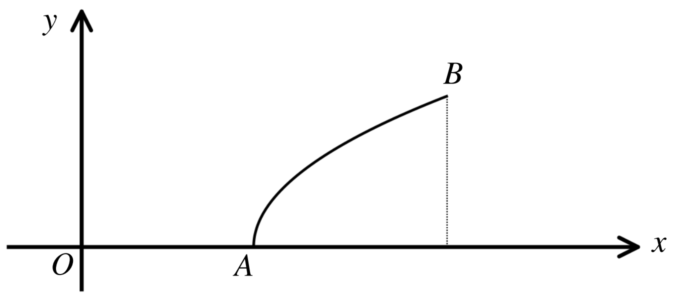
**Question 7**




The figure above shows the curve $C$, given parametrically by the equations

$$x = \frac{1}{2}\cosh 2t, \quad y = 2\sinh t, \; t \in \mathbb{R}.$$

**a)** Show that

$$\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} = 2\cosh^2 t$$ **(4)**

The arc of $C$ from the point $A\left(\frac{1}{2}, 0\right)$ to the point $B\left(\frac{17}{16}, \frac{3}{2}\right)$ is rotated through $2\pi$ radians about the $x$ axis.

**b)** Show that the area of the surface generated is $\frac{61}{24}\pi$ square units. **(5)**

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**Question 8**

A transformation $T$ maps the point $x+iy$ from the $z$ plane to the point $u+iv$ in the $w$ plane, and is defined by

$$w=\frac{z+i}{z}, \; z\in\mathbb{C}, \; z\neq 0.$$

$T$ transforms the line with equation $y=x$ in the $z$ plane, except the origin, into the straight line $L_1$ in the $w$ plane.

**a)** Find a Cartesian equation for $L_1$. **(5)**

$T$ transforms the circle $C_1$ in the $z$ plane, into the circle $C_2$ in the $w$ plane.

**b)** Find the coordinates of the centre of $C_1$ and the length of its radius, given the Cartesian equation of $C_2$ is

$$u^2+v^2=4u.$$ **(7)**

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