IYGB - FS1 PAPER B - QUESTION 1

a) WRITE THE DISTRIBUTION IN "TABLE" FORM

x	1	2	3	4
$P(X=x)$	$4k$	$6k$	$6k$	$4k$

$$4k + 6k + 6k + 4k = 1$$
$$20k = 1$$
$$k = \frac{1}{20}$$

b) AS THE PROBABILITIES ARE SYMMETRICAL & THE "GAPS" IN x ARE EQUAL, BY SYMMETRY

$$E(X) = 2.5$$

c) FIND $E(X^2) = \sum x^2 P(X=x)$

$$E(X^2) = (1^2 \times 4k) + (2^2 \times 6k) + (3^2 \times 6k) + (4^2 \times 4k)$$
$$= 4k + 24k + 54k + 64k$$
$$= 146k$$
$$= 7.3$$

$\text{Var}(X) = E(X^2) - (E(X))^2$

$$= 7.3 - (2.5)^2$$
$$= 1.05$$

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d) $E(4X-5) = 4E(X) - 5$

$= 4 \times 2.5 - 5$

$= 5$

IYGB – FS1 PAPER R – QUESTION 2

a) X = NUMBER OF CARS FROM LONDON

$X \sim B(37, 0.42)$

$$P(X > 22) = P(X \geq 23) = 1 - P(X \leq 22)$$
$$= 1 - 0.9893466...$$
$$= 0.01065...$$
$$\simeq 0.01 \quad \text{IE } 1\%$$

b) Y = NUMBER OF CARS FROM FRANCE

$Y \sim B(80, 0.005)$

AS n IS LARGE & p IS SMALL APPROXIMATE BY POISSON $Po(0.4)$

$$P(Y = 2) = \frac{e^{-0.4} \times 0.4^2}{2!} = 0.0536$$

IYGB - FS1 PAPER R - QUESTION 3

X = NO OF APPLICANTS PER YEAR

$X \sim Po(12)$

SETTING UP SUITABLE HYPOTHESES

$H_0 : \lambda = 12$

$H_1 : \lambda > 12$

WHERE λ IS THE RATE OF APPLICANTS PER YEAR, IN GENERAL

TESTING AT THE 1% LEVEL OF SIGNIFICANCE ON THE BASIS THAT $x = 19$

$$P(X \geq 19) = 1 - P(X \leq 18)$$

...tables/calculator...

$$= 1 - 0.9626$$

$$= 0.0376$$

$$= 3.76\% > 1\%$$

THERE IS NO SIGNIFICANT EVIDENCE, AT 1% LEVEL, THAT THERE HAS BEEN AN INCREASE IN THE NUMBER OF APPLICANTS PER YEAR.

NO SUFFICIENT EVIDENCE TO REJECT H_0

IYGB – FS1 PAPER R – QUESTION 4

H_0 : DATA CAN BE MODELLED BY $B(4, 0.4)$

H_1 : DATA CANNOT BE MODELLED BY $B(4, 0.4)$

FORMING A TABLE

x_i	OBSERVED $= O_i$	EXPECTED $= E_i = P(X = x_i) \times 50$	$\frac{(O_i - E_i)^2}{E_i}$
0	4	$\binom{4}{0} 0.4^0\, 0.6 \times 50^{\cdot} = 6.48$	0.949...
1	20	$\binom{4}{1} 0.4^1 \times 0.6^3 \times 50 = 17.28$	0.428...
2	15	$\binom{4}{2} 0.4^2 \times 0.6^2 \times 50 = 17.28$	0.301....
3	10 $\}$ 11	$\binom{4}{3} 0.4^3 \times 0.6^1 \times 50 = 7.68$ $\}$ 8.96	0.464
4	1	$\binom{4}{4} 0.4^4 \times 0.6^0 \times 50 = 1.28$ LESS THAN 5	

- $\nu = 4 - 1 = 3$
- $\chi^2_3(10\%) = 6.251$
- $\sum_{i=1}^{4} \frac{(O_i - E_i)^2}{E_i} = 2.142$

AS $2.142 < 6.251$ THE IS SUFFICIENT EVIDENCE TO SUPPORT THE CLAIM OF THE MANAGER – REJECT H_1

IYGB – FS1 PAPER R – QUESTION 5

a) THE HIGHEST VALUE CAN ONLY BE 7 (HIGHEST POWER OF t)
THE LEAST VALUE IS ZERO, AS t^0 = CONSTANT IS POSSIBLE

b) LOOKS LIKE A BINOMIAL — COMPARE WITH

$$G_X(t) = (1-p+pt)^n$$

$p = \frac{1}{2}$
$n = 7$

$$G_X(t) = \left(1-\tfrac{1}{2}+\tfrac{1}{2}t\right)^7$$
$$= \left(\tfrac{1}{2}+\tfrac{1}{2}t\right)^7$$
$$= \left(\tfrac{1}{2}\right)^7(1+t)^7$$
$$= \tfrac{1}{128}(1+t)^7$$

$$\therefore\ X \sim B(7, 0.5)$$

[ALSO NOTE THAT IF $G_X(1)=1 \implies k \times 2^7 = 1$
$\implies k = \frac{1}{128}$]

c) USING $X \sim B(7, \frac{1}{2})$

$$P(X=5) = \binom{7}{5}\left(\tfrac{1}{2}\right)^5\left(\tfrac{1}{2}\right)^2 = \frac{21}{128}$$

OR THE COEFFICIENT OF t^5 IN $\frac{1}{128}(1+t)^7$

d) USING THE P.G.F & $E(X) = G_X'(1)$ & $Var(X) = G_X''(1) + G_X'(1) - [G_X'(1)]^2$

$$G_X(t) = \tfrac{1}{128}(1+t)^7$$
$$G_X'(t) = \tfrac{7}{128}(1+t)^6$$
$$G_X''(t) = \tfrac{21}{64}(1+t)^5$$

& $G_X'(1) = \frac{7}{2}$, $G_X''(1) = \frac{21}{2}$

$$E(X) = G_X'(1) = 7/2 = 3.5$$

$$Var(X) = \tfrac{21}{2} + \tfrac{7}{2} - \left(\tfrac{7}{2}\right)^2 = 14 - \tfrac{49}{4} = \tfrac{7}{4} = 1.75$$

IYGB – FS1 PAPER R – QUESTION 6

X = NUMBER OF PHONES SOLD PER DAY

$X \sim Po(3)$

a) I) $P(X=3) = \dfrac{e^{-3} \times 3^3}{3!} = \dfrac{e^{-3} \times 27}{6} = \dfrac{9}{2}e^{-3} = \dfrac{9}{2e^3}$

II) $P(X \geqslant 4) = 1 - P(X \leqslant 3) = 1 - P(X=0,1,2,3)$

$$= 1 - \left[\frac{e^{-3} \times 3^0}{0!} + \frac{e^{-3} \times 3^1}{1!} + \frac{e^{-3} \times 3^2}{2!} + \frac{e^{-3} \times 3^3}{3!} \right]$$

$$= 1 - \left[e^{-3} + 3e^{-3} + \frac{9}{2}e^{-3} + \frac{9}{2}e^{-3} \right]$$

$$= 1 - 13e^{-3}$$

b) WE REQUIRE $P(X=7 \mid X \geqslant 4)$

$$\frac{P(X=7)}{P(X \geqslant 4)} = \frac{\frac{e^{-3} \times 3^7}{7!}}{1 - 13e^{-3}} = \frac{e^{-3} \times 3^7}{7!} \div (1 - 13e^{-3})$$

$$= \frac{243}{560}e^{-3} \times \frac{1}{1 - 13e^{-3}} = \frac{243}{560e^3} \times \frac{1}{1 - 13e^{-3}}$$

$$= \frac{243}{560(e^3 - 13e^{-3}e^3)} = \frac{243}{560(e^3 - 13e^0)} = \frac{243}{560(e^3 - 13)}$$

AS REQUIRED

$(k = 560)$

IYGB - FS1 PAPER R - QUESTION 7

LET X = NUMBER OF PICKS UNTIL A £2 COIN IS SELECTED

LET p = THE PROBABILITY OF PICKING A £2 COIN

HENCE $X \sim Geo(p)$

$$P(X=2) = \frac{3}{16}$$
$$(1-p)p = \frac{3}{16}$$

$$P(X>3) = P(X \geqslant 4) = \frac{27}{64}$$
$$P(X=4,5,6,7,\dots) = \frac{27}{64}$$
$$P(X=1,2,3) = 1 - \frac{27}{64}$$
$$p + (1-p)p + (1-p)^2 p = \frac{37}{64}$$

SOLVING THE FIRST EQUATION FOR p

$$\Rightarrow (1-p)p = \frac{3}{16}$$
$$\Rightarrow p - p^2 = \frac{3}{16}$$
$$\Rightarrow 16p - 16p^2 = 3$$
$$\Rightarrow 0 = 16p^2 - 16p + 3$$
$$\Rightarrow (4p-3)(4p-1) = 0$$
$$\Rightarrow p = \begin{cases} 3/4 \\ 1/4 \end{cases}$$

VERIFY EACH VALUE WITH THE SECOND EQUATION

- IF $p = \frac{3}{4}$ $\quad \frac{3}{4} + \frac{1}{4} \times \frac{3}{4} + \frac{1}{16} \times \frac{3}{4} = \frac{63}{64} \neq \frac{37}{64}$
- IF $p = \frac{1}{4}$ $\quad \frac{1}{4} + \frac{3}{4} \times \frac{1}{4} + \frac{9}{16} \times \frac{1}{4} = \frac{37}{64}$

$\therefore p = \frac{1}{4}$

FINALLY WE HAVE

$$P(X=5) = (1-p)^4 p = \left(\frac{3}{4}\right)^4 \times \frac{1}{4} = \frac{81}{1024} \simeq 0.0791$$

(P.T.O)

IYGB - FS1 PAPER R - QUESTION 7

ALTERNATIVE SOLUTION OF THE EQUATIONS

$$p(1-p)=\frac{3}{16}$$

$$p+(1-p)p+(1-p)^2p=\frac{37}{64}$$

$$p+p(1-p)+p(1-p)(1-p)=\frac{37}{64}$$

$$p+\frac{3}{16}+\frac{3}{16}(1-p)=\frac{37}{64}$$

$$p+\frac{3}{16}+\frac{3}{16}-\frac{3}{16}p=\frac{37}{64}$$

$$\frac{13}{16}p=\frac{13}{64}$$

$$\frac{1}{16}p=\frac{1}{64}$$

$$p=\frac{1}{4}$$

(THIS APPROACH GIVES $\frac{1}{4}$ WITHOUT THE NEED FOR VERIFICATION)

IYGB - FS1 PAPER R - QUESTION 8

- A FAIR DICE FOLLOWS A DISCRETE UNIFORM DISTRIBUTION WITH $n=6$

$$E(X) = \frac{n+1}{2} = \frac{6+1}{2} = 3.5$$

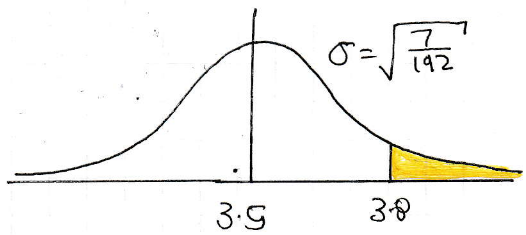
$$\text{Var}(X) = \frac{n^2-1}{12} = \frac{35}{12}$$

- BY THE CENTRAL LIMIT THEOREM, THE MEAN OF 80 OBSERVATIONS, WILL HAVE AN APPROXIMATE DISTRIBUTION

$$\bar{X}_{80} \sim N\left(3.5, \frac{35/12}{80}\right)$$

SIMPLIFIES TO $\frac{7}{192}$

- HENCE WE NOW HAVE



$$\Rightarrow P(\bar{X}_{80} > 3.8)$$

$$= 1 - P(\bar{X}_{80} < 3.8)$$

$$= 1 - P\left(z < \frac{3.8 - 3.5}{\sqrt{7/192}}\right)$$

$$= 1 - \Phi(1.5712)$$

$$= 1 - 0.9419$$

$$= 0.0581$$

IE 5.81%