# IYGB – FS2 PAPER R – QUESTION 1

## CALCULATING SAMPLE STATISTICS

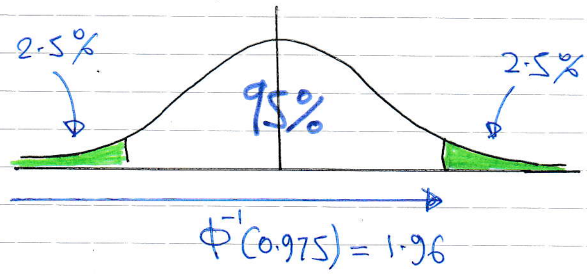
| No of Fossils | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|
| No of Rocks | 11 | 45 | 56 | 66 | 47 | 23 | 9 | 1 |

- $\sum x = 719$ • $\sum x^2 = 2563$ • $n = 258$

$$\bar{x} = \frac{\sum x}{n} = \frac{719}{258} = 2.7868\ldots$$

$$s = \sqrt{\frac{1}{n-1}\left[\sum x^2 - \frac{\sum x \sum x}{n}\right]} = \sqrt{\frac{1}{257}\left[2563 - \frac{719^2}{258}\right]} = 1.47518\ldots$$

## LOOKING AT DIAGRAM BELOW




$$\mu = \bar{x} \pm \frac{s}{\sqrt{n}}\Phi^{-1}(0.975)$$

$$\mu = 2.7868 \pm \frac{1.47518\ldots}{\sqrt{258}} \times 1.96$$

$$\mu = 2.7868 \pm 0.18071\ldots$$

$$\therefore \ CI = (2.608, 2.966)$$

# IYGB - FS2 PAPER R - QUESTION 2

a) TRANSFORMING THE TABLE

| $t$ | 151 | 154 | 157 | 163 | 169 |
|---|---|---|---|---|---|
| $V$ | 8800 | 7800 | 7400 | 6500 | 3100 |

| $x$ | $-2$ | $-1$ | 0 | 2 | 4 |
|---|---|---|---|---|---|
| $y$ | 88 | 78 | 74 | 65 | 31 |

DETERMINING THE SUMMARY STATISTICS

- $\sum x = 3$
- $\sum x^2 = 25$
- $\sum xy = 0$
- $\sum y = 336$
- $\sum y^2 = 24490$
- $n = 5$

CALCULATE $S_{xx}$, $S_{yy}$, $S_{xy}$

$$S_{xx} = \sum x^2 - \frac{\sum x \sum x}{n} = 25 - \frac{3 \times 3}{5} = 23.2$$

$$S_{yy} = \sum y^2 - \frac{\sum y \sum y}{n} = 24490 - \frac{336 \times 336}{5} = 1910.8$$

$$S_{xy} = \sum xy - \frac{\sum x \sum y}{n} = 0 - \frac{3 \times 336}{5} = -201.6$$

b) CALCULATE THE P.M.C.C

$$r = \frac{S_{xy}}{\sqrt{S_{xx} S_{yy}}} = \frac{-201.6}{\sqrt{23.2 \times 1910.8}} = -0.957500\ldots \approx -0.958$$

c) UNCHANGED AT $-0.958$, AS THE P.M.C.C IS INDEPENDENT OF SCALING $(\div 100)$ OR CHOICE OF ORIGIN $(-157)$

# IYGB - FS2 PAPER R - QUESTION 2

d) OBTAIN ALL THE AUXILLIARIES FIRST

$$\bar{x} = \frac{\Sigma x}{n} = \frac{3}{5} = 0.6$$

$$\bar{y} = \frac{\Sigma y}{n} = \frac{336}{5} = 67.2$$

$$b = \frac{S_{xy}}{S_{xx}} = \frac{-201.6}{23.2} = -\frac{252}{29} = -8.68965\ldots$$

$$a = \bar{y} - b\bar{x} = 67.2 - (-8.6865\ldots)(0.6) = 72.41379\ldots \quad \left(\frac{2100}{29}\right)$$

$$\therefore \quad y = a + bx$$

$$\Rightarrow \left(\frac{V}{100}\right) = a + b\left(\frac{t-157}{3}\right)$$

$\times 100$

$$\Rightarrow V = 100a + \frac{100b}{3}(t-157)$$

$$\Rightarrow V = 100a + \frac{100b}{3}t - \frac{15700b}{3}$$

$$\Rightarrow V = 52717 - 290t$$

# IYGB – FS2 PAPER R – QUESTION 3

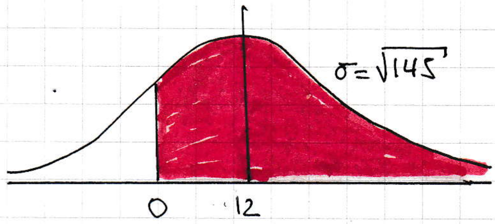
$$\boxed{S \sim N(24, 3^2) \quad \text{AND} \quad T \sim N(30, 4^2)}$$

a) DEFINE A NEW VARIABLE $3S - 2T$ AS $X$

- $E(X) = E(3S - 2T) = 3E(S) - 2E(T) = 3 \times 24 - 2 \times 30 = 12$
- $\text{Var}(X) = \text{Var}(3S - 2T) = 3^2\text{Var}(S) + 2^2\text{Var}(T) = (9 \times 9) + (4 \times 16) = 145$

THUS $X = 3S - 2T \sim N(12, 145)$

$$
\begin{aligned}
& P(3S > 2T) \\
&= P(3S - 2T > 0) \\
&= P(X > 0) \\
&= P\left(Z > \frac{0 - 12}{\sqrt{145}}\right) \\
&= \Phi(-0.9965) \\
&= 0.8405
\end{aligned}
$$




NOTE THAT AS IF WE DO NOT KNOW WHAT PHYSICAL QUANTITIES (IF ANY) THESE VARIABLES REPRESENT, WE MAY ASSUME THAT $X$ CAN TAKE ANY VALUE FROM $-\infty$ TO $\infty$

# IYGB - FS2 PAPER R - QUESTION 4

FORMING A TABLE OF DIFFERENCES (d)

| ATHLETE | A | B | C | D | E | F | G | H | I |
|---|---|---|---|---|---|---|---|---|---|
| TIME ON OLD TRACK | 10.7 | 11.2 | 11.5 | 10.9 | 11.8 | 12.0 | 10.6 | 13.1 | 12.1 |
| TIME ON NEW TRACK | 10.8 | 11.0 | 11.4 | 11.1 | 11.4 | 11.6 | 10.7 | 12.5 | 12.5 |
| DIFFERENCES (d) | 0.1 | −0.2 | −0.1 | 0.2 | −0.4 | −0.4 | 0.1 | −0.6 | 0.4 |

"d = NEW − OLD"

CALCULATE UNBIASED ESTIMATORS FOR THE MEAN & VARIANCE OF d

- $\bar{d} = \frac{\sum d}{n} = \frac{-0.9}{9} = -0.1$
- $s_d^2 = \frac{1}{n-1}\left[\sum d^2 - \frac{\sum d \sum d}{n}\right] = \frac{1}{8}\left[0.95 - \frac{(-0.9)(-0.9)}{9}\right] = \frac{43}{400}$

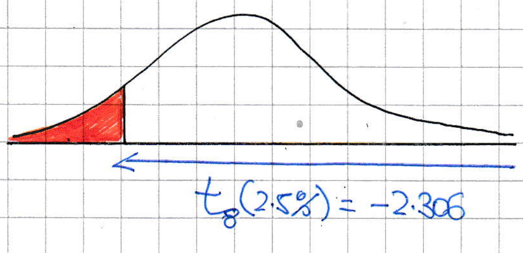
SETTING HYPOTHESES, WHERE $\mu_d$ IS THE POPULATION MEAN DIFFERENCE

- $H_0 : \mu_d = 0$
- $H_1 : \mu_d < 0$

OR

- $H_0 : \mu_{BEF} = \mu_{AFTER}$
- $H_1 : \mu_{BEF} > \mu_{AFTER}$

USING A t DISTRIBUTION, WITH $\nu = 8$, AT 2.5% SIGNIFICANCE




$$t\text{-STATISTIC} = \frac{\bar{d} - \mu_d}{s_d/\sqrt{n}}$$

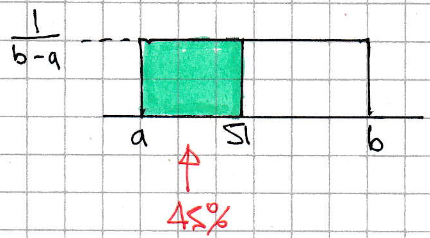
$$t\text{-STATISTIC} = \frac{-0.1 - 0}{\sqrt{\frac{43/400}{9}}}$$

$$t\text{-STATISTIC} = -0.91499\ldots$$

AS $-2.306 < -0.915 < 2.306$, THERE IS NO SIGNIFICANT EVIDENCE TO SUPPORT THE CLAIM — INSUFFICIENT EVIDENCE TO REJECT $H_0$

# IYGB - FS2 PAPER R - QUESTION 5

a) LOOKING AT THE DIAGRAMS BELOW





$$(51-a)\times\frac{1}{b-a}=0.45$$

$$(b-78)\times\frac{1}{b-a}=0.1$$

OR BETTER

$$(78-a)\times\frac{1}{b-a}=0.9$$

ELIMINATE $b-a$ AS A WHOLE

$$\left.\begin{aligned}\frac{51-a}{0.45}&=b-a\\ \frac{78-a}{0.9}&=b-a\end{aligned}\right\}\Rightarrow \frac{51-a}{0.45}=\frac{78-a}{0.9}$$

$$\Rightarrow 45.9-0.9a=35.1-0.45a$$

$$10.8=0.45a$$

$$a=24$$

USING EITHER EQUATION WITH $a=24$

$$\frac{78-a}{0.9}=b-a$$

$$60=b-24$$

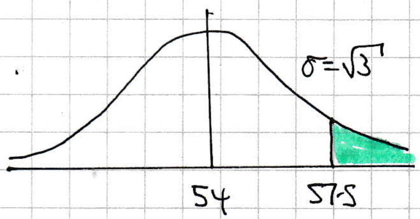
$$b=84$$

$$\therefore f(x)=\begin{cases}\frac{1}{60} & 24<x<84\\ 0 & \text{OTHERWISE}\end{cases}$$

# IYGB - FS2 PAPER R - QUESTION 5

b) METHOD A – USING THE SAMPLE MEAN (CENTRAL LIMIT THEOREM APPLIES AS $n=100$ IS LARGE)

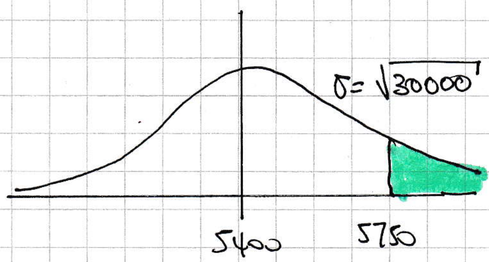
- $E(X) = \frac{84+24}{2} = 54$
- $Var(X) = \frac{(84-24)^2}{12} = 300$
- $\bar{X}_{100} \sim N\left(54, \sqrt{\frac{300}{100}}\right)$




$$P(\$ > 5750) = P(\bar{X}_{100} > 57.5)$$
$$= 1 - P(\bar{X}_{100} < 57.5)$$
$$= 1 - P\left(Z < \frac{57.5-54}{\sqrt{3}}\right)$$
$$= 1 - \Phi(2.0207)$$
$$= 1 - 0.9783$$
$$= 0.0217$$

METHOD B – USING THE SUM DIRECTLY (CENTRAL THEOREM APPLIES ALSO)

$$\$ = X_1 + X_2 + X_3 + \dots + X_{100}$$
$$E(\$) = 100 \times 54 = 5400$$
$$Var(\$) = 100 \times 300 = 30000$$




$$P(\$ > 5750) = 1 - P(\$ < 5750)$$
$$= 1 - P\left(Z < \frac{5750-5400}{\sqrt{30000}}\right)$$
$$= 1 - \Phi(2.0207)$$
$$= 1 - 0.9783$$
$$= 0.0217$$

# IYGB – FS2 PAPER R – QUESTION 6

a) USING $f(x) = \int_a^b f(x)\,dx = 1$

$$\int_2^5 \frac{2+x}{k}\,dx = 1 \implies \frac{1}{k}\int_2^5 2+x\,dx = 1$$

$$\implies \left[2x + \frac{1}{2}x^2\right]_2^5 = k$$

$$\implies k = \left(10 + \frac{25}{2}\right) - (4+2)$$

$$\implies k = \frac{33}{2}$$

b) $E(X) = \int_a^b x f(x)\,dx$

$$E(X) = \int_2^5 x \times \frac{2}{33}(2+x) = \frac{2}{33}\int_2^5 2x + x^2\,dx = \frac{2}{33}\left[x^2 + \frac{1}{3}x^3\right]_2^5$$

$$= \frac{2}{33}\left[\left(25 + \frac{125}{3}\right) - \left(4 + \frac{8}{3}\right)\right] = \frac{2}{33} \times 60 = \frac{40}{11} \simeq 3.64$$

c) COMPUTING $E(X^2) = \int_a^b x^2 f(x)\,dx$

$$E(X^2) = \int_2^5 x^2 \times \frac{2}{33}(2+x)\,dx = \frac{2}{33}\int_2^5 2x^2 + x^3\,dx = \frac{2}{33}\left[\frac{2}{3}x^3 + \frac{1}{4}x^4\right]_2^5$$

$$= \frac{2}{33}\left[\left(\frac{250}{3} + \frac{625}{4}\right) - \left(\frac{16}{3} + 4\right)\right] = \frac{2}{33} \times \frac{921}{4} = \frac{307}{22}$$

$\mathrm{Var}(X) = E(X^2) - [E(X)]^2$

$$= \frac{307}{22} - \left(\frac{40}{11}\right)^2$$

$$= \frac{177}{242} \simeq 0.731$$

AS REQUIRED

# IYGB - FS2 PAPER R - QUESTION 6

d) USING $F(x) = \int_a^x f(x)\,dx$

$$F(x) = \int_2^x \frac{2}{33}(2+x)\,dx = \frac{2}{33}\int_2^x 2+x\,dx = \frac{2}{33}\left[2x+\frac{1}{2}x^2\right]_2^x$$

$$= \frac{2}{33}\left[\left(2x+\frac{1}{2}x^2\right)-(4+2)\right] = \frac{2}{33}\left[\frac{1}{2}x^2+2x-6\right]$$

$$\therefore F(x) = \begin{cases} 0 & x<2 \\ \frac{1}{33}(x^2+4x-12) = \frac{1}{33}(x-4)(x-6) & 2 \leq x \leq 5 \\ 1 & x>5 \end{cases}$$

e) SOLVING $F(x) = \frac{1}{2}$

$$\Rightarrow \frac{1}{33}(x^2+4x-12) = \frac{1}{2}$$

$$\Rightarrow x^2+4x-12 = 16.5$$

$$\Rightarrow 2x^2+8x-24 = 33$$

$$\Rightarrow 2x^2+8x-57=0$$

BY THE QUADRATIC FORMULA

$$x = \frac{-8 \pm \sqrt{8^2-4\times 2(-57)}}{2\times 2} = \frac{-8\pm\sqrt{520}}{4} = \begin{cases} 3.70 \\ \cancel{-7.70} \end{cases}$$

$\therefore$ MEDIAN $= 3.70$

# IYGB - FS2 PAPER R - QUESTION 7

LABELLING THE POINTS AS "A - F" FROM LEFT TO RIGHT

| POINT | A | B | C | D | E | F |
|---|---|---|---|---|---|---|
| Rank in $x$ | 6 | 5 | 4 | 3 | 2 | 1 |
| Rank in $y$ | 6 | 5 | 3 | 4 | 1 | 2 |
| $d^2$ | 0 | 0 | 1 | 1 | 1 | 1 |

USING THE STANDARD FORMULA WITH $\sum d^2 = 4$

$$r_s = 1 - \frac{6\sum d^2}{n(n^2-1)} = 1 - \frac{\cancel{6} \times 4}{\cancel{6} \times 35} = 1 - \frac{4}{35} = \frac{31}{35} \approx 0.886$$