YGB GCE

Mathematics SYNF

Advanced Level

Synoptic Paper A

Difficulty Rating: 3.525/0.5657

Time: 3 hours

Candidates may use any calculator allowed by the regulations of this examination.

Information for Candidates

This synoptic practice paper follows closely the Advanced Level Further Pure Mathematics Syllabus, suitable for first assessment Summer 2018.

The standard booklet "Mathematical Formulae and Statistical Tables" may be used.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 24 questions in this question paper.
The total mark for this paper is 200.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
Non exact answers should be given to an appropriate degree of accuracy.
The examiner may refuse to mark any parts of questions if deemed not to be legible.

Question 1

The vectors $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ are not parallel.

Simplify fully

$$\mathbf{a} \cdot \left[\mathbf{b} \wedge (\mathbf{c}+\mathbf{a}) \right] \qquad (2)$$

Question 2

Find the general solution of the following trigonometric equation

$$\tan 2x + \tan 4x = 0,$$

where x is measured in radians. (5)

Question 3

The curve C has polar equation

$$r = 6\cos 3\theta, \ -\pi < \theta \le \pi.$$

a) Sketch the graph of C. (3)

b) Find the exact value of area enclosed by the C, for $-\dfrac{\pi}{6} < \theta \le \dfrac{\pi}{6}$. (5)

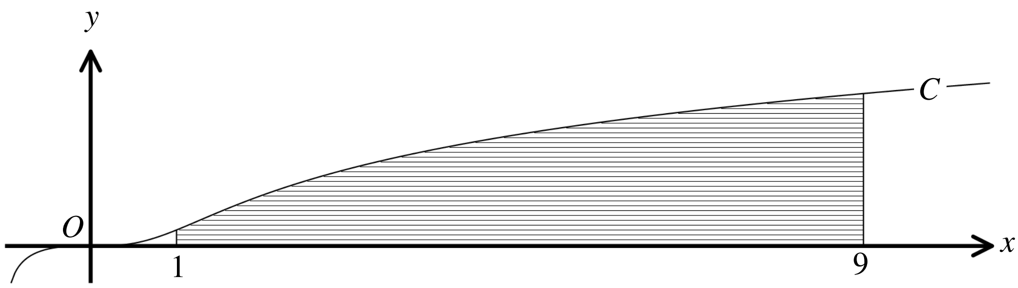
Question 4

$$z^4 = -16, \ z \in \mathbb{C}.$$

a) Determine the solutions of the above equation, giving the answers in the form $a + b\mathrm{i}$, where a and b are real numbers. (6)

b) Plot the roots of the equation as points in an Argand diagram. (2)

Question 5



The figure above shows part of the curve C with equation

$$y = \ln\left(1 + x^3\right), \quad x > -1 .$$

The area of the shaded region bounded by C, the x axis and the straight lines with equations $x = 1$ and $x = 9$ is to be estimated by the mid-ordinate rule using 4 equally spaced strips.

Find an estimate for the area of this region.

All steps in the calculation must be shown and the final answer must be correct to 3 significant figures. **(5)**

Question 6

Find, in exact simplified logarithmic form, the y coordinate of the stationary point of the curve with equation

$$y = 5 - 12x + 4\operatorname{arcosh}(4x) .$$

Detailed workings must be shown. **(8)**

Question 7

The general point $P\left(9t^2, 18t\right)$, where t is a parameter, lies on the parabola with Cartesian equation

$$y^2 = 36x.$$

a) Show that the equation of a tangent at the point P is given by

$$x - ty + 9t^2 = 0. \qquad (4)$$

The tangent to the parabola $y^2 = 36x$ at the point $Q\,(1,6)$ crosses the directrix of the parabola at the point D.

b) Find the coordinates of D. **(4)**

Question 8

The three planes defined by the equations

$$\begin{aligned} x + 2y + z &= 2 \\ 2x + a\,y + z &= 2 \\ x + y + 2z &= b \end{aligned}$$

where a and k are constants, intersect along a straight line L.

Determine an equation of L. **(8)**

Question 9

By considering the derivatives of $\mathrm{e}^x \sin x$ and $\mathrm{e}^x \cos x$, find

$$\int \mathrm{e}^x \left(2\cos x - 3\sin x\right)\, dx. \qquad (8)$$

Question 10

Bernoulli's inequality asserts that if $a \in \mathbb{R}$, $a > -1$ and $n \in \mathbb{N}$, $n \geq 2$, then

$$(1+a)^n > 1 + an.$$

Prove, by induction, the validity of Bernoulli's identity. **(6)**

Question 11

The three roots of the cubic equation

$$x^3 + 2x - 1 = 0,$$

are denoted by α, β and γ.

Determine the exact value of $\dfrac{1}{\alpha^4} + \dfrac{1}{\beta^4} + \dfrac{1}{\gamma^4}$. **(10)**

Question 12

$$\frac{2t}{(t+1)(t^2+1)} \equiv \frac{A}{t+1} + \frac{Bt+C}{t^2+1}.$$

a) Determine the values of A, B and C in the above identity. **(3)**

b) Hence find an exact simplified value for

$$\int_0^{\frac{\pi}{2}} \sqrt{\frac{1-\cos x}{1+\sin x}}\, dx.$$ **(9)**

Question 13

The straight line l has the following vector equation

$$\mathbf{r} = -2\mathbf{i} - 12\mathbf{j} - 9\mathbf{k} + \lambda(\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}),$$

where λ is a scalar parameter.

The point $P(a,b,3)$ lies on l.

a) Find the value of each of the scalar constants a and b. **(2)**

The point O represents a fixed origin.

The point Q lies on l, so that OQ is perpendicular to l.

b) Show that the coordinates of Q are $(2,0,-1)$.

You may not verify this fact by using the coordinates of Q. **(4)**

c) Find the exact area of the triangle OPQ. **(3)**

Question 14

$$\frac{d^2y}{dx^2} + y = \sin 2x, \text{ with } y = 0, \ \frac{dy}{dx} = 0 \ \text{ at } x = \frac{\pi}{2}.$$

Show that a solution of the above differential equation is

$$y = \frac{2}{3}\cos x(1 - \sin x).$$ **(11)**

Question 15

A plane Π has Cartesian equation

$$2x+3y+4z=24.$$

Determine a Cartesian equation for the transformation of Π under the matrix

$$\begin{pmatrix} 2 & 3 & 1 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \end{pmatrix}.$$

(8)

Question 16

$$y = \arcsin x, \quad -1 \le x \le 1.$$

a) Show that

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}.$$

(4)

The point $P\left(\frac{1}{6},k\right)$, where k is a constant lies on the curve with equation

$$\arcsin 3x + 2\arcsin y = \frac{\pi}{2}, \quad |x| \le \frac{1}{3}, \quad |y| \le 1.$$

b) Find the value of the gradient at P. **(6)**

Question 17

By considering the area of two different rectangles of unit width under and above the graph of $y=\frac{1}{x}$, show that

$$\left(1+\frac{1}{n}\right)^n < \mathrm{e} < \left(1+\frac{1}{n}\right)^{n+1}$$

(8)

Question 18

$$f(x) \equiv \frac{3x-1}{x+2}, \; x \in \mathbb{R}, \; x \neq -2.$$

a) Sketch the graph of $f(x)$.

The sketch must include the coordinates of any points where the graph meets the coordinate axes and the equations of any asymptotes. **(5)**

b) Hence, or otherwise, solve the inequality

$$\left| \frac{3x-1}{x+2} \right| < 2.$$ **(7)**

Question 19

A curve C, with equation $y = f(x)$, meets the y axis the point with coordinates $(0,1)$.

It is further given that the equation of C satisfies the differential equation

$$\frac{dy}{dx} = x - 2y.$$

a) Determine an equation of C. **(7)**

b) Sketch the graph of C.

The graph must include in exact simplified form the coordinates of the stationary point of the curve and the equation of its asymptote. **(6)**

Question 20

Find an exact simplified value for the following integral.

$$\int_0^{\frac{1}{4}\pi} \frac{1}{1+3\sin^2 x} \, dx.$$ **(8)**

Question 21

When some positive integer N is divided by 4, the quotient is 3 times as large as the remainder.

Determine the possible values of N. **(4)**

Question 22

The complex numbers z_1 and z_2 are given by

$$z_1 = 1 + \mathrm{i}\sqrt{3} \quad \text{and} \quad z_2 = \mathrm{i}z_1 .$$

a) Label the points representing z_1 and z_2, in an Argand diagram. **(1)**

b) On the same Argand diagram, sketch the locus of the points z satisfying …

i. … $|z - z_1| = |z - z_2|$. **(2)**

ii. … $\arg(z - z_1) = \arg z_2$. **(2)**

c) Determine, in the form $x + \mathrm{i}y$, the complex number z_3 represented by the intersection of the two loci of part **(b)**. **(7)**

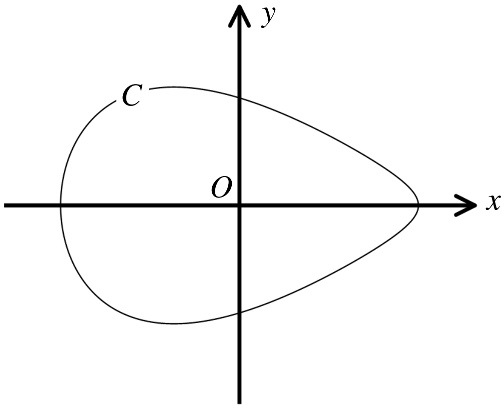
Question 23

The 2×2 matrix $\mathbf{M}$ is defined by

$$\mathbf{M} = \begin{pmatrix} 0 & 3 \\ 3 & 0 \end{pmatrix}.$$

Find, by calculation, the equations of the two lines which pass through the origin, that remain invariant under the transformation represented by $\mathbf{M}$. **(6)**

Question 24



The figure above shows the closed curve C with parametric equations

$$x = \cos\theta\,,\ \ y = \sin\theta - \tfrac{1}{4}\sin 2\theta\,,\ \ 0 \le \theta < 2\pi\,.$$

The curve is symmetrical about the x axis.

The finite region enclosed by C is revolved by π radians about the x axis, forming a solid of revolution S.

Show that the volume of S is given by

$$\pi\int_0^{\pi} \sin^3\theta\left(1-\tfrac{1}{2}\cos\theta\right)^2\ d\theta\,,$$

and by using the substitution $u = \cos\theta$, or otherwise, determine an exact value for the volume of S.

(11)

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