OCR
Oxford Cambridge and RSA

GCE

Further Mathematics B (MEI)

Unit **Y414/01**: Numerical Methods

Advanced Subsidiary GCE

Mark Scheme for June 2018

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

Annotations and abbreviations

Annotation in scoris	**Meaning**
✓and ✖	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining a result or establishing a given result
dep*	Mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This indicates that the instruction **In this question you must show detailed reasoning** appears in the question.

Subject-specific Marking Instructions for AS Level Mathematics B (MEI)

a Annotations should be used whenever appropriate during your marking. The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded. For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
If you are in any doubt whatsoever you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep*' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case please, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is numerically correct and expressed either in SI or in the units of the question. (e.g. lengths will be assumed to be in metres unless in a particular question all the lengths are in km, when this would be assumed to be the unspecified unit.) We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so. When a value is given in the paper only accept an answer correct to at least as many significant figures as the given value. This rule should be applied to each case. When a value is not given in the paper accept any answer that agrees with the correct value to 2 s.f. Follow through should be used so that only one mark is lost for each distinct accuracy error, except for errors due to premature approximation which should be penalised only once in the examination. There is no penalty for using a wrong value for *g*. E marks will be lost except when results agree to the accuracy required in the question.

g Rules for replaced work: if a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests; if there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others. NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question. Marks designated as cao may be awarded as long as there are no other errors. E marks are lost unless, by chance, the given results are established by equivalent working. 'Fresh starts' will not affect an earlier decision about a misread. Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

i If a graphical calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers (provided, of course, that there is nothing in the wording of the question specifying that analytical methods are required). Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

j If in any case the scheme operates with considerable unfairness consult your Team Leader.

Question			Answer	Marks	AOs		Guidance
1	(i)		$\frac{3.085-2.705}{1.065-1.055}$ for maximum	M1	1.1	allow **M1** if either numerator or denominator is correct	
			$\frac{3.075-2.715}{1.075-1.045}$ for minimum	M1	1.1	allow **M1** if either numerator or denominator is correct	
			12 and 38 identified	A1	1.1		
			the denominator involves the difference between two numbers which are very close in value, leading to instability in the calculation	B1	2.4		must mention subtraction of nearly equal numbers
				[4]			
2	(i)		$\frac{0.25}{3}[0.630930 + 4(0.738140 + 0.920799) + 2 \times 0.834044 + 1]$	B1	1.1	$h = 0.25$ used in correct formula	or **B1** for $T_2 = 0.8247545$ or $M_2 = 0.8294695$ and **M1** for their $(2M_2 + T_2)/3$ evaluated if **B0M0** allow **SC2** for $S_2 = 0.827851$ www NB $T_1 = 0.815465$ $M_1 = 0.834044$
				M1	1.1a	formula has correct structure and values correctly placed; allow bracket error – eg omission of outer brackets	
			0.827898	A1	1.1		
				[3]			
2	(ii)		eg the values in the table are only given to 6 dp eg h is probably too large to achieve such accuracy oe	B1	2.4		eg Simpson's rule is only an approximation scores 0
				[1]			

Question			Answer	Marks	AOs		Guidance
3	(i)		fixes the column of the second term in subsequent calculations	B1 [1]	1.2	allow fixes the second term in the subtraction as the value in cell B2 as the formula is dragged across oe	do not allow "dragged down"
3	(ii)		forward difference oe isw	B1 [1]	1.2		
3	(iii)		$-0.14281171 - 0.00000143402 \times \frac{0.1}{1-0.1}$ $-0.142811869(336\ldots)$ so -0.14281187 is probable	M1 M1 A1 A1 [4]	3.1a 1.1 1.1 2.2b	for extrapolation from -0.14281171 with 1.43402×10^{-6} using $r = 0.1$ allow -0.1428119 or -0.142812	
3	(iv)		$f(2.05) \approx f(2) + 0.05 \times f'(2)$ soi 0.632859	M1 A1FT [2]	3.1a 1.1	**NB** $0.64 - 0.05 \times 0.142812$; allow any value for the derivative from row 6 **FT** their -0.14281187 correct to 3 or more dp if unsupported, allow **B2** for 0.632859 or 0.63286	If **M0** allow **SC1** for 0.632938…isw from linear interpolation correct to 4 or more dp; may be unsupported

Question			Answer	Marks	AOs		Guidance
4	(i)		$e^x - x^2 = 2x$ $x = \frac{e^x - x^2}{2}$ $x_{r+1} = \frac{e^{x_r} - x_r^2}{2}$	**M1** **A1** **[2]**	**1.1** **2.5**	at least one step shown **NB AG** suffixes needed	
4	(ii)		$\lvert g'(\alpha)\rvert < 1$ oe in words $\lvert g'(\beta)\rvert \geq 1$ oe in words	**B1** **B1** **[2]**	**2.4** **2.4**	allow $0 < g'(\alpha) < 1$ but do not allow eg $g'(\alpha) < 1$	inequality must be strict for convergence allow reference to "gradient of curve", but not just "gradient" in each case
4	(iii)		1 0.859141 0.811504 0.796376 0.791638 0.790159 0.789698 0.789554 0.789509 0.789495 0.789491 0.78949 0.789489 0.789489 0.789489	**M1** **A1** **[2]**	**1.1** **1.1**	0.5 0.699360635 0.761680149 0.780857607 0.786802656 0.78865212 0.789228173 0.789407666 0.789463602 0.789481033 0.789486466 0.789488159 0.789488686 0.789488851	x_0 must be stated and two more iterates found; allow any x_0 which converges **0** if unsupported
4	(iv)		with $\lambda = -0.8$	**M1**	**1.1a**	$x_0 = 2$ and two more iterates found	

Question	Answer	Marks	AOs		Guidance
	2 2.24437756 2.28095534 2.27241255 2.27477369 2.27414368 2.27431347 2.27426783 2.27428011 2.27427681 2.27427769 2.27427745 2.27427752 2.2742775 2.27427751 2.27427751	**A1**	**1.1**	any two correct iterates	**0** if unsupported
	eg 2.24437756 and eg 2.28095534 seen to 3 or more dp	**A1**	**1.1**		
	2.274278 cao	**[3]**			
	with $\lambda = 0.8$	**M1**	**1.1**		

Question			Answer	Marks	AOs		Guidance
			2 1.75562 1.43306 1.14175 0.95981 0.86795 0.82506 0.80553 0.79671 0.79273 0.79095 0.79014 0.78978 0.78962 0.78955 0.78952 0.7895 0.78949 0.78949 converges to α	**A1** **[2]**	**1.1**	$x_0 = 2$ and two more iterates seen	**0** if unsupported
5	**(i)**		$\frac{(t-1)(t-3)}{(0-1)(0-3)} \times 2.21 + \frac{t(t-3)}{(1-0)(1-3)} \times 4$ $+ \frac{(t-0)(t-1)}{(3-0)(3-1)} \times 7.34$ $[x =] -0.04t^2 + 1.83t + 2.21$ cao	**M1** **A1** **A1** **A1** **[4]**	**3.1a** **3.3** **1.1** **1.1**	use of Lagrange with 3 terms all terms correctly placed two of three terms correct all three correct	If x or other variable used instead of t throughout, allow a maximum of **M1A1A1** ; but allow recovery if final answer given in terms of t; if all the coefficients are correct but any are left unsimplified , allow **M1A1A1**
5	**(ii)**		substitution of $t = 2$ in their $[x =] -0.04t^2 + 1.83t + 2.21$ $[x] = 5.71$ so good fit cao	**M1** **A1** **[2]**	**3.4** **1.1**	correct answer stated for their polynomial implies **M1** must see a comment	
5	**(iii)**		(After peaking) they will decrease in value (until they eventually become worthless)	**B1** **[1]**	**3.2b**	**B0** for exponential decay	**FT** their quadratic

Question			Answer	Marks	AOs		Guidance
5	(iv)		The third column of differences contains approximately equal values, so the proposal is reasonable	B1 [1]	3.2a	or 4th column all approximately 0	
5	(v)		$[x =] 2.21 + 1.79t - 0.06 \times \frac{t(t-1)}{2!} -$ $0.06 \times \frac{t(t-1)(t-2)}{3!}$ $2.21 + 1.79t - 0.03t^2 + 0.03t - 0.01t^3$ $+0.03t^2 - 0.02t$ completion to $x = -0.01t^3 + 1.8t + 2.21$ www	M1 A1 M1 A1 [4]	1.1a 1.1 1.1 1.1	Newton's interpolation formula used, allow one slip all four terms correct or equivalent intermediate step **NB AG**	if other variable used allow a maximum of **M1A0M1A0** allow one error allow full marks if recovery from use of other variable with supporting comment
5	(vi)		(since the coefficient of t^3 is negative), the model predicts that in the long run the value of a unit will decrease (and eventually be less than zero).	B1 [1]	3.5b	allow eg can't extrapolate beyond $t = 7$ oe	
6	(i)		f(3) = – 0.3 and f(4) = 1.4 seen	B1 [1]	1.1		
6	(ii)		(a first) approximation to the root [using the algorithm for false position] oe	B1 [1]	1.1	if method referenced it must be false position	allow eg where chord between (3,– 0.3) and (4, 1.4) cuts the *x*-axis
6	(iii)		=0.1*F2^3–2*F2+3	B1 [1]	1.1	allow superfluous brackets as long as order of operation is not compromised; must see = sign	**B0** for eg G2=0.1*F2^3–2*F2+3
6	(iv)		sign check isw	B1 [1]	2.4	allow any explanation based on sign check or ensuring the straddling of the root	references to a, b and c do not score unless a, b and c are defined in terms of the spreadsheet variables
6	(v)		(a logical) test to determine whether the value in B3 should be the value in B2 (3) or the value in F2 (3.176471)	B1 [1]	2.4	may be supported by reference to sign of value in G2	

Question			Answer	Marks	AOs		Guidance
6	(vi)		3.305	B1 [1]	2.2b	allow 3.3049	
6	(vii)		the values in column L are decreasing slowly which suggests convergence is slightly faster than first order. the values in column M are increasing, so convergence is slower than/not second order.	B1 B1 [2]	2.2b 2.2a	allow approximately constant so first order convergence must mention convergence is not 2nd order	

Question			Answer	Marks	AOs		Guidance
7	(i)		over-estimate with explanation based on position of trapezia relative to curve or on increasing gradient over the interval	B1 [1]	1.2	allow explanation based on diagram including concave up or convex down	"concave" or "convex" scores **0**
7	(ii)		=(N2+O2)/2	M1 A1 [2]	1.1 1.1	**M1** for use of correct formula for T_2 soi eg O3 =(N2+O2)/2 scores **M1A0**	or =0.5*(N2+O2) or =1/2*(N2+O2) for 2 marks
7	(iii)		$2.20808\ldots < I < 2.20851\ldots$ or comparison of M_{32} and T_{32} 2.21	B1 B1 [2]	1.1 2.2b	or calculation of $S_{64} = 2.2082(2699467)$ or error in M is half error in T so allow 2.208	
7	(iv)		it is reasonable to make this assumption since it is nearly always the case that the ratios behave in the same way for all three methods oe	B1 [1]	2.4		
7	(v)	(A)	$\frac{2\times 2.207650029 + 2.209380966}{3}$ or $\frac{4\times 2.209380966 - 2.212842207}{3}$ $S_{32} = 2.208227008$ $S_{64} = 2.208226994(67)$	M1 A1 A1 [3]	3.1a 2.1 1.1	or $\frac{2\times 2.208082743 + 2.208515498}{3}$ or $\frac{4\times 2.208515498 - 2.209380966}{3}$ correct to 8 or 9 dp correct to 8 or 9 dp	allow **M1A1** if S_{32} found correctly but incorrectly attributed; the second **A1** is then not available
		(B)	assumes $r = 0.0625$ and extrapolates accordingly 2.20822699(378) so 2.20822699(4) justified eg by comparison with S_{64}	M1 A1 A1 [3]	3.1a 1.1 2.2b	$(16S_{64} - S_{32})/15$ oe FT correct to 8 or more dp allow 2.2082270	allow justification based on extrapolation improving accuracy

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