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AS LEVEL

Examiners' report

FURTHER MATHEMATICS B (MEI)

H635
For first teaching in 2017

Y416/01 Summer 2018 series

Version 1

Contents

Introduction

Our examiners' reports are produced to offer constructive feedback on candidates' performance in the examinations. They provide useful guidance for future candidates. The reports will include a general commentary on candidates' performance, identify technical aspects examined in the questions and highlight good performance and where performance could be improved. The reports will also explain aspects which caused difficulty and why the difficulties arose, whether through a lack of knowledge, poor examination technique, or any other identifiable and explainable reason.

Where overall performance on a question/question part was considered good, with no particular areas to highlight, these questions have not been included in the report. A full copy of the question paper can be downloaded from OCR.

Paper Y416/01 overview

Y416 (Statistics b) is an optional examination component for the new revised AS Level examination for GCE Further Mathematics B (MEI). This component extends the content of Y412 (Statistics a) to include continuous random variables, confidence intervals, further hypothesis tests and simulation. To do well on this paper, candidates need to be familiar with the content of Y412 and be comfortable applying their knowledge and understanding to new and unfamiliar contexts.

As with Y412, many of the questions in Y416 are in context and require interpretation in addition to understanding. Questions may also require candidates to comment about the modelling assumptions underlying their answers. Candidates should also have gained experience of using a spreadsheet for simulating random variables.

Candidate performance overview

Candidates who did well on this paper generally did the following.

- Performed standard calculations showing clear working and rounding appropriately.
- Produced clear and concise responses to questions asking for explanation and/or interpretation, e.g. 1(iv), 4(iv), 4(vi), 4(vii), 4(viii), 5(i), 6(ii), 6(iii), 6(iv).
- Found the parameters of Normal distributions for combinations of Normal variables (question 2).
- Linked different areas of the specification (question 4).
- Applied knowledge and understanding to questions set in a novel context.

Candidates who did less well on this paper generally did the following.

- Showed insufficient working and over-rounding, e.g. 2, 3(iii), 6(i).
- Found it difficult to apply what they had learnt when dealing with new contexts.
- Produced responses that lacked depth in questions asking for explanation, sometimes relying on stock phases such as 'the sample is large', e.g. 4(iv), 4(vi), 4(viii), 6(ii), 6(iii), 6(iv).
- Made errors in dealing with probability density functions: (question 3).

Question 1(i)

1 The birth weights, in kilograms, of a random sample of 9 captive-bred elephants are as follows.

94 138 130 118 146 165 82 115 69

A researcher uses software to produce a 99% confidence interval for the mean birth weight of captive-bred elephants. The output from the software is shown in Fig. 1.

T Estimate of a Mean

Confidence Level 0.99

Sample

Mean 117.444

s 31.2574

N 9

Result

T Estimate of a Mean

Mean	117.444
s	31.2574
SE	10.4191
N	9
df	8
Lower limit	82.4838
Upper limit	152.4042
Interval	117.444 ± 34.9602

Fig. 1

(i) State an assumption about the distribution of the population from which these weights come that is necessary in order to produce this interval. **[1]**

Question 1(ii)

(ii) State the confidence interval which the software gives, in the form $a < \mu < b$. **[1]**

Question 1(iii)

(iii) Explain

- what the label df means,
- how the value of df is calculated for a confidence interval produced using the t distribution. **[2]**

Almost all candidates correctly explained the meaning of the label df. Most gained only one of the two marks available however, since although they stated that df = n – 1, they did not say what n represented and so could not be credited the second mark. In the software output there is a label N, but it is not defined in the output and so the second mark was only credited if there was a clear definition of n as the sample size. Some candidates defined n incorrectly as 'the number of samples'.

Question 1(iv)

(iv) State two ways in which the researcher could have obtained a narrower confidence interval. **[2]**

The majority of candidates scored both marks, however, some stated that 'a different confidence level' was required, rather than a lower confidence level.

Question 2(i)

2 A supermarket sells oranges. Their weights are modelled by the random variable X which has a Normal distribution with mean 345 grams and standard deviation 15 grams. When the oranges have been peeled, their weights in grams, Y, are modelled by $Y = 0.7X$.

(i) Find the probability that a randomly chosen peeled orange weighs less than 250 grams. **[3]**

This question was well answered, and almost all candidates found the new mean correctly. A number of candidates used a wrong variance of 0.7×15^2 rather than $(0.7 \times 15)^2$ and so could only gain the first mark for the Normal distribution and the correct mean. Candidates should be advised that in questions involving the calculation of Normal probabilities, they should state the distribution that they are using in the form $N(\mu, \sigma^2)$. They should also show how they have calculated the new mean and variance, so that if they have not got the correct final values of these they may still at least be credited some method marks.

Question 2(ii)

I randomly choose 5 oranges to buy.

(ii) Find the probability that the total weight of the 5 unpeeled oranges is at least 1800 grams. **[2]**

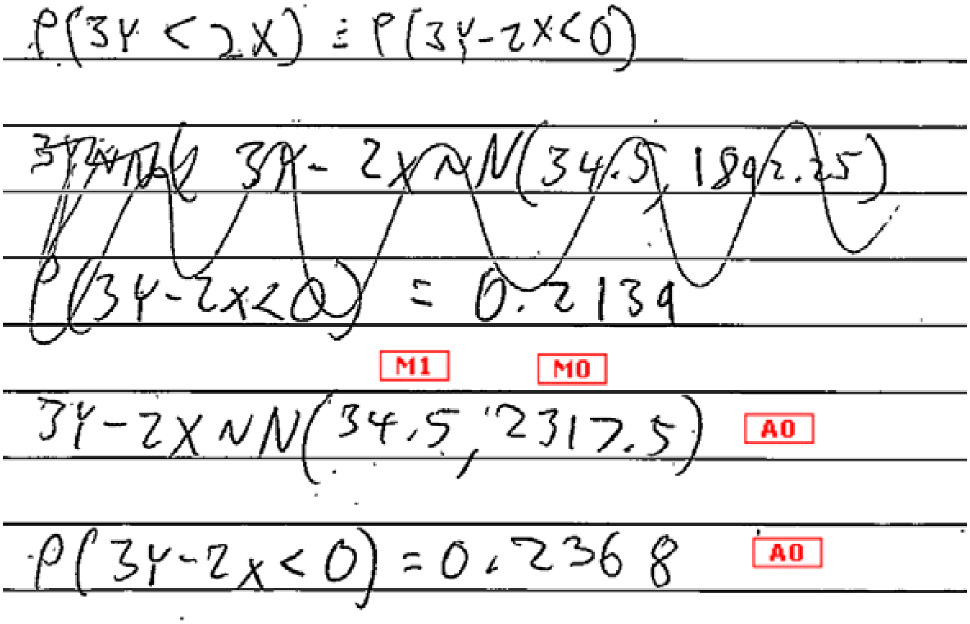
This question was again well answered, but some candidates again used the wrong variance (usually $(5\times15)^2$ rather than 5×15^2).

Question 2(iii)

(iii) I peel three of the oranges and leave the remaining two unpeeled. Find the probability that the total weight of the two unpeeled oranges is greater than the total weight of the three peeled ones. **[4]**

Although this was reasonably well answered, many candidates either scored zero or only scored the first mark for a correct mean. In this part in particular, it would have been pleasing to see more candidates show their working for the new variance.

Exemplar 1



This is a one of several responses in which the candidate gains the first mark for stating that the distribution is Normal with mean -34.5. The variance is however then simply stated to be 2317.5, without any working at all. If the stated variance is wrong, as it is in this case, then no further marks are available. If there had been working, then another mark might have been scored.

Question 3(i)(a)

3 The probability density function of the continuous random variable X is given by

$$f(x) = \begin{cases} c + x & -c \leqslant x \leqslant 0, \\ c - x & 0 \leqslant x \leqslant c, \\ 0 & \text{otherwise,} \end{cases}$$

where c is a positive constant.

(i) (A) Sketch the graph of the probability density function. **[2]**

The majority of candidates only scored one mark out of the two available due to omitting axis labels, more often on the vertical axis.

Question 3(i)(b)

(B) Show that $c = 1$. **[2]**

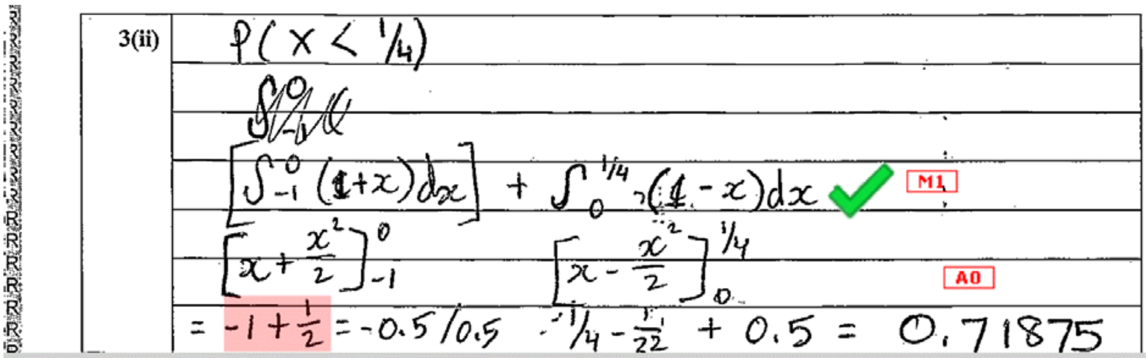
Almost all candidates used an integration method to try to answer this question. In questions such as this one where there are only straight lines involved, it is far easier to use the formula for the area of a triangle. A sizeable proportion did not produce a fully correct response and if they had used the latter method, it is likely that many more would have gained both marks.

Question 3(ii)

(ii) Find $\mathrm{P}\left(X < \frac{1}{4}\right)$. **[2]**

Although very well answered, most candidates again used integration for both parts of the area, instead of realising that the area below $x = 0$ was ½. A few candidates made errors with negative values.

Exemplar 2



Integral evaluated incorrectly so A0.

This response gives a correct expression for $\mathrm{P}(X < ¼)$, but then there is an error in the evaluation. The candidate incorrectly evaluates the left hand tail, getting an answer of -0.5. Having realised that this is wrong, the negative sign is dropped. As the final answer comes from incorrect working, the second mark was not credited. A number of candidates made errors of this type in part (i)B, (ii) and (iii). As mentioned above, integration for the left hand tail was not even necessary as it is clear that the area = ½. Of course the whole of the area can be found without integration, but no candidates attempted any permutation of this method. The simplest method is to subtract $\frac{1}{2} \times \left(\frac{3}{4}\right)^2$ from 1.

Question 3(iii)

(iii) Find
- the mean of X,
- the standard deviation of X. **[4]**

As in the previous parts of the question, nearly all candidates used integration to find $\mathrm{E}(X)$ rather than simply using symmetry to state the value. In finding the variance (in which integration had to be used) a number of candidates again made errors with negative values. A few gave the variance as their final answer, rather than standard deviation.

Question 4(i)

4 The random variable X has a continuous uniform distribution on $[0, 10]$.

(i) Find $\mathrm{P}(3 < X < 6)$. **[1]**

Less than half of the candidates gave a correct answer to this question. The main error made was to think that the distribution was discrete and so give an answer of $^{2}/_{11}$, $^{3}/_{11}$ or $^{4}/_{11}$. Candidates should be advised to make sure that they use the specified distribution.

Question 4(ii)

(ii) Find each of the following.

- $E(X)$
- $Var(X)$ **[2]**

There were many more correct responses to this part, even from candidates that in part (i) had used a discrete distribution.

Question 4(iii)

Marisa is investigating the sample mean, Y, of 8 independent values of X. She designs a simulation shown in the spreadsheet in Fig. 4.1. Each of the 25 rows below the heading row consists of 8 values of X together with the value of Y. All of the values in the spreadsheet have been rounded to 2 decimal places.

	A	B	C	D	E	F	G	H	I	J
1	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	Y	
2	6.31	2.45	3.27	3.06	4.16	1.53	0.43	7.99	3.65	
3	1.70	1.52	7.10	8.93	6.44	2.70	9.96	7.83	5.77	
4	9.15	0.52	4.95	6.99	6.52	3.15	0.81	5.35	4.68	
5	0.65	2.71	7.92	9.65	0.50	4.87	6.46	2.67	4.43	
6	3.09	6.11	3.96	0.09	0.18	4.67	0.67	6.20	3.12	
7	7.06	5.84	1.97	3.60	9.36	1.97	4.48	3.47	4.72	
8	1.46	1.57	5.45	0.37	3.76	7.56	8.48	9.12	4.72	
9	9.42	1.85	4.91	1.61	1.94	8.00	1.77	5.34	4.36	
10	2.98	5.32	2.91	4.12	9.16	1.76	9.97	6.88	5.39	
11	2.83	3.44	3.28	7.85	1.00	0.93	8.77	4.03	4.01	
12	4.51	0.59	5.84	9.87	8.65	3.94	7.18	0.23	5.10	
13	4.49	0.69	3.65	8.78	4.96	8.96	3.77	1.43	4.59	
14	6.57	8.08	4.85	6.75	7.92	0.27	9.69	4.04	6.02	
15	8.35	1.09	8.63	8.04	7.23	2.12	2.57	9.59	5.95	
16	5.24	9.53	6.08	8.21	3.61	7.07	6.65	7.63	6.75	
17	7.89	5.50	3.09	0.71	6.47	5.49	6.47	4.95	5.07	
18	8.36	7.27	2.35	9.04	0.58	2.26	3.01	7.90	5.10	
19	3.76	1.01	9.61	9.65	7.89	9.98	6.28	4.34	6.56	
20	9.94	6.84	3.38	5.53	0.26	8.53	5.72	5.12	5.66	
21	7.25	9.10	0.34	2.88	4.66	2.65	6.37	7.63	5.11	
22	7.18	7.14	5.38	0.04	4.09	6.47	4.96	4.23	4.94	
23	8.69	5.04	4.90	2.94	2.00	4.23	4.13	0.97	4.11	
24	3.46	6.33	0.48	9.35	0.23	1.18	7.97	6.37	4.42	
25	2.37	7.26	7.16	1.24	5.26	2.80	3.55	3.84	4.19	
26	2.16	8.30	7.17	3.32	2.96	1.30	9.11	0.31	4.33	
27										

Fig. 4.1

(iii) Use the spreadsheet to estimate $P(3 < Y < 6)$. **[2]**

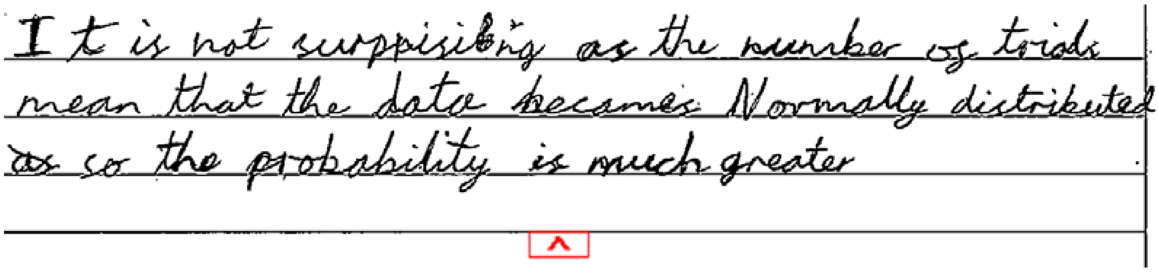
This question was usually answered correctly although some candidates miscounted the number of values between 3 and 6. Some seemed to have no idea how to tackle this question, which simply involved counting. As mentioned in the overview above candidates should have gained experience of using a spreadsheet for simulating random variables.

Question 4(iv)

(iv) Explain why it is not surprising that this estimated probability is substantially greater than the value which you calculated in part **(i)**. **[2]**

Most candidates discussed a 'larger sample size' or a 'larger number of trials'. Some stated that as the sample size increased the distribution of the data would become closer to a Normal distribution. Very few mentioned anything about the variance of *Y* as compared to that of *X*, which was the essential point.

Exemplar 3


It is not surppisibing as the number of trials mean that the data becomes Normally distributed as so the probability is much greater

This is a typical incorrect response, in which the candidate states that as the number of trials (increases?) the data becomes Normally distributed and so the probability is much greater. This seems to be referring to the Central Limit Theorem, although the use of the word 'data' here is incorrect. No marks were credited since this response does not explain why the estimated probability is greater.

Question 4(v)

Marisa wonders whether, even though the sample size is only 8, use of the Central Limit Theorem will provide a good approximation to $P(3 < Y < 6)$.

(v) Calculate an estimate of $P(3 < Y < 6)$ using the Central Limit Theorem. **[3]**

A number of candidates seemed to have no idea how to tackle this question. Of those who did, many did not correctly find the new variance, usually simply using the original variance of *X*, forgetting to divide it by 8. These then only scored the first mark for stating that the distribution was N(5,...) or in other words Normal with mean 5.

Question 4(vi)

A Normal probability plot of the 25 simulated values of Y is shown in Fig. 4.2.

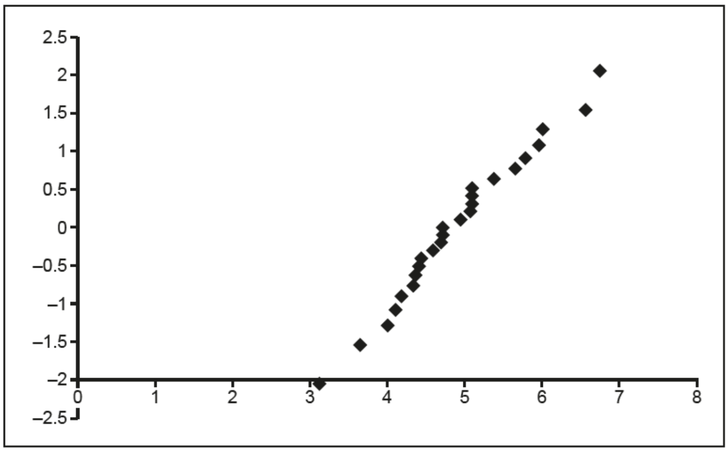


Fig. 4.2

(vi) Explain what the Normal probability plot suggests about the use of the Central Limit Theorem to approximate $P(3 < Y < 6)$. **[2]**

Most candidates realised that the essential point was that the line was approximately straight, and so gained a mark. Most did not give a complete answer, with relatively few going on to state that the distribution is probably Normal and so the Central Limit Theorem may provide a good approximation.

Question 4(vii)

Marisa now decides to use a spreadsheet with 1000 rows below the heading row, rather than the 25 which she used in the initial simulation shown in Fig. 4.1. She uses a counter to count the number of values of Y between 3 and 6. This value is 808.

(vii) Explain whether the value 808 supports the suggestion that the Central Limit Theorem provides a good approximation to $P(3 < Y < 6)$. **[1]**

This question was not very well answered. To gain the mark, candidates had first to state the correct estimated probability of 0.808 and then compare this to the correct probability of 0.811 found in part (v). There were a number of responses where 0.808 was not found and a number where it was found, but then compared to 0.88 rather than 0.811.

Question 4(viii)

Marisa decides to repeat each of her two simulations many times in order to investigate how variable the probability estimates are in each case.

(viii) Explain whether you would expect there to be more, the same or less variability in the probability estimates based on 1000 rows than in the probability estimates based on 25 rows. **[2]**

In this last part of the question a large majority of candidates scored an easy mark for stating that there was less variation, however very few of them gave a convincing reason for why there was less. Many candidates discussed the sample size, but this did not gain the second mark as the sample size was 8, not 1000. What was required was to say that as the number of rows increases, the estimated probability tends to get closer to the true probability.

Exemplar 4

B1

There should be less variability in the probability estimates based on 1000 rows because the bigger the sample, the smaller the variation (in general)

^

In this response the candidate correctly states that less variation would be expected, however their reason refers to 'the bigger the sample'. In order to score the second mark it had to be absolutely clear that the candidate was discussing the number of rows. In this case, although the candidate mentioned the number of rows, it was not absolutely clear that the candidate was not discussing the sample size of 8 and so the second mark was not credited.

Question 5(i)

5 The flight time between two airports is known to be Normally distributed with mean 3.75 hours and standard deviation 0.21 hours. A new airline starts flying the same route. The flight times for a random sample of 12 flights with the new airline are shown in the spreadsheet (Fig. 5), together with the sample mean.

	A	B	C	D	E	F	G	H	I	J	K	L
1	3.595	3.723	3.584	3.643	3.669	3.697	3.550	3.674	3.924	3.563	3.330	3.706
2												
3	Mean	3.638										
4												

Fig. 5

(i) In this question you must show detailed reasoning.

You should assume that:

- the flight times for the new airline are Normally distributed,
- the standard deviation of the flight times is still 0.21 hours.

Carry out a test at the 5% significance level to investigate whether the mean flight time for the new airline is less than 3.75 hours. **[7]**

Most candidates knew how to tackle this question, but many did not mention 'population' in their definition of μ and so did not score this mark. Almost all candidates had correct hypotheses and test statistic, but some then used the wrong critical value and so did not gain the last 3 marks. Wrong critical values included the two-tailed 1.96 and also values from a *t*-distribution (often despite having used the correct Normal distribution to calculate the test statistic).

Question 5(ii)

(ii) If both of the assumptions in part **(i)** were false, name an alternative test that you could carry out to investigate average flight times, stating any assumption necessary for this test. **[2]**

Many fully correct responses were seen, however some candidates simply mentioned 'Wilcoxon test' rather than 'Wilcoxon signed rank test'. Some also did not gain the second mark as their assumption was 'median = mean' without mentioning anything about symmetry. Whilst symmetry implies that the median is equal to the mean, the reverse is not the case.

Question 5(iii)

(iii) If instead the flight times were still Normally distributed but the standard deviation was not known to be 0.21 hours, name another test that you could carry out. **[1]**

Question 6(i)

6 A company has a large fleet of cars. It is claimed that use of a fuel additive will reduce fuel consumption. In order to test this claim a researcher at the company randomly selects 40 of the cars. The fuel consumption of each of the cars is measured, both with and without the fuel additive. The researcher then calculates the difference d litres per kilometre between the two figures for each car, where d is the fuel consumption without the additive minus the fuel consumption with the additive. The sample mean of d is 0.29 and the sample standard deviation is 1.64.

(i) Showing your working, find a 95% confidence interval for the population mean difference. **[4]**

This question was generally well answered. Of those candidates who did not gain full credit, the majority used an incorrect value in place of 1.96, usually 1.645 or a value from a *t*-distribution. A few candidates did not show working as the question required and so could only score a maximum of 2 marks for a fully correct confidence interval.

Question 6(ii)

(ii) Explain whether the confidence interval suggests that, on average, the fuel additive does reduce fuel consumption. **[2]**

Many candidates did not state that zero was within the bounds of the confidence interval and so could only gain at most one of the marks. It is good practice for candidates to state that the value of interest is or is not within the bounds. Quite a number of candidates thought that the fact that 'the mean was above zero' or similar meant that the confidence interval does suggest that the fuel additive reduces fuel consumption, which indicates that they do not realise that a confidence interval can in effect be used to conduct a hypothesis test.

Exemplar 5

On average the fuel will reduce fuel consumption as most of the confidence interval is positive so the average fuel consumption is less

This candidate responds that the fuel additive reduces fuel consumption as most of the confidence interval is positive. There were several similar responses, implying a lack of understanding of the nature of a confidence interval.

Question 6(iii)

(iii) Explain why you can construct the interval in part **(i)** despite not having any information about the distribution of the population of differences. **[2]**

In order to gain credit candidates had to say more than just 'the sample is large'; they had to mention either the Normal distribution or the Central Limit Theorem. For full credit candidates had to give a full answer involving both, which many did manage to do.

Question 6(iv)

(iv) Explain why the sample used was random. **[2]**

Very few fully correct responses were seen, but the majority of candidates scored 1 mark for mentioning that using a random sample would make it more likely that the sample was representative of the population or less likely to be biased.

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