5b. Accessibility

Reasonable adjustments and access arrangements allow learners with special educational needs, disabilities or temporary injuries to access the assessment and show what they know and can do, without changing the demands of the assessment. Applications for these should be made before the examination series. Detailed information about eligibility for access arrangements can be found in the JCQ *Access Arrangements and Reasonable Adjustments.*

The A level qualification and subject criteria have been reviewed in order to identify any feature which could disadvantage learners who share a protected Characteristic as defined by the Equality Act 2010. All reasonable steps have been taken to minimise any such disadvantage.

5c. Mathematical notation

The tables below set out the notation that must be used by AS and A Level Mathematics and Further Mathematics specifications. Students will be expected to understand this notation without need for further explanation. Any additional notation is listed in section 2 of the specification.

1	**Set Notation**	
1.1	$\in$	is an element of
1.2	$\notin$	is not an element of
1.3	$\subseteq$	is a subset of
1.4	$\subset$	is a proper subset of
1.5	$\{x_1, x_2, \ldots\}$	the set with elements $x_1, x_2, \ldots$
1.6	$\{x : \ldots\}$	the set of all x such that …
1.7	$\mathrm{n}(A)$	the number of elements in set A
1.8	$\varnothing$	the empty set
1.9	ε	the universal set
1.10	A'	the complement of the set A
1.11	$\mathbb{N}$	the set of natural numbers, $\{1, 2, 3, \ldots\}$
1.12	$\mathbb{Z}$	the set of integers, $\{0, \pm 1, \pm 2, \pm 3, \ldots\}$
1.13	$\mathbb{Z}^+$	the set of positive integers, $\{1, 2, 3, \ldots\}$
1.14	$\mathbb{Z}_0^+$	the set of non-negative integers, $\{0, 1, 2, 3, \ldots\}$
1.15	$\mathbb{R}$	the set of real numbers
1.16	$\mathbb{Q}$	the set of rational numbers, $\left\{\frac{p}{q} : p \in \mathbb{Z},\ q \in \mathbb{Z}^+\right\}$
1.17	$\cup$	union

1.18	$\cap$	intersection
1.19	$(x,\ y)$	the ordered pair x, y
1.20	$[a,\ b]$	the closed interval $\{x \in \mathbb{R} : a \leq x \leq b\}$
1.21	$[a,\ b)$	the interval $\{x \in \mathbb{R} : a \leq x < b\}$
1.22	$(a,\ b]$	the interval $\{x \in \mathbb{R} : a < x \leq b\}$
1.23	$(a,\ b)$	the open interval $\{x \in \mathbb{R} : a < x < b\}$
1	**Set Notation (Further Mathematics only)**	
1.24	$\mathbb{C}$	the set of complex numbers
2	**Miscellaneous Symbols**	
2.1	$=$	is equal to
2.2	$\neq$	is not equal to
2.3	$\equiv$	is identical to or is congruent to
2.4	$\approx$	is approximately equal to
2.5	∞	infinity
2.6	$\propto$	is proportional to
2.7	$\therefore$	therefore
2.8	$\because$	because
2.9	$<$	is less than
2.10	$\leqslant, \leq$	is less than or equal to, is not greater than
2.11	$>$	is greater than
2.12	$\geqslant, \geq$	is greater than or equal to, is not less than
2.13	$p \Rightarrow q$	p implies q (if p then q)
2.14	$p \Leftarrow q$	p is implied by q (if q then p)
2.15	$p \Leftrightarrow q$	p implies and is implied by q (p is equivalent to q)
2.16	a	first term for an arithmetic or geometric sequence
2.17	l	last term for an arithmetic sequence
2.18	d	common difference for an arithmetic sequence
2.19	r	common ratio for a geometric sequence
2.20	S_n	sum to n terms of a sequence
2.21	S_∞	sum to infinity of a sequence

3	Operations	
3.1	$a + b$	a plus b
3.2	$a - b$	a minus b
3.3	$a \times b,\ ab,\ a.b$	a multiplied by b
3.4	$a \div b,\ \frac{a}{b}$	a divided by b
3.5	$\sum_{i=1}^{n} a_i$	$a_1 + a_2 + \ldots + a_n$
3.6	$\prod_{i=1}^{n} a_i$	$a_1 \times a_2 \times \ldots \times a_n$
3.7	$\sqrt{a}$	the non-negative square root of a
3.8	$\lvert a \rvert$	the modulus of a
3.9	$n!$	n factorial: $n! = n \times (n-1) \times \ldots \times 2 \times 1,\ n \in \mathbb{N};\ 0!=1$
3.10	$\binom{n}{r},\ {}^{n}C_{r},\ {}_{n}C_{r}$	the binomial coefficient $\frac{n!}{r!(n-r)!}$ for $n, r \in \mathbb{Z}_0^+, r \leqslant n$ or $\frac{n(n-1)\ldots(n-r+1)}{r!}$ for $n \in \mathbb{Q}, r \in \mathbb{Z}_0^+$
4	**Functions**	
4.1	$\mathrm{f}(x)$	the value of the function f at x
4.2	$\mathrm{f} : x \mapsto y$	the function f maps the element x to the element y
4.3	f^{-1}	the inverse function of the function f
4.4	gf	the composite function of f and g which is defined by $\mathrm{gf}(x) = \mathrm{g}(\mathrm{f}(x))$
4.5	$\lim_{x \to a} \mathrm{f}(x)$	the limit of $\mathrm{f}(x)$ as x tends to a
4.6	$\Delta x,\ \delta x$	an increment of x
4.7	$\frac{\mathrm{d}y}{\mathrm{d}x}$	the derivative of y with respect to x
4.8	$\frac{\mathrm{d}^n y}{\mathrm{d}x^n}$	the nth derivative of y with respect to x
4.9	$\mathrm{f}'(x),\ \mathrm{f}''(x),\ \ldots,\ \mathrm{f}^{(n)}(x)$	the first, second, ..., n^{th} derivatives of $\mathrm{f}(x)$ with respect to x
4.10	$\dot{x},\ \ddot{x},\ \ldots$	the first, second, ... derivatives of x with respect to t
4.11	$\int y\,\mathrm{d}x$	the indefinite integral of y with respect to x
4.12	$\int_a^b y\,\mathrm{d}x$	the definite integral of y with respect to x between the limits $x = a$ and $x = b$

5	**Exponential and Logarithmic Functions**	
5.1	e	base of natural logarithms
5.2	e^x, $\exp x$	exponential function of x
5.3	$\log_a x$	logarithm to the base a of x
5.4	$\ln x$, $\log_e x$	natural logarithm of x
6	**Trigonometric Functions**	
6.1	$\left.\begin{matrix}\sin, \cos, \tan \\ \text{cosec}, \sec, \cot\end{matrix}\right\}$	the trigonometric functions
6.2	$\left.\begin{matrix}\sin^{-1}, \cos^{-1}, \tan^{-1} \\ \arcsin, \arccos, \arctan\end{matrix}\right\}$	the inverse trigonometric functions
6.3	°	degrees
6.4	rad	radians
6	**Trigonometric and Hyperbolic Functions (Further Mathematics only)**	
6.5	$\left.\begin{matrix}\text{cosec}^{-1}, \sec^{-1}, \cot^{-1} \\ \text{arccosec}, \text{arcsec}, \text{arccot}\end{matrix}\right\}$	the inverse trigonometric functions
6.6	$\left.\begin{matrix}\sinh, \cosh, \tanh, \\ \text{cosech}, \text{sech}, \coth\end{matrix}\right\}$	the hyperbolic functions
6.7	$\left.\begin{matrix}\sinh^{-1}, \cosh^{-1}, \tanh^{-1} \\ \text{cosech}^{-1}, \text{sech}^{-1}, \coth^{-1}\end{matrix}\right\}$ $\left.\begin{matrix}\text{arsinh}, \text{arcosh}, \text{artanh}, \\ \text{arcosech}, \text{arsech}, \text{arcoth}\end{matrix}\right\}$	the inverse hyperbolic functions
7	**Complex Numbers (Further Mathematics only)**	
7.1	i, j	square root of –1
7.2	$x + \text{i}y$	complex number with real part x and imaginary part y
7.3	$r(\cos\theta + \text{i}\sin\theta)$	modulus argument form of a complex number with modulus r and argument θ
7.4	z	a complex number, $z = x + \text{i}y = r(\cos\theta + \text{i}\sin\theta)$
7.5	$\text{Re}(z)$	the real part of z, $\text{Re}(z) = x$
7.6	$\text{Im}(z)$	the imaginary part of z, $\text{Im}(z) = y$
7.7	$\lvert z \rvert$	the modulus of z, $\lvert z \rvert = \sqrt{x^2 + y^2}$
7.8	$\arg(z)$	the argument of z, $\arg(z) = \theta$, $-\pi < \theta \leq \pi$
7.9	z^*	the complex conjugate of z, $x - \text{i}y$

8	**Matrices (Further Mathematics only)**	
8.1	$\mathbf{M}$	a matrix **M**
8.2	$\mathbf{0}$	zero matrix
8.3	$\mathbf{I}$	identity matrix
8.4	$\mathbf{M}^{-1}$	the inverse of the matrix **M**
8.5	$\mathbf{M}^{\mathrm{T}}$	the transpose of the matrix **M**
8.6	Δ, det **M** or $\lvert\mathbf{M}\rvert$	the determinant of the square matrix **M**
8.7	**Mr**	Image of column vector **r** under the transformation associated with the matrix **M**
9	**Vectors**	
9.1	$\mathbf{a}$, $\underline{a}$, $\underline{\mathbf{a}}$	the vector **a**, $\underline{a}$, $\underline{\mathbf{a}}$; these alternatives apply throughout section 9
9.2	$\overrightarrow{\mathrm{AB}}$	the vector represented in magnitude and direction by the directed line segment AB
9.3	$\hat{\mathbf{a}}$	a unit vector in the direction of **a**
9.4	$\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$	unit vectors in the directions of the cartesian coordinate axes
9.5	$\lvert\mathbf{a}\rvert$, a	the magnitude of **a**
9.6	$\lvert\overrightarrow{\mathrm{AB}}\rvert$, AB	the magnitude of $\overrightarrow{\mathrm{AB}}$
9.7	$\begin{pmatrix} a \\ b \end{pmatrix}$, $a\mathbf{i} + b\mathbf{j}$	column vector and corresponding unit vector notation
9.8	$\mathbf{r}$	position vector
9.9	$\mathbf{s}$	displacement vector
9.10	$\mathbf{v}$	velocity vector
9.11	$\mathbf{a}$	acceleration vector
9	**Vectors (Further Mathematics only)**	
9.12	$\mathbf{a.b}$	the scalar product of **a** and **b**
10	**Differential Equations (Further Mathematics only)**	
10.1	ω	angular speed
11	**Probability and Statistics**	
11.1	A, B, C, etc.	events
11.2	$A \cup B$	union of the events A and B
11.3	$A \cap B$	intersection of the events A and B
11.4	$\mathrm{P}(A)$	probability of the event A

11.5	A'	complement of the event A
11.6	$\mathrm{P}(A \mid B)$	probability of the event A conditional on the event B
11.7	X, Y, R, etc.	random variables
11.8	x, y, r, etc.	values of the random variables X, Y, R etc.
11.9	$x_1, x_2, \ldots$	observations
11.10	$f_1, f_2, \ldots$	frequencies with which the observations $x_1, x_2, \ldots$ occur
11.11	$\mathrm{p}(x)$, $\mathrm{P}(X = x)$	probability function of the discrete random variable X
11.12	$p_1, p_2, \ldots$	probabilities of the values $x_1, x_2, \ldots$ of the discrete random variable X
11.13	$\mathrm{E}(X)$	expectation of the random variable X
11.14	$\mathrm{Var}(X)$	variance of the random variable X
11.15	$\sim$	has the distribution
11.16	$\mathrm{B}(n,\ p)$	binomial distribution with parameters n and p, where n is the number of trials and p is the probability of success in a trial
11.17	q	$q = 1 - p$ for binomial distribution
11.18	$\mathrm{N}(\mu, \sigma^2)$	Normal distribution with mean μ and variance σ^2
11.19	$Z \sim N(0, 1)$	standard Normal distribution
11.20	ϕ	probability density function of the standardised Normal variable with distribution $\mathrm{N}(0,\ 1)$
11.21	Φ	corresponding cumulative distribution function
11.22	μ	population mean
11.23	σ^2	population variance
11.24	σ	population standard deviation
11.25	$\bar{x}$	sample mean
11.26	s^2	sample variance
11.27	s	sample standard deviation
11.28	H_0	Null hypothesis
11.29	H_1	Alternative hypothesis
11.30	r	product moment correlation coefficient for a sample
11.31	ρ	product moment correlation coefficient for a population
12	**Mechanics**	
12.1	kg	kilograms
12.2	m	metres
12.3	km	kilometres
12.4	m/s, m s^{-1}	metres per second (velocity)
12.5	m/s^2, m s^{-2}	metres per second per second (acceleration)

12.6	F	force or resultant force
12.7	N	newton
12.8	N m	newton metre (moment of a force)
12.9	t	time
12.10	s	displacement
12.11	u	initial velocity
12.12	v	velocity or final velocity
12.13	a	acceleration
12.14	g	acceleration due to gravity
12.15	μ	coefficient of friction

5d. Mathematical formulae, identities and statistical tables

Learners must be able to use the following formulae and identities for AS and A Level Further Mathematics, without these formulae and identities being provided, either in these forms or in equivalent forms. These formulae and identities may only be provided where they are the starting point for a proof or as a result to be proved.

Pure Mathematics

Quadratic Equations

$ax^2 + bx + c = 0$ has roots $\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Laws of Indices

$$a^x a^y \equiv a^{x+y}$$
$$a^x \div a^y \equiv a^{x-y}$$
$$(a^x)^y \equiv a^{xy}$$

Laws of Logarithms

$x = a^n \Leftrightarrow n = \log_a x$ for $a > 0$ and $x > 0$

$$\log_a x + \log_a y \equiv \log_a (xy)$$
$$\log_a x - \log_a y \equiv \log_a \left(\frac{x}{y}\right)$$
$$k \log_a x \equiv \log_a (x^k)$$

Coordinate Geometry

A straight line graph, gradient m passing through (x_1, y_1) has equation

$y - y_1 = m(x - x_1)$

Straight lines with gradients m_1 and m_2 are perpendicular when $m_1 m_2 = -1$

Sequences

General term of an arithmetic progression:

$u_n = a + (n-1)d$

General term of a geometric progression:

$u_n = ar^{n-1}$

Trigonometry

In the triangle ABC

Sine rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Cosine rule: $a^2 = b^2 + c^2 - 2bc\cos A$

Area $= \frac{1}{2}ab\sin C$

$\cos^2 A + \sin^2 A \equiv 1$

$\sec^2 A \equiv 1 + \tan^2 A$

$\operatorname{cosec}^2 A \equiv 1 + \cot^2 A$

$\sin 2A \equiv 2\sin A\cos A$

$\cos 2A \equiv \cos^2 A - \sin^2 A$

$\tan 2A \equiv \frac{2\tan A}{1 - \tan^2 A}$

Mensuration

Circumference and area of circle, radius r and diameter d:

$C = 2\pi r = \pi d \qquad A = \pi r^2$

Pythagoras' Theorem: In any right-angled triangle where a, b and c are the lengths of the sides and c is the hypotenuse:

$c^2 = a^2 + b^2$

Area of a trapezium $= \frac{1}{2}(a+b)h$, where a and b are the lengths of the parallel sides and h is their perpendicular separation.

Volume of a prism = area of cross section × length

For a circle of radius r, where an angle at the centre of θ radians subtends an arc of length l and encloses an associated sector of area a:

$l = r\theta \qquad a = \frac{1}{2}r^2\theta$

Complex Numbers

For two complex numbers $z_1 = r_1 e^{i\theta_1}$ and $z_2 = r_2 e^{i\theta_2}$:

$z_1 z_2 = r_1 r_2 e^{i(\theta_1 + \theta_2)}$

$\frac{z_1}{z_2} = \frac{r_1}{r_2} e^{i(\theta_1 - \theta_2)}$

Loci in the Argand diagram:

$|z - a| = r$ is a circle radius r centred at a

$\arg(z - a) = \theta$ is a half line drawn from a at angle θ to a line parallel to the positive real axis

Exponential Form:

$e^{i\theta} = \cos\theta + i\sin\theta$

Matrices

For a 2 by 2 matrix $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ the determinant $\Delta = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$

the inverse is $\frac{1}{\Delta}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

The transformation represented by matrix **AB** is the transformation represented by matrix **B** followed by the transformation represented by matrix **A**.

For matrices **A**, **B**:

$(\mathbf{AB})^{-1} = \mathbf{B}^{-1}\mathbf{A}^{-1}$

Algebra

$\sum_{r=1}^{n} r = \frac{1}{2}n(n+1)$

For $ax^2 + bx + c = 0$ with roots α and β:

$$\alpha + \beta = \frac{-b}{a} \qquad \alpha\beta = \frac{c}{a}$$

For $ax^3 + bx^2 + cx + d = 0$ with roots α, β and γ:

$$\sum \alpha = \frac{-b}{a} \qquad \sum \alpha\beta = \frac{c}{a} \quad \alpha\beta\gamma = \frac{-d}{a}$$

Hyperbolic Functions

$\cosh x \equiv \frac{1}{2}(e^x - e^{-x})$

$\sinh x \equiv \frac{1}{2}(e^x - e^{-x})$

$\tanh x \equiv \frac{\sinh x}{\cosh x}$

Calculus and Differential Equations

Differentiation

Function	Derivative
x^n	nx^{n-1}
$\sin kx$	$k\cos kx$
$\cos kx$	$-k\sin kx$
$\sinh kx$	$k\cosh kx$
$\cosh kx$	$k\sinh kx$
e^{kx}	ke^{kx}
$\ln x$	$\frac{1}{x}$
$f(x)+g(x)$	$f'(x)+g'(x)$
$f(x)g(x)$	$f'(x)g(x)+f(x)g'(x)$
$f(g(x))$	$f'(g(x))g'(x)$

Integration

Function	Integral
x^n	$\frac{1}{n+1}x^{n+1}+c,\ n\neq -1$
$\cos kx$	$\frac{1}{k}\sin kx+c$
$\sin kx$	$-\frac{1}{k}\cos kx+c$
$\cosh kx$	$\frac{1}{k}\sinh kx+c$
$\sinh kx$	$\frac{1}{k}\cosh kx+c$
e^{kx}	$\frac{1}{k}e^{kx}+c$
$\frac{1}{x}$	$\ln\lvert x\rvert+c,\ x\neq 0$
$f'(x)+g'(x)$	$f(x)+g(x)+c$
$f'(g(x))g'(x)$	$f(g(x))+c$

Area under a curve $= \int_a^b y\,dx (y \geq 0)$

Volumes of revolution about the x and y axes:

$V_x = \pi\int_a^b y^2\,dx$ $\qquad$ $V_y = \pi\int_c^d x^2\,dy$

Simple Harmonic Motion:

$\ddot{x} = -\omega^2 x$

Vectors

$|x\mathbf{i}+y\mathbf{j}+z\mathbf{k}|=\sqrt{(x^2+y^2+z^2)}$

Scalar product of two vectors $\mathbf{a}=\begin{pmatrix}a_1\\a_2\\a_3\end{pmatrix}$ and $\mathbf{b}=\begin{pmatrix}b_1\\b_2\\b_3\end{pmatrix}$ is

$$\begin{pmatrix}a_1\\a_2\\a_3\end{pmatrix}.\begin{pmatrix}b_1\\b_2\\b_3\end{pmatrix}=a_1b_1+a_2b_2+a_3b_3=|\mathbf{a}\|\mathbf{b}|\cos\theta$$

where θ is the acute angle between the vectors **a** and **b**

The equation of the line through the point with position vector **a** parallel to vector **b** is:

$\mathbf{r}=\mathbf{a}+t\mathbf{b}$

The equation of the plane containing the point with position vector **a** and perpendicular to vector **n** is:

$(\mathbf{r}-\mathbf{a}).\mathbf{n}=0$

Mechanics

Forces and Equilibrium

Weight = mass $\times g$

Friction: $F\leq\mu R$

Newton's second law in the form: $F=ma$

Kinematics

For motion in a straight line with variable acceleration:

$v=\dfrac{\mathrm{d}r}{\mathrm{d}t}\qquad a=\dfrac{\mathrm{d}v}{\mathrm{d}t}=\dfrac{\mathrm{d}^2r}{\mathrm{d}t^2}$

$r=\int v\,\mathrm{d}t\qquad v=\int a\,\mathrm{d}t$

Statistics

The mean of a set of data: $\bar{x}=\dfrac{\sum x}{n}=\dfrac{\sum fx}{\sum f}$

The standard Normal variable: $Z=\dfrac{X-\mu}{\sigma}$ where $X\sim N(\mu,\sigma^2)$