Qualification Accredited

A LEVEL

Examiners' report

FURTHER MATHEMATICS B (MEI)

H645
For first teaching in 2017

Y434/01 Summer 2019 series

Version 1

www.ocr.org.uk/maths

Contents

Introduction

Our examiners' reports are produced to offer constructive feedback on candidates' performance in the examinations. They provide useful guidance for future candidates. The reports will include a general commentary on candidates' performance, identify technical aspects examined in the questions and highlight good performance and where performance could be improved. The reports will also explain aspects that caused difficulty and why the difficulties arose, whether through a lack of knowledge, poor examination technique, or any other identifiable and explainable reason.

Where overall performance on a question/part was considered good, with no particular areas to highlight, these questions have not been included in the report. A full copy of the question paper can be downloaded from OCR.

Paper Y434 series overview

Y434 Numerical Methods can count for 16⅔% of the qualification H645 OCR A Level Further Mathematics B (MEI). There is one examination paper, which lasts 1 hour 15 minutes. Candidates are expected to know content from A Level Mathematics as well as from the mandatory H645 Core Pure (Y420).

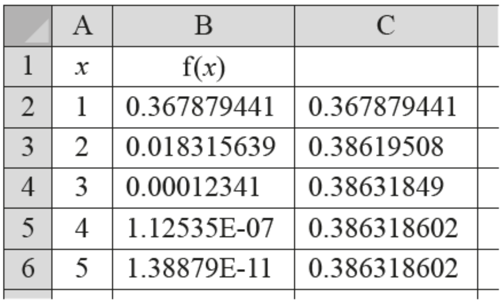
It is expected that a calculator is used in the examination. In Numerical Methods, candidates are expected to show evidence of working through methods, rather than just writing down solutions provided by their calculator’s equation solver/numerical differentiation/integration functions.

It is expected that candidates will have experience of using a spreadsheet for implementing Numerical Methods; in the examination candidates will be given output from spreadsheets and may be asked what certain cells represent, to explain/give formulae for certain cells, to give solutions and justify their accuracy, or to comment on the error due to the numerical method employed as well as the appropriateness of the model.

Candidates who did well in this paper were used to working with spreadsheets and were familiar with how to input cell formulae and to interpret spreadsheet output. They were familiar with the algorithms on the course and were able to use their calculators efficiently to implement them. They were able to clearly and concisely articulate their understanding of the relative merits of different algorithms, how and why they might fail and their interpretation of the associated spreadsheet output.

Question 1

1 Fig. 1 shows some spreadsheet output concerning the values of a function, $f(x)$.



	A	B	C
1	x	$f(x)$	
2	1	0.367879441	0.367879441
3	2	0.018315639	0.38619508
4	3	0.00012341	0.38631849
5	4	1.12535E-07	0.386318602
6	5	1.38879E-11	0.386318602

Fig. 1

The formula in cell B2 is `=EXP(-(A2^2))`
and equivalent formulae are in cells B3 to B6.

The formula in cell C2 is `=B2`.

The formula in cell C3 is `=C2+B3`.

Equivalent formulae are in cells C4 to C6.

(a) Use sigma notation to express the formula in cell C5 in standard mathematical notation. **[2]**

(b) Explain why the same value is displayed in cells C5 and C6. **[2]**

Now suppose that the value in cell C2 is chopped to 3 decimal places and used to approximate the value in cell C2.

(c) Calculate the relative error when this approximation is used. **[1]**

Suppose that the values in cells B4, B5 and B6 are chopped to 3 decimal places and used as approximations to the original values in cells B4, B5 and B6 respectively.

(d) Explain why the relative errors in these approximations are all the same. **[1]**

Candidates who did well in Question 1 were familiar with spreadsheet notation and recognised that the exponential function was involved. They gave concise explanations in part (b) and referred to the magnitude of the value in cell B6 in conjunction with the accuracy displayed in cells C5 and C6. They understood how to find relative error and supported their explanation in part (d) with a suitable calculation.

Candidates who did less well gave an incomplete answer in part (a), or thought that the sum in cell C5 was of 5 values, not 4. In part (b) they often displayed some understanding of what was going on, but were unable to articulate their reasoning clearly. Sometimes in part (c) they made an arithmetical slip, and in part (d) a circular argument was sometimes presented.

Candidates who did not do well were either unable to use sigma notation or did not recognise the exponential function in part (a). In part (b) sometimes they wrote things that were correct, but irrelevant and in part (c) they sometimes made slips in the arithmetic or calculated the absolute error instead. In part (d) the explanation was often incomplete or a circular argument was presented.

Question 2 (a)

2 Fig. 2.1 shows the graph of $y = x^2 e^{2x} - 5x^2 + 0.5$.

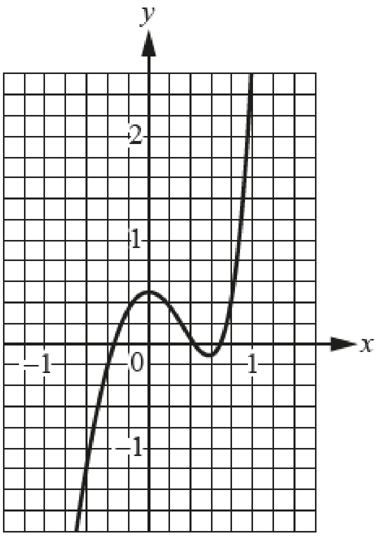


Fig. 2.1

There are three roots of the equation $x^2 e^{2x} - 5x^2 + 0.5 = 0$. The roots are α, β and γ, where $\alpha < \beta < \gamma$.

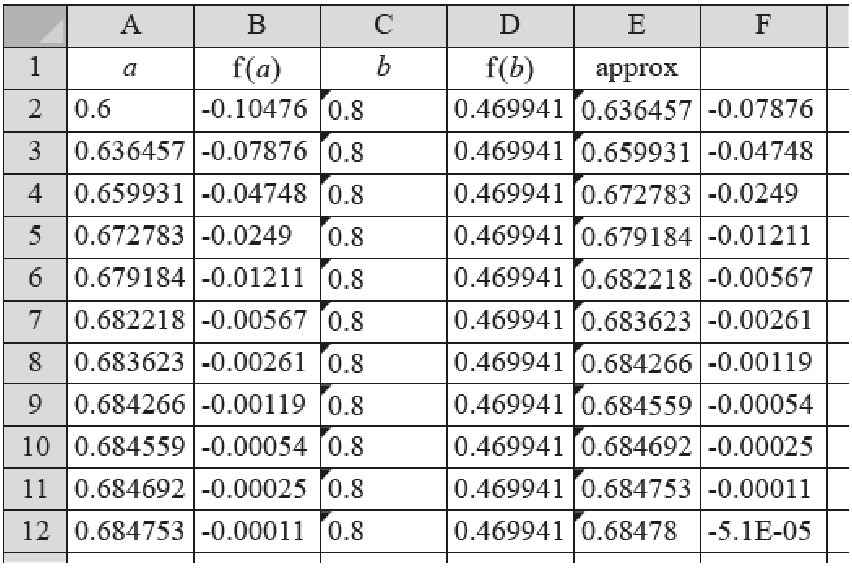
(a) Explain why it is not possible to use the method of false position with $x_0 = 0$ and $x_1 = 1$ to find β and γ. **[1]**

Candidates who did well in this question noted that using these two values to start with does not introduce a sign change, so the method can't be used.

Candidates who did not do well commented that the process would converge to one root only, that it wouldn't work because the gradient is bigger than 1 or that it wouldn't work because it is near a turning point.

Question 2(b)

The graph of the function indicates that the root γ lies in the interval [0.6, 0.8]. Fig. 2.2 shows some spreadsheet output using the method of false position using these values as starting points.



	A	B	C	D	E	F
1	*a*	f(*a*)	*b*	f(*b*)	approx	
2	0.6	-0.10476	0.8	0.469941	0.636457	-0.07876
3	0.636457	-0.07876	0.8	0.469941	0.659931	-0.04748
4	0.659931	-0.04748	0.8	0.469941	0.672783	-0.0249
5	0.672783	-0.0249	0.8	0.469941	0.679184	-0.01211
6	0.679184	-0.01211	0.8	0.469941	0.682218	-0.00567
7	0.682218	-0.00567	0.8	0.469941	0.683623	-0.00261
8	0.683623	-0.00261	0.8	0.469941	0.684266	-0.00119
9	0.684266	-0.00119	0.8	0.469941	0.684559	-0.00054
10	0.684559	-0.00054	0.8	0.469941	0.684692	-0.00025
11	0.684692	-0.00025	0.8	0.469941	0.684753	-0.00011
12	0.684753	-0.00011	0.8	0.469941	0.68478	-5.1E-05

Fig. 2.2

(b) **Without** doing any further calculation, write down the smallest possible interval which is certain to contain γ. **[1]**

Candidates who did well in this question were able to interpret the spreadsheet correctly and write down the correct interval.

Candidates who did not do well often gave the interval as 0.694753 < γ < 0.8.

Question 2(c)

(c) State what is being calculated in column F. **[1]**

Candidates who did well in this question referred to the value of the function.

Candidates who did not do well displayed a variety of misconceptions.

Question 2 (d)

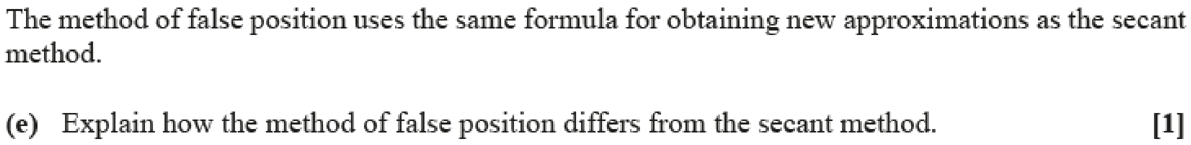
The formula in cell A3 is `=IF (F2<0,E2,A2)`.

(d) Explain the purpose of this formula in the application of the method of false position. **[2]**

Candidates who did well in this question commented that the purpose of the formula is to check the sign of the value of the function at the new approximation, in order to establish a new interval for the root. They went on to give details of which values might be placed in cell A3.

Candidates who did less well successfully described just one of the points given above.

Question 2(e)



The method of false position uses the same formula for obtaining new approximations as the secant method.

(e) Explain how the method of false position differs from the secant method. **[1]**

Candidates who did well in this question identified the incorporation of a sign change as the distinguishing feature.

Candidates who did not do well either did not grasp this point or were unable to articulate it clearly.

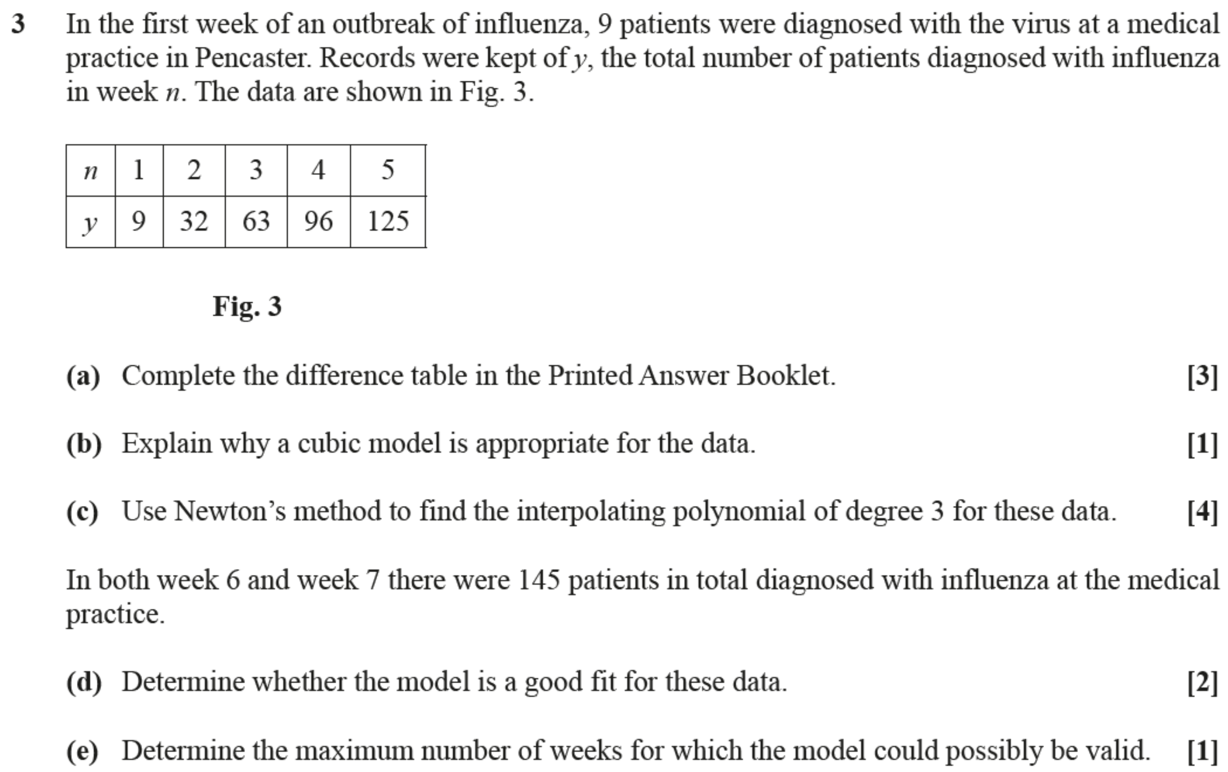
Question 2(f)

(f) Give one **advantage** and one **disadvantage** of using the method of false position instead of the secant method. **[2]**

Candidates who did well in this question commented on speed of convergence and ease of implementation.

Candidates who did less well identified either an advantage or a disadvantage successfully.

Question 3



3 In the first week of an outbreak of influenza, 9 patients were diagnosed with the virus at a medical practice in Pencaster. Records were kept of y, the total number of patients diagnosed with influenza in week n. The data are shown in Fig. 3.

n	1	2	3	4	5
y	9	32	63	96	125

Fig. 3

(a) Complete the difference table in the Printed Answer Booklet. **[3]**

(b) Explain why a cubic model is appropriate for the data. **[1]**

(c) Use Newton’s method to find the interpolating polynomial of degree 3 for these data. **[4]**

In both week 6 and week 7 there were 145 patients in total diagnosed with influenza at the medical practice.

(d) Determine whether the model is a good fit for these data. **[2]**

(e) Determine the maximum number of weeks for which the model could possibly be valid. **[1]**

Candidates who did well in this question produced accurate working through parts (a) and (c) and articulated the reason why a cubic model was appropriate in part (b). In part (d) they generally used their model correctly and commented accordingly. In part (e) they understood that the command word ‘determine’ indicated the need to provide supporting calculations and considered when *y* would be negative.

Candidates who did less well worked in *x* instead of *n* and simply stated an answer to part (e). Some made slips in generating the polynomial in part (b).

Candidates who did not do well made slips in the difference table and/or went astray in finding the polynomial in part (b). In part (d) they often added the two values together before making a comparison and rarely understood what was required in part (e).

i	**OCR support**	OCR’s command words poster is available to download in both A2 and A4 versions from ‘Assessment guides’ at https://www.ocr.org.uk/qualifications/as-a-level-gce/further-mathematics-b-mei-h635-h645-from-2017/assessment.

Question 4

4 Fig. 4 shows the graph of $y = x^5 - 6\sqrt{x} + 4$.

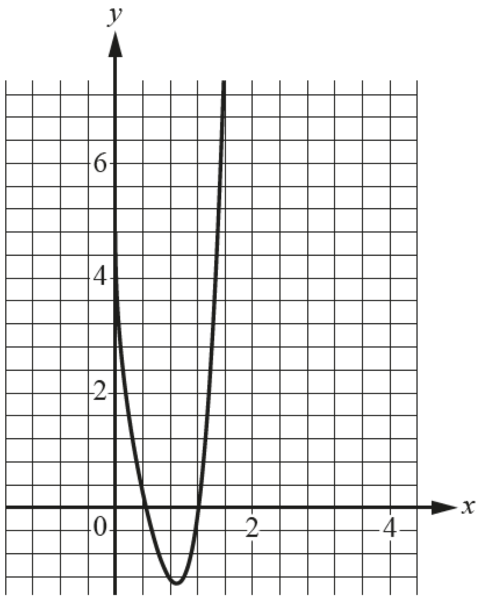


Fig. 4

There are two roots of the equation $x^5 - 6\sqrt{x} + 4 = 0$. The roots are α and β, such that $\alpha < \beta$.

(a) Show that $0 < \alpha < 1$ and $1 < \beta < 2$. **[2]**

(b) Obtain the Newton-Raphson iterative formula

$$x_{n+1} = x_n - \frac{x_n^{\frac{11}{2}} - 6x_n + 4\sqrt{x_n}}{5x_n^{\frac{9}{2}} - 3}.$$ **[3]**

(c) Use the iterative formula found in part **(b)** with a starting value of $x_0 = 1$ to obtain β correct to 6 decimal places. **[2]**

(d) Use the iterative formula found in part **(b)** with a starting value of $x_0 = 0$ to find x_1. **[1]**

(e) Give a geometrical explanation of why the Newton-Raphson iteration fails to find α in part **(d)**. **[1]**

(f) Obtain the iterative formula

$$x_{n+1} = \left(\frac{x_n^5 + 4}{6}\right)^2.$$ **[2]**

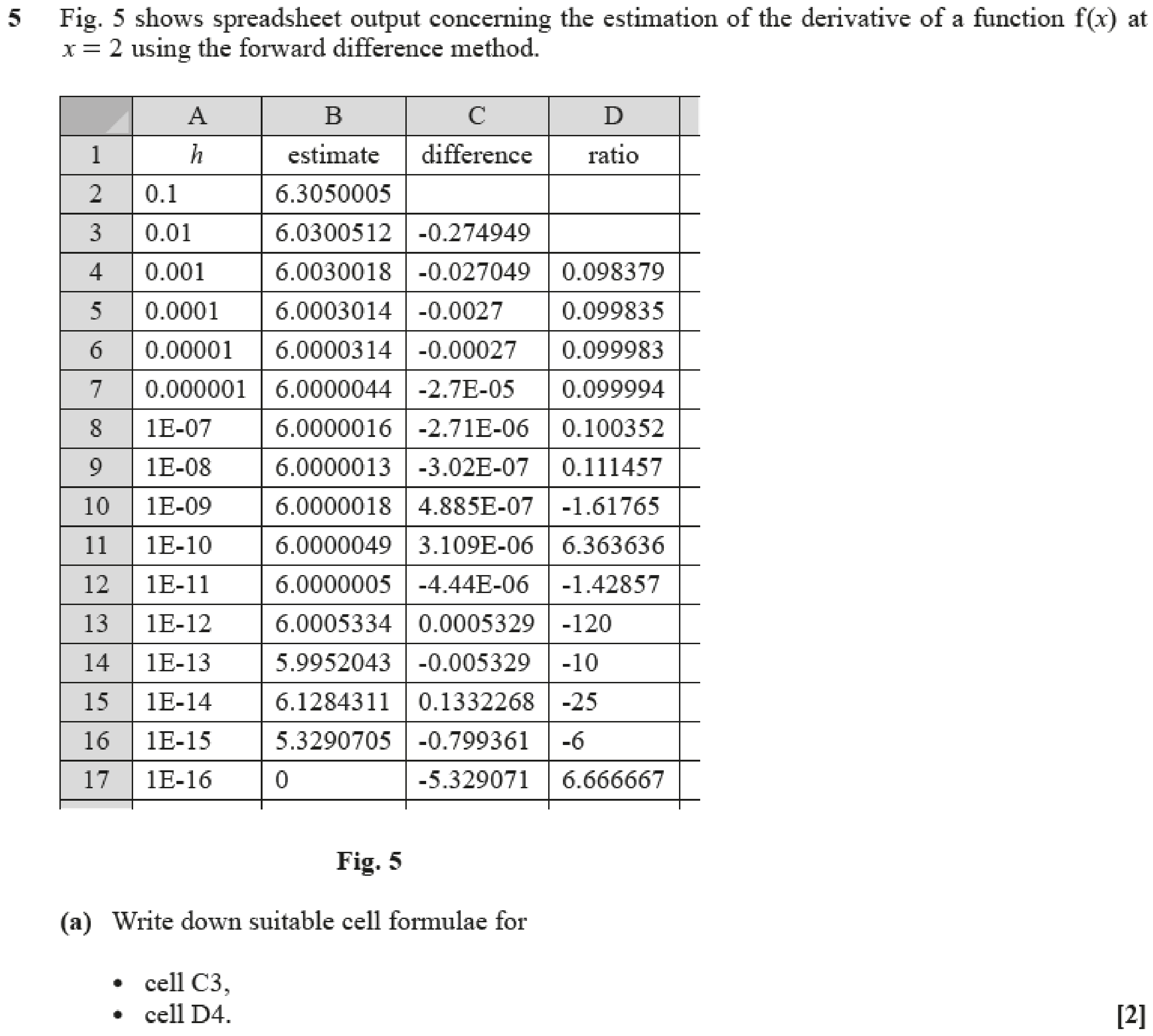
(g) Use the iterative formula found in part **(f)** with a starting value of $x_0 = 0$ to obtain α correct to 6 decimal places. **[2]**

Candidates who did well in this question used their calculators efficiently to demonstrate two sign changes in part (a) and the ANS function to provide iterates in parts (c), (d) and (g). They correctly interpreted their output in each case. They were confident with the algebra in parts (b) and (f), and referred to the behaviour of the tangent in part (e).

Candidates who did less well made slips with the algebra, usually in part (b), but occasionally subscripts were missed off in part (f). They misinterpreted their output in parts (c) and (g), often giving the 6th decimal place incorrectly. In part (d) they sometimes displayed some understanding of why the iterative formula wouldn't work in this case, but were unable to articulate it clearly.

Candidates who did not do well made slips in part (a) and either made slips in inputting the formulae in parts (c) and (g), or used their equation solver. It is worth noting that they are expected to show a sufficient number of iterates to convince the examiner that the correct method has been used. In parts (b) and (f) slips were not uncommon and the requirement of part (d) was not clearly understood.

Question 5(a)



5 Fig. 5 shows spreadsheet output concerning the estimation of the derivative of a function $f(x)$ at $x = 2$ using the forward difference method.

	A	B	C	D
1	h	estimate	difference	ratio
2	0.1	6.3050005		
3	0.01	6.0300512	-0.274949	
4	0.001	6.0030018	-0.027049	0.098379
5	0.0001	6.0003014	-0.0027	0.099835
6	0.00001	6.0000314	-0.00027	0.099983
7	0.000001	6.0000044	-2.7E-05	0.099994
8	1E-07	6.0000016	-2.71E-06	0.100352
9	1E-08	6.0000013	-3.02E-07	0.111457
10	1E-09	6.0000018	4.885E-07	-1.61765
11	1E-10	6.0000049	3.109E-06	6.363636
12	1E-11	6.0000005	-4.44E-06	-1.42857
13	1E-12	6.0005334	0.0005329	-120
14	1E-13	5.9952043	-0.005329	-10
15	1E-14	6.1284311	0.1332268	-25
16	1E-15	5.3290705	-0.799361	-6
17	1E-16	0	-5.329071	6.666667

Fig. 5

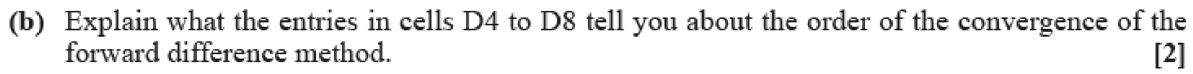
(a) Write down suitable cell formulae for

- cell C3,
- cell D4. **[2]**

Candidates who did well in this question wrote down correct formulae and clearly identified which cell they were assigned to.

Candidates who did less well spoiled their answers by including C3 or D4 in their formula.

Question 5(b)



(b) Explain what the entries in cells D4 to D8 tell you about the order of the convergence of the forward difference method. **[2]**

Candidates who did well in this question noted that the ratios were converging and allied this to first order convergence.

Candidates who did less well identified one of these points correctly.

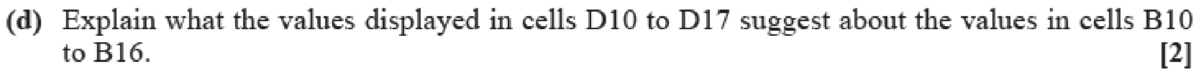
Question 5(c)

(c) Write the entry in cell A10 in standard mathematical notation. **[1]**

Candidates who did well in this question usually wrote their answer in standard form correctly.

Candidates who did not do well were generally out by a factor of 10.

Question 5(d)



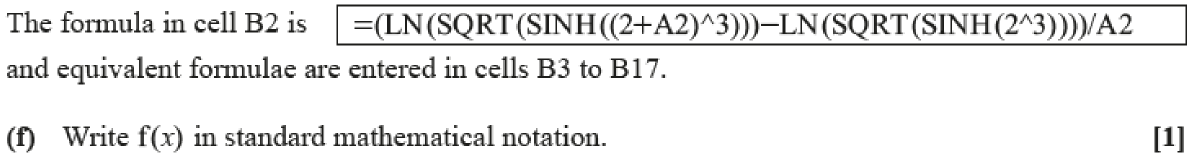
(d) Explain what the values displayed in cells D10 to D17 suggest about the values in cells B10 to B16. **[2]**

Candidates who did well in this question noted that the ratio of differences is diverging, suggesting that the approximations are becoming increasingly inaccurate.

Candidates who did less well identified one of these points correctly.

Candidates who did not do well wrote in generalities or made (sometimes correct) irrelevant comments.

Question 5(f)



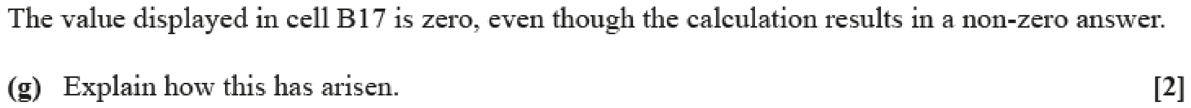
The formula in cell B2 is `=(LN(SQRT(SINH((2+A2)^3)))−LN(SQRT(SINH(2^3))))/A2`
and equivalent formulae are entered in cells B3 to B17.

(f) Write f(x) in standard mathematical notation. **[1]**

Candidates who did well in this question were able to extract the formula for the function from the forward difference method.

Candidates who did not do well included 2^3 and the denominator.

Question 5(g)



The value displayed in cell B17 is zero, even though the calculation results in a non-zero answer.

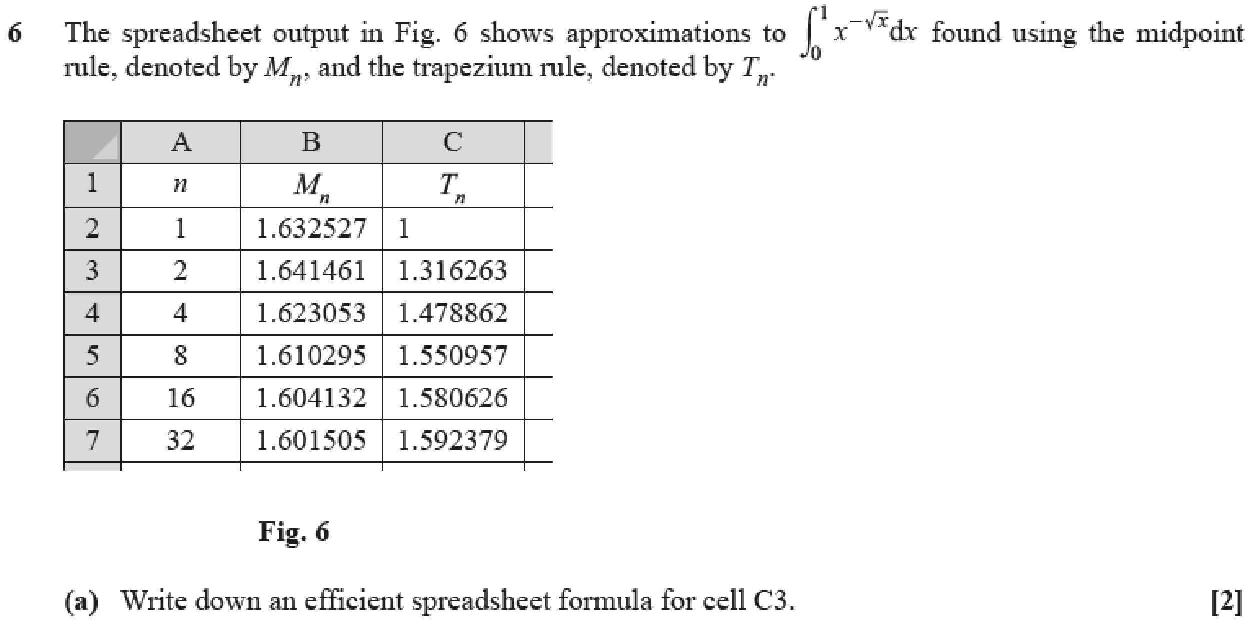
(g) Explain how this has arisen. **[2]**

Candidates who did well in this question commented on the spreadsheet storing two similar values as the same (binary) number and that the subtraction of nearly equal values led to the discrepancy.

Candidates who did less well identified one of these points.

Candidates who did not do well sometimes made correct observations without actually addressing the question.

Question 6(a)



6 The spreadsheet output in Fig. 6 shows approximations to $\int_0^1 x^{-\sqrt{x}}\mathrm{d}x$ found using the midpoint rule, denoted by M_n, and the trapezium rule, denoted by T_n.

	A	B	C
1	n	M_n	T_n
2	1	1.632527	1
3	2	1.641461	1.316263
4	4	1.623053	1.478862
5	8	1.610295	1.550957
6	16	1.604132	1.580626
7	32	1.601505	1.592379

Fig. 6

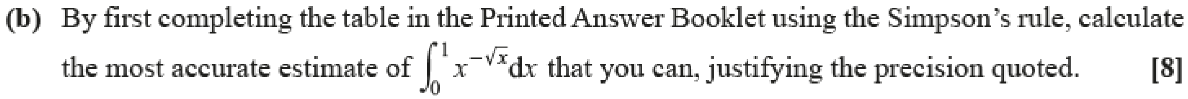
(a) Write down an efficient spreadsheet formula for cell C3. **[2]**

Candidates who did well in this question were familiar with spreadsheet syntax and wrote a correct answer.

Candidates who did less well included a reference to C3 or used mathematical rather than spreadsheet notation (e.g. using '×' instead of '*').

Candidates who did not do well attempted to produce a formula for T_2 in terms of the function values.

Question 6(b)



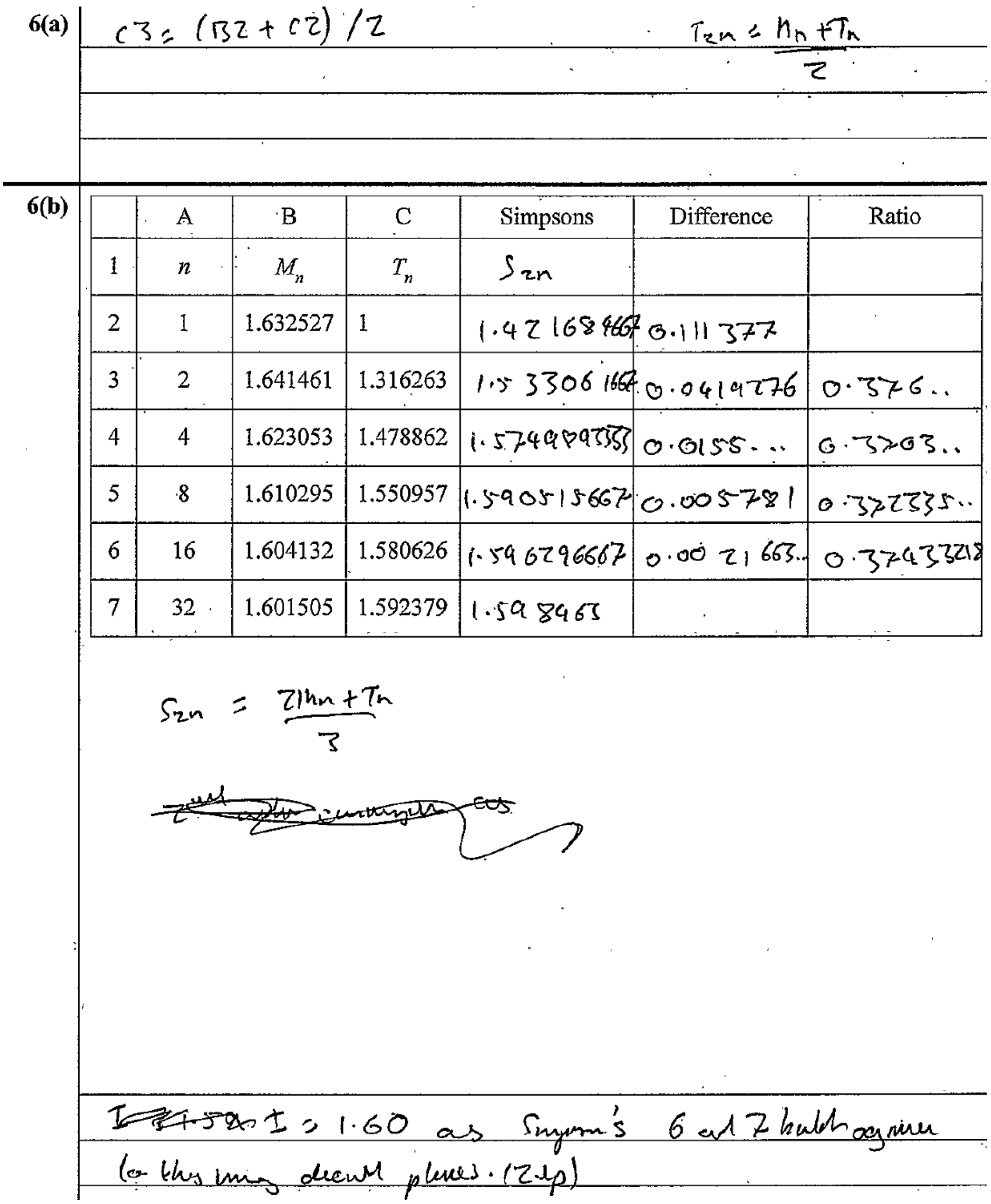
(b) By first completing the table in the Printed Answer Booklet using the Simpson's rule, calculate the most accurate estimate of $\int_0^1 x^{-\sqrt{x}}\mathrm{d}x$ that you can, justifying the precision quoted. **[8]**

Candidates who did well in this question calculated the values of S_{2n} and the associated differences and ratios correctly. They went on to extrapolate with the correct values and were often able to reach a correct conclusion.

Candidates who did less well made slips in the arithmetic in the first part or in extrapolating. Some were over-optimistic in terms of the precision they believed was appropriate for the final answer. A few candidates ignored the work on ratios and extrapolated using r = 1/16.

Candidates who did not do well made more than one slip in obtaining the Simpson's estimates and/or made errors in calculating the differences and their associated ratios. They made little or no progress with extrapolating.

Exemplar 1



6(a) $C3 = (B2 + C2)/2$ $T_{2n} = \frac{M_n + T_n}{2}$

6(b)

	A	B	C	Simpsons	Difference	Ratio
1	n	M_n	T_n	S_{2n}		
2	1	1.632527	1	1.421684667	0.111377	
3	2	1.641461	1.316263	1.533061667	0.0419276	0.376..
4	4	1.623053	1.478862	1.574949333	0.0155...	0.3703..
5	8	1.610295	1.550957	1.590515667	0.005781	0.372335..
6	16	1.604132	1.580626	1.596296667	0.0021663..	0.3743321
7	32	1.601505	1.592379	1.598463		

$S_{2n} = \frac{2M_n + T_n}{3}$

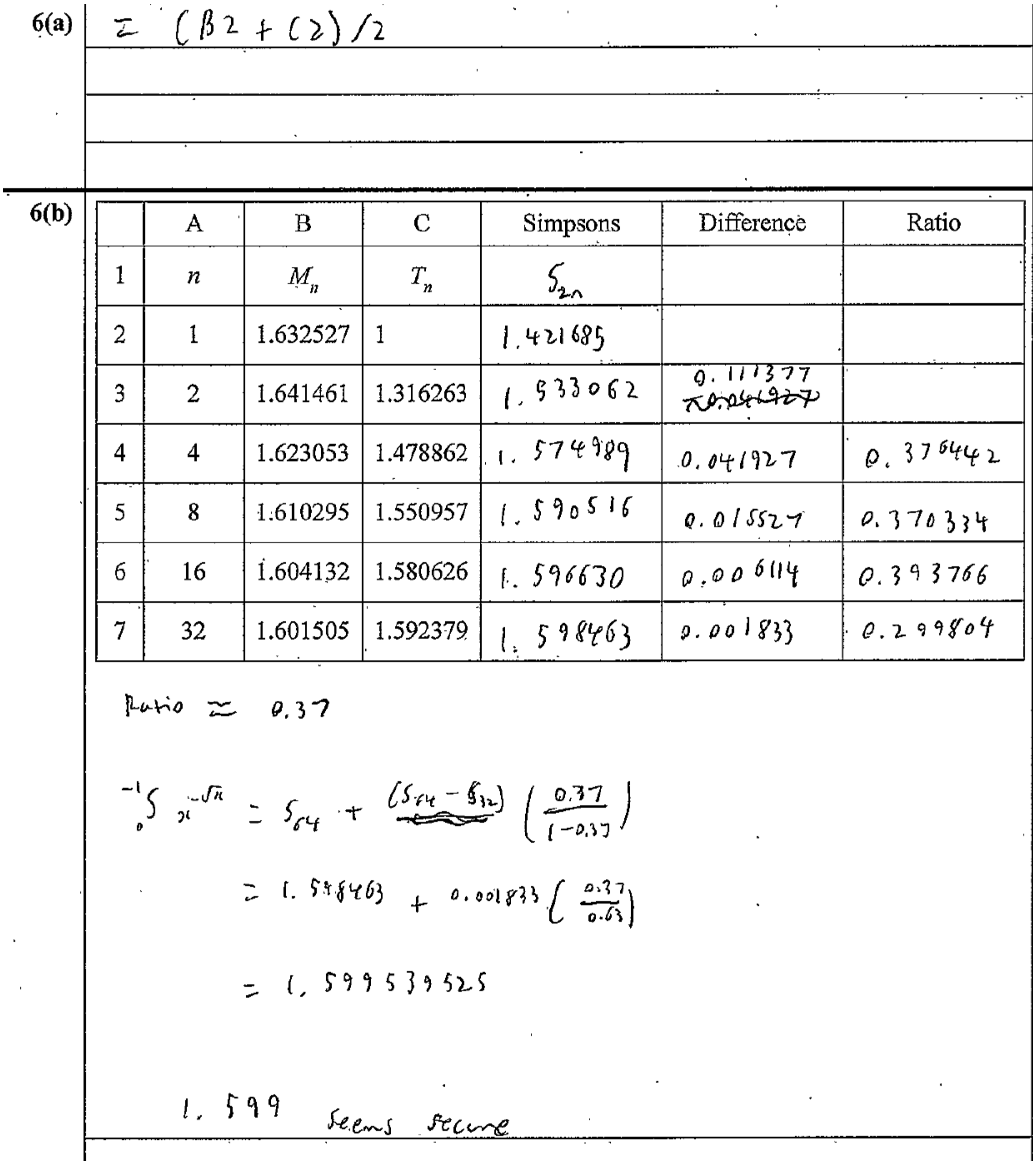
~~[illegible]~~

~~[illegible]~~ I = 1.60 as Simpson's 6 and 7 both agree (to this many decimal places. (2dp)

In part (a) of the above exemplar, beginning the formula with 'C3' means the response cannot be given the A1 mark, because if this is typed into the cell it won't work. There is sufficient evidence for the award of M1, however.

In part (b) the calculations are all carried out correctly and M1A1M1M1 is earned. As there was no attempt to use this work to extrapolate, no further credit was given.

Exemplar 2



6(a) =(B2+C2)/2

6(b)

	A	B	C	Simpsons	Difference	Ratio
1	n	M_n	T_n	S_{2n}		
2	1	1.632527	1	1.421685		
3	2	1.641461	1.316263	1.533062	0.111377 ~~0.041927~~	
4	4	1.623053	1.478862	1.574989	0.041927	0.376442
5	8	1.610295	1.550957	1.590516	0.015527	0.370334
6	16	1.604132	1.580626	1.596630	0.006114	0.393766
7	32	1.601505	1.592379	1.598463	0.001833	0.299804

Ratio $\simeq$ 0.37

$$\int_0^{1} x^{-\sqrt{x}} = S_{64} + (S_{64} - S_{32})\left(\frac{0.37}{1-0.37}\right)$$

$$= 1.598463 + 0.001833\left(\frac{0.37}{0.63}\right)$$

$$= 1.599539525$$

1.599 seems secure

In Exemplar 2 both marks are earned in part (a).

In part (b) one Simson's estimate is incorrect and so the work in the table earns M1A0M1M1. There is an attempt to extrapolate that earns M1, but with an incorrect value for the difference resulting in A0. The final M1 is not earned as the value obtained is outside the allowed range.

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