

**A LEVEL**

**Examiners' report**

**FURTHER  
MATHEMATICS B  
(MEI)**

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**H645**

For first teaching in 2017

**Y422/01 Summer 2022 series**

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## Introduction

Our examiners' reports are produced to offer constructive feedback on candidates' performance in the examinations. They provide useful guidance for future candidates.

The reports will include a general commentary on candidates' performance, identify technical aspects examined in the questions and highlight good performance and where performance could be improved. A selection of candidate answers are also provided. The reports will also explain aspects which caused difficulty and why the difficulties arose, whether through a lack of knowledge, poor examination technique, or any other identifiable and explainable reason.

Where overall performance on a question/question part was considered good, with no particular areas to highlight, these questions have not been included in the report.

A full copy of the question paper and the mark scheme can be downloaded from OCR.

### Advance Information for Summer 2022 assessments

To support student revision, advance information was published about the focus of exams for Summer 2022 assessments. Advance information was available for most GCSE, AS and A Level subjects, Core Maths, FSMQ, and Cambridge Nationals Information Technologies. You can find more information on our [website](#).

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## Paper Y422/01 series overview

Y422/01 (Statistics Major) is an optional examined component for GCE Further Mathematics B (MEI). The component provides two thirds of the total mark for the applied part of the course and includes the content of Y432 (Statistics Minor) together with various other topics. The component focuses on:

- Discrete random variables, including the Poisson, geometric and discrete uniform distributions.
- Bivariate data, including Pearson's product moment correlation coefficient, Spearman's rank correlation coefficient and regression analysis.
- Chi-squared tests for contingency tables and for goodness of fit.
- Continuous random variables, including probability density and cumulative distribution functions, the Normal distribution and the continuous uniform distribution.
- Statistical inference including confidence intervals and hypothesis tests, using the Central Limit Theorem if necessary.
- Simulation.

Candidates are expected to know the content of A Level Mathematics and the Core Pure mandatory paper for Further Mathematics (Y420). Candidates should have gained experience during their course of spreadsheets or other software to explore data sets and to conduct hypothesis tests and construct confidence intervals. They should also have had experience of using a spreadsheet to simulate a random variable.

Most of the questions in Y422/01 are in context and many require interpretation in addition to understanding. Questions may also require candidates to comment about the modelling assumptions underlying their answers.

In general candidates did well in questions involving calculations, but rather less well in questions requiring reasons or explanations.

Candidates who did well on this paper generally did the following:	Candidates who did less well on this paper generally did the following:
<ul style="list-style-type: none"> <li>• identified the correct hypothesis test to carry out and then used correct terminology and gave non-assertive conclusions</li> <li>• identified appropriate probability distributions to use</li> <li>• used calculators effectively to calculate probabilities, find the equation of a regression line and to find means and standard deviations</li> <li>• applied their knowledge and understanding to new and unfamiliar contexts</li> <li>• chose appropriate levels of accuracy to give answers to and in particular give probabilities as exact answers or to four decimal places.</li> </ul>	<ul style="list-style-type: none"> <li>• correctly carried out calculations involving the Poisson and geometric distributions</li> <li>• found the equation of a regression line and used it to predict values</li> <li>• correctly carried out a chi-squared test.</li> </ul>

## Section A overview

This section consists of more straightforward questions. Most questions in this section were very well answered apart from Question 2 (c) which required an explanation and Question 4 (b) which involved the use of a non-standard probability distribution (although Question 4 (b) could be done using a binomial distribution).

### Question 1 (a)

1 During a meteor shower, the number of meteors that can be seen at a particular location can be modelled by a Poisson distribution with mean 1.2 per minute.

(a) Find the probability that exactly 2 meteors are seen in a period of 1 minute. [1]

Almost all candidates answered this correctly.

### Question 1 (b)

(b) Find the probability that more than 3 meteors are seen in a period of 1 minute. [2]

Most candidates answered this correctly. The most common error was to use  $P(X > 3) = 1 - P(X < 3)$  rather than  $P(X > 3) = 1 - P(X \leq 3)$

### Question 1 (c)

(c) Find the probability that no more than 8 meteors are seen in a period of 10 minutes. [2]

The vast majority of candidates answered this correctly.

### Question 1 (d)

(d) Explain what the fact that the number of meteors seen can be modelled by a Poisson distribution tells you about the occurrence of meteors. [2]

Many correct answers were seen, but although most candidates mentioned that occurrences must be random and independent, a number missed out 'uniform' or 'average' in their explanation of 'uniform average rate'.

## Question 2 (a)

- 2 A manufacturer is testing how long coloured LED lights will last before the battery runs out, using two different battery types. The times in hours before the battery runs out are modelled by independent Normal distributions with means and standard deviations as shown in the table.

Type	Time	
	Mean	Standard deviation
A	23	2.8
B	35	3.6

- (a) In a particular test, a battery of type A is used and the time taken for it to run out is recorded. This process is repeated until a total of 5 randomly selected batteries have been used.

Determine the probability that the total time the 5 batteries last is at least 120 hours. [3]

This question on the Normal distribution was usually answered correctly. The main error was to use a variance of  $5^2 \times 2.8^2$  rather than  $5 \times 2.8^2$ . Some candidates knew that the mean was 115 but did not state that the distribution was  $N(115, \sigma^2)$ , and so if they did not get the variance correct, they could not be credited with the first mark.

## Question 2 (b)

- (b) In a similar test, 3 randomly selected batteries of type A are used, one after the other. Then 2 randomly selected batteries of type B are used, one after the other.

Determine the probability that the 3 type A batteries last longer in total than the 2 type B batteries. [3]

This part was slightly more difficult than the previous part since it involved the difference of two Normal distributions. Most candidates coped very well with this part also. Almost all found the correct mean of  $\pm 1$ , but rather more had problems in finding the variance, with some multiplying by 9 and 4 instead of 3 and 2, but it was good to note that hardly any candidates subtracted rather than added the variances.

## Question 2 (c)

- (c) Explain why it is necessary that the Normal distributions are independent in order to be able to find the probability in part (b). [1]

This part was found to be very difficult with few correct responses seen. Many candidates made statements such as 'If they were not independent then the time for one could influence the time for the other.' This type of answer is simply stating what independence is rather than explaining why it is necessary. Some candidates who stated that it would not be possible to add the variances to give the combined variance, also stated something similar about the expectations, which is of course not correct, but this error was condoned.

## Question 3 (a)

- 3 The table shows the probability distribution of the random variable  $X$ , where  $a$  and  $b$  are constants.

$r$	0	1	2	3	4
$P(X=r)$	$a$	$b$	0.24	0.32	$b^2$

- (a) Given that  $E(X) = 1.8$ , determine the values of  $a$  and  $b$ . [3]

Almost all candidates answered this correctly.

## Question 3 (b)

The random variable  $Y$  is given by  $Y = 10 - 3X$ .

- (b) Using the values of  $a$  and  $b$  which you found in part (a), find each of the following.
- $E(Y)$
  - $\text{Var}(Y)$

[3]

Most candidates answered this correctly. A few multiplied the variance of  $X$  by 3 instead of by 9.

**Question 4 (a)**

4 A pack of  $k$  cards is labelled 1, 2, ...,  $k$ . A card is drawn at random from the pack. The random variable  $X$  represents the number on the card.

(a) Given that  $k > 10$ , find  $P(X \geq 10)$ . [2]

This was well answered by most candidates. A few had a numerator of  $k - 10$  rather than  $k - 9$ .

**Question 4 (b)**

You are now given that  $k = 20$ .

(b) A card is drawn at random from the pack and the number on it is noted. The card is then returned to the pack. This process is repeated until the second occasion on which the number noted is less than 9.

Find the probability that no more than 4 cards have to be drawn. [3]

Candidates found this part more challenging, with well under half giving a fully correct response. Most candidates realised that the probability of one card having a number less than 9 was 0.4 which gained them a mark. However many either made no more progress or had correct terms but wrong coefficients. Successful responses used the distribution  $B(4, 0.4)$  and simply found the probability of at least two successes.

## Section B overview

This section consists of less and more straightforward questions. Questions 5, 7 and 10 (apart from 10 (d)) were generally very well answered, whereas Questions 8, 9 and 11 (apart from 9 (b) and 9 (c)) were found to be the most challenging.

### Question 5 (a)

- 5 A motorist is investigating the relationship between tyre pressure and temperature. As the temperature increases during a hot day, she records the pressure (measured in bars) of one of her car tyres at specific temperatures of  $20^{\circ}\text{C}$ ,  $22^{\circ}\text{C}$ , ...,  $36^{\circ}\text{C}$ . The results are shown in **Table 5.1**.

Temperature ( $t^{\circ}\text{C}$ )	20	22	24	26	28	30	32	34	36
Tyre pressure ( $P$ bar)	2.012	2.036	2.065	2.074	2.114	2.140	2.149	2.176	2.192

**Table 5.1**

- (a) Calculate the equation of the regression line of pressure on temperature. Give your answer in the form  $P = at + b$ , giving the values of  $a$  and  $b$  to 4 significant figures. [2]

This part was very well answered. However, a number of candidates wasted time by working out  $S_{xx}$  and  $S_{xy}$  instead of simply putting the data into their calculator to find the equation of the regression line.

### Question 5 (b)

- (b) **Table 5.2** shows the residuals for most of the data values. Complete the copy of the table in the Printed Answer Booklet. [3]

Temperature	20	22	24	26	28	30	32	34	36
Residual tyre pressure	-0.003	-0.002	0.004	-0.010		0.011	-0.003	0.001	

**Table 5.2**

This part was fairly well answered with most candidates knowing how to calculate the residuals. Some subtracted the wrong way around but only a very few did not attempt the question.

### Question 5 (c)

- (c) With reference to the values of the residuals, comment on the goodness of fit of the regression line. [2]

The modal mark in this question was 1, with many candidates stating that the residuals were all small, or that the sum of squares of the residuals was small. Some stated that the sum of the residuals was small, which of course was not given any credit as the sum of the residuals is always zero. Very few candidates were credited with the second mark for 'no discernible pattern to suggest that the fit is non-linear'.

### Question 5 (d)

- (d) Use your answer to part (a) to calculate an estimate of the pressure in the tyre at each of the following temperatures, giving your answers to 3 decimal places. [2]
- 25°C
  - 10°C

This part was very well answered.

### Question 5 (e)

- (e) Comment on the reliability of each of your estimates. [2]

Almost all candidates realised that one of the temperatures involved interpolation whereas the other involved extrapolation and so were given 1 mark. Only a quarter of candidates mentioned the goodness of fit of the line or discussed the size of the residuals which was required to gain the second mark.

## Question 6 (a)

### 6 In this question you must show detailed reasoning.

A particular type of skin cream is meant to contain an average of 12.2 g of liquid paraffin in a 200 g tub. A researcher believes that the average may differ from this. A random sample of 40 tubs is chosen, and the weight of liquid paraffin,  $x$  g, in each tub is measured. You are given that

$$\sum x = 491.84, \quad \sum x^2 = 6050.3.$$

- (a) Determine a 95% confidence interval for the mean weight of liquid paraffin in a tub. [6]

Two thirds of candidates were credited with full marks in this question. Some made an error in finding the variance and others used the variance instead of the standard deviation in calculating the interval. Almost all used the correct inverse Normal distribution probability of 1.96 or occasionally the corresponding inverse  $t$  value of 2.023. A few candidates used the value of 2.2 in working out their confidence interval, although these same candidates had usually nevertheless calculated the sample mean, so this error only cost them 1 mark.

## Question 6 (b)

- (b) Explain whether the confidence interval supports the researcher's belief. [1]

Almost all candidates realised how to answer this question and following through their confidence interval if it was incorrect. However, over a third gave an answer of the form 'It does not as 12.2 does not lie within the interval', which is of course incorrect.

## Question 6 (c)

- (c) Explain why the sample has to be random in order to construct the confidence interval. [1]

In previous years in questions of this nature, very few candidates mentioned 'inference'. This year there were rather more mentions of it, although still less than a third of candidates scored the mark.

### Question 6 (d)

- (d) A 95% confidence interval for the mean weight in grams of another ingredient in the skin cream is [1.202, 1.398]. This confidence interval is based on a large sample and the unbiased estimate of the population variance calculated from the sample is 0.25.

Find each of the following.

- The mean of the sample
- The size of the sample

[3]

Almost all candidates found the sample mean correctly. Rather less found the correct variance as, despite having used the standard deviation correctly in part (a), many did not square root the variance in this part and therefore got a final answer of 25 rather than 100. These candidates could only score the first mark for the mean.

### Assessment for learning



In many questions, candidates have to use (and sometimes calculate) the standard deviation or variance. It is essential to make sure that the correct one of the two is being used. This applies to questions such as this, and to questions involving the Normal or  $t$  distribution.

### Question 7 (a)

- 7 Amir is trying to thread a needle. On each attempt the probability that he is successful is 0.3, independently of any other attempt. The random variable  $X$  represents the number of attempts that he takes to thread the needle.

- (a) Find  $P(X = 5)$ .

[2]

Almost all candidates answered this correctly.

### Question 7 (b)

- (b) During the course of a day, Amir has to thread 6 needles.

Determine the probability that it takes him more than 3 attempts to be successful for at least 4 of the 6 needles.

[3]

This question was found to be surprisingly difficult, with only about half of candidates getting the correct final answer. Of those who did not, many scored 1 mark, for either the correct probability of taking more than 3 attempts to be successful for one needle, or for stating the distribution as  $B(6, \text{their } 0.343)$ . Some candidates who found 0.343, did not realise that all that they had to do was to find  $0.7^3 = 0.343$ , and instead worked out the probability of 1, 2 or 3 attempts and then took the sum of these from 1.

### Question 7 (c)

- (c) Amaya is also trying to thread a needle. On each attempt the probability that she is successful is  $p$ , independently of any other attempt. The probability that Amaya takes 2 attempts to thread a particular needle is  $\frac{28}{121}$ .

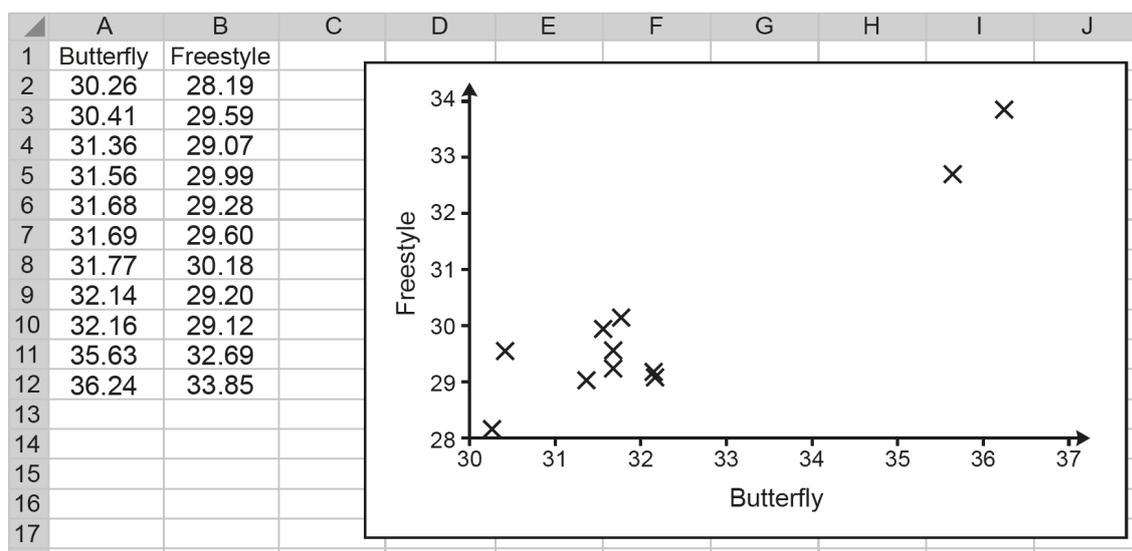
Determine the possible values of  $p$ .

[3]

This question was very well answered, with most candidates scoring full marks.

### Question 8 (a)

- 8 A swimming coach is investigating whether there is correlation between the times taken by teenage swimmers to swim 50m Butterfly and 50m Freestyle. The coach selects a random sample of 11 teenage swimmers and records the times that each of them take for each event. The spreadsheet shows the data, together with a scatter diagram to illustrate the data.



- (a) In the scatter diagram, Butterfly times have been plotted on the horizontal axis and Freestyle times on the vertical axis. A student states that the variables should have been plotted the other way around.

Explain whether the student is correct.

[2]

Candidates were more or less equally divided between those who scored both marks and those who scored zero, with few gaining just 1 mark. Those who did not gain any marks made a wide variety of errors, such as saying that both variables were dependent, or that the dependence was the other way around, or that the student was correct.

## Question 8 (b)

The student decides to carry out a hypothesis test to investigate whether there is any correlation between the times taken for the two events.

- (b) Explain why the student decides to carry out a test based on Spearman's rank correlation coefficient. [2]

Candidates found this question to be fairly difficult, with few candidates mentioning outliers and only about half stating that the scatter diagram does not appear to be elliptical (and so the distribution is probably not bivariate Normal).

## Question 8 (c)

- (c) **In this question you must show detailed reasoning.**

Carry out the test at the 5% significance level. [8]

The modal mark in this question was 7 with very few gaining full marks. Of those who scored 7 marks, almost all had everything correct other than mentioning 'population' in their hypotheses. Almost all of those who scored the second most common mark of 5 made a different error in that their hypotheses were given in terms of 'correlation' instead of 'association'. Most candidates made a good attempt at calculating Spearman's coefficient, even if some made a slight error in the rankings. Some however used the wrong formula. Almost all gave the correct critical value. Most candidates gave their conclusion in context, which is required, although a few were over-assertive in expressing their conclusion.

### Assessment for learning



In questions involving hypothesis testing, the hypotheses should always mention the population, either by stating as in this case 'in the population' at the end of each hypothesis, or if a parametric test is being carried out, defining the parameter (usually  $\mu$ ) as the population parameter. Thus, in Question 11 (c) the definition of  $\mu$  should be ' $\mu$  is the population mean increase in body mass' rather than be ' $\mu$  is the mean increase in body mass'.

The conclusions in such tests should always be given in context. In this case 'There is insufficient evidence to suggest that there is association between Butterfly and Freestyle times in the population', rather than just 'There is insufficient evidence to suggest that there is association'.

The conclusions should have an element of doubt. A statement such as 'There is insufficient evidence to suggest that there is association, and so Butterfly and Freestyle times are not associated' is too assertive. Such a statement would not be credited with the mark for the conclusion due to the final part of the sentence.

Exemplar 1

8(c)  $H_0$ : there is ~~no~~ correlation between butterfly swimming times to swim 50m and freestyle swimming times to swim 50m.  
 $H_1$ : there is ~~a~~ correlation between butterfly swimming times to swim 50m and freestyle swimming times to swim 50m.

Butterfly rank	Freestyle rank	$d$	$d^2$
1		0	0
2	<del>6</del> 6	4	16
3	<del>2</del> 2	1	1
4	<del>8</del> 8	4	16
5	<del>5</del> 5	0	0
6	<del>7</del> 7	1	1
7	9	2	4
8	4	4	16

(answer space continued on next page)

8(c) (continued)

9	3	6	36
10	10	0	0
11	11	0	0
			<u>90</u>

$$r_s = \frac{6 \times 90}{11(11^2 - 1)} = 0.4090909 \dots$$

$$\approx \underline{\underline{0.4091}}$$

$$CV = 0.6182$$

$\therefore 0.4091 < 0.6182$   
 $\therefore r_s < CV$   
 $\therefore$  insignificant result. Sufficient evidence to reject  $H_0$  would suggest that there is a correlation between butterfly and freestyle swimming times for 50m. ( $H_1$ )

This response scored 4 marks out of 8. The candidate's response is easy to follow as is required for a 'detailed reasoning' question. The hypotheses are given in terms of correlation instead of association, and do not mention population, so no marks can be given. The ranks are then found correctly and the squares of the differences between them, but the formula for Spearman's coefficient of rank correlation is wrong, despite it being given in the Formulae Booklet. The critical value is correct, and the mark is given for the comparison, despite the formula being incorrect. The final answer mark is not given as there are three errors: the value of Spearman's coefficient is wrong, the answer is given in terms of correlation rather than association, and the conclusion should be that there is no association.

### Question 8 (d)

(d) The student concludes that there is definitely no correlation between the times.

Comment on the student's conclusion.

[2]

Responses to this question were almost equally divided between the three possible marks of 0, 1 or 2. Those who scored 1 mark often made statements such as 'The test only suggests that there is no association', which is true but is not a full answer. To score both marks candidates needed to discuss the significance level of the test, or to state that a different sample might produce a different result.

### Question 9 (a)

9 The random variable  $X$  has a discrete uniform distribution over the values  $\{0, 1, 2, \dots, 20\}$ .

(a) Find  $P(X \leq 7)$ .

[1]

Two thirds of candidates had the correct answer. Almost all of the rest gave an incorrect answer of  $\frac{7}{21}$  or  $\frac{7}{20}$ .

### Question 9 (b)

(b) Find each of the following.

- $E(X)$
- $\text{Var}(X)$

[3]

Full marks were given to half of candidates. Some candidates thought that the expectation of  $X$  was 10.5 or 11. Others used the correct formula for the variance but used a value of  $n = 20$  whereas, since the first value of  $X$  is 0 rather than 1, the value of  $n$  should have been 21.

### Question 9 (c)

The spreadsheet shows a simulation of the distribution of  $X$ . Each of the 25 rows of the spreadsheet below the heading row shows a simulation of 10 independent values of  $X$  together with the value of the mean of the 10 values, denoted by  $Y$ .

	A	B	C	D	E	F	G	H	I	J	K	L
1	$X_1$	$X_2$	$X_3$	$X_4$	$X_5$	$X_6$	$X_7$	$X_8$	$X_9$	$X_{10}$	$Y$	
2	1	6	2	1	18	6	4	9	11	11	6.9	
3	13	14	12	2	4	11	16	0	16	0	8.8	
4	4	17	1	16	4	10	12	2	18	13	9.7	
5	2	8	12	1	4	16	12	2	15	8	8.0	
6	7	15	16	0	4	7	1	13	0	20	8.3	
7	15	13	10	1	12	0	20	15	16	6	10.8	
8	14	13	17	12	2	18	16	18	9	4	12.3	
9	20	2	12	3	17	3	0	18	15	13	10.3	
10	2	12	5	12	2	6	0	9	10	15	7.3	
11	5	11	13	10	9	17	10	4	20	15	11.4	
12	14	9	9	7	6	20	2	2	11	16	9.6	
13	15	19	18	19	7	6	6	20	3	8	12.1	
14	5	10	6	4	1	19	15	8	17	18	10.3	
15	0	3	15	15	11	12	0	3	9	16	8.4	
16	1	12	1	15	0	4	11	11	9	2	6.6	
17	12	5	0	8	3	8	12	19	13	12	9.2	
18	9	5	1	13	5	4	18	1	1	19	7.6	
19	16	2	20	20	12	17	2	7	8	20	12.4	
20	18	17	3	2	8	18	7	0	11	6	9.0	
21	15	10	7	20	4	0	5	6	11	14	9.2	
22	3	9	10	14	2	1	8	6	0	7	6.0	
23	11	10	11	10	19	11	3	7	10	0	9.2	
24	12	14	6	6	5	20	11	18	10	14	11.6	
25	1	11	5	14	11	10	1	1	2	0	5.6	
26	0	14	7	11	18	5	10	20	11	9	10.5	
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(c) Use the spreadsheet to estimate  $P(Y \leq 7)$ . [1]

The vast majority of candidates answered correctly. The main error was to count wrongly and give an answer of  $\frac{3}{25}$  or  $\frac{5}{25}$ , or occasionally to think that there were 26 rows so giving an answer of  $\frac{4}{26}$ .

### Question 9 (d)

(d) Explain why the true value of  $P(Y \leq 7)$  is less than  $P(X \leq 7)$ , relating your answer to  $\text{Var}(X)$  and  $\text{Var}(Y)$ . [2]

Although of the majority of candidates attempted this part, under half scored any marks. Many made an error in finding the variance of  $Y$ , often stating that it was more rather than less than that of  $X$ , despite the fact that it is the mean of 10 values of  $X$ . Of those who did have the correct value of the variance of  $Y$ , most then gave an adequate final explanation, this explanation being generously marked.

### Question 9 (e)

(e) The random variable  $W$  is the mean of 30 independent values of  $X$ .

Determine an estimate of  $P(W \leq 7)$ .

[4]

Under half of the candidates did not gain any marks in this question, and of the rest the marks were more or less equally divided between those who scored 1, 2, 3 or 4 marks. For those who did score any marks, the first mark was a follow through from their variance of  $X$  and this was often correct. The second mark for stating the distribution as  $N(\text{their mean, their variance})$  was usually then scored. Those candidates who had the correct mean and variance usually used these correctly, either with or without a continuity correction, to score 4 or 3 marks. A few candidates used the wrong continuity correct, either  $7 - \frac{1}{60}$  or  $7 + \frac{1}{30}$ . A small number of candidates used a new variable  $30W$ , and often came to the correct answer.

### Question 10 (a)

10 A scientist is researching dietary fat intake and cholesterol level. A random sample of 60 people is selected and their dietary fat intakes and cholesterol levels are measured. Dietary fat intakes are classified as low, medium and high, and cholesterol levels are classified as normal and high.

The scientist decides to carry out a chi-squared test to investigate whether there is any association between dietary fat intake and cholesterol level. **Tables 10.1** and **10.2** show the data and some of the expected frequencies for the test.

		Dietary fat intake			Total
		Low	Medium	High	
Cholesterol level	Normal	9	18	5	32
	High	3	13	12	28
Total		12	31	17	60

**Table 10.1**

Expected frequency		Dietary fat intake		
		Low	Medium	High
Cholesterol level	Normal			9.0667
	High			7.9333

**Table 10.2**

(a) Complete the table of expected frequencies in the Printed Answer Booklet.

[2]

This question was very well answered by almost all candidates.

## Question 10 (b)

- (b) Determine the contribution to the chi-squared test statistic for people with normal cholesterol level and high dietary fat intake, giving your answer to 4 decimal places. [2]

Similarly, to part (a), this question was also very well answered.

## Question 10 (c)

The contributions to the chi-squared test statistic for the remaining categories are shown in Table 10.3.

Contribution to the test statistic		Dietary fat intake		
		Low	Medium	High
Cholesterol level	Normal	1.0563	0.1301	
	High	1.2071	0.1487	2.0846

Table 10.3

- (c) In this question you must show detailed reasoning.

Carry out the test at the 5% significance level.

[6]

This question was found to be very accessible with well over half of the candidature gaining full marks. Of those who did not, most scored at least 3 marks and often 4 or 5 marks. For such candidates there was a wide variety of errors including: having an over-assertive final conclusion, using the wrong number of degrees of freedom, making a mistake in calculating the test statistic, giving the hypotheses the wrong way around and stating them in terms of correlation instead of association.

## Question 10 (d)

- (d) For each level of dietary fat intake, give a brief interpretation of what the data suggest about the level of cholesterol. [3]

This question was found to be very challenging, with only just over half of candidates scoring any marks at all, and the majority of those who did only scored 1 mark. Many candidates gave vague statements about those with high dietary fat tending to have high cholesterol and vice-versa. Candidates can only be credited with marks if they compare the observed numbers with the expected numbers. Many who did this did not however refer to the contributions and so could only score at most 1 mark out of 3. Rather more candidates than hoped for stated that for medium dietary fat levels there were differences between the observed and the expected, despite both of the contributions being well below 1.

Exemplar 2

10(d)	<p><del>For low dietary fat intake a normal cholesterol</del>  <del>for each level of dietary fat intake</del></p> <p>for low + medium levels of dietary fat intake          normal cholesterol is more than expected.          and high cholesterol is less than expected</p> <p>for high level of dietary fat intake          normal cholesterol is less than expected          and high cholesterol is more than expected</p>
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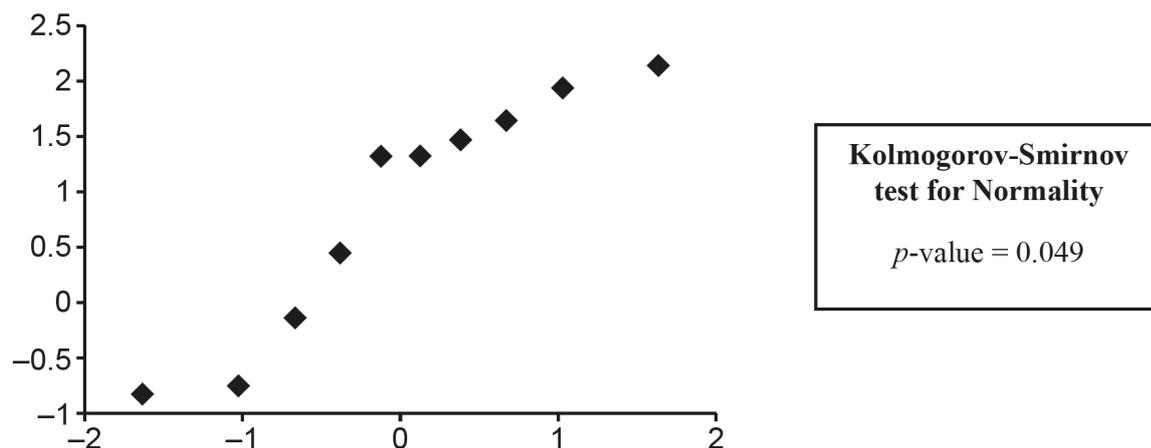
This response scored 1 mark out of 3. The candidate has two correct comments for 'High cholesterol', although contributions are not mentioned. The comment for 'Low cholesterol' is correct, but that for 'Medium cholesterol' is not, since the contributions for Medium are both low, and so the candidate should say that the numbers are as expected. Since the candidate does not mention the size of the contributions, the maximum mark that is available is just 1. Candidates should **always** refer to the sizes of the contributions when interpreting the results of a chi-squared test. If the contributions are much less than 1, they should say that the numbers are as expected.

## Question 11 (a)

- 11 A particular dietary supplement, when taken for a period of 1 month, is claimed to increase lean body mass of adults by an average of 1 kg. A researcher believes that this claim overestimates the increase. She selects a random sample of 10 adults who then each take the supplement for a month. The increases in lean body masses in kg are as follows.

−0.84    −0.76    −0.16    0.43    1.31    1.32    1.47    1.64    1.93    2.14

A Normal probability plot and the  $p$ -value of the Kolmogorov-Smirnov test for these data are shown below.



- (a) The researcher decides to carry out a hypothesis test in order to investigate the claim.

Comment on the type of hypothesis test that should be used. You should refer to

- The Normal probability plot
- The  $p$ -value of the Kolmogorov-Smirnov test

[3]

Although there were many correct answers, over a fifth of candidates scored zero, almost always stating that a  $t$  distribution or a distribution based on the Normal distribution should be used. Most of these suggested that the Normal probability plot was almost a straight line, and that the Kolmogorov-Smirnov test gave a value that suggested that a  $t$  test would be appropriate. Some candidates even stated that the fact that the  $p$ -value was less than 0.05 meant that a  $t$  test could be carried out.

## Question 11 (b)

- (b) Carry out a test at the 5% significance level to investigate whether the researcher's belief may be correct.

[7]

This question proved to be very challenging, with only a third of candidates scoring more than 4 marks out of 7. Some candidates used a  $t$  test, for which no marks were available. Others did not subtract the hypothesised population median from the data values or subtracted the median or mean of the data from the values. Such candidates could only score a maximum of 4 marks. A few had the wrong critical value. Of those who found the Wilcoxon test statistic correctly and had the correct critical value, approximately half included 'population' in their hypotheses and so could get full marks.

Exemplar 3

11(b)

$H_0$ : The median lean weight gain is 1 kg  
 $H_1$ : The median lean weight gain is less than 1 kg.

~~-0.84~~ ~~-0.76~~

Sample Median =  $1.31 + 1.32 = 1.315$

<del><math>x</math></del>	$M - x$	<del><math>K</math></del> $W_+$	$W_-$
<del>-0.84</del>	2.155	10	1
-0.76	2.075	9	
-0.16	1.475	8	
0.43	0.885	7	
1.31	0.005	1.5	
1.32	-0.005		1.5
1.49	-0.155		3
1.66	-0.325		4
1.93	-0.615		5
2.19	-0.825		6 (answer space continued on next page)

$\sum W_+ = 35.5$  } 5  
 $\sum W_- = 19.5$  }

$\frac{1}{2}(10)(9) = 45$

11(b) (continued)

Test statistic = 19.5

at 5% significance level,  $n=10$ , 1 tail  $\Rightarrow$  CV = 5

Since  $19.5 > 5$  accept  $H_0$ .

There is insufficient evidence at the 5% significance level to suggest the median ~~weight~~ lean weight gain is ~~greater~~ <sup>less</sup> than 1 kg.

This response scored 1 mark out of 7. The candidate's hypotheses are correct, but they do not mention population and so the candidate only gets 1 of the 2 marks for the hypotheses. They then calculate the sample median and use this instead of the hypothesised median to carry out a Wilcoxon test. Therefore, the candidate gets no marks for the calculation. The critical value is wrong and so no more marks can be credited to the candidate.

### Question 11 (c)

- (c) If the Normal probability plot had been different, giving a  $p$ -value of 0.65 for the Kolmogorov-Smirnov test, a different procedure could have been used to investigate the researcher's belief.
- State what alternative test could have been used in this case.
  - State what the hypotheses would have been. [3]

Responses were more successful in this part than in part (b), with over a third of candidates scoring full marks. Some candidates thought that a high value of  $p$  meant that the parent population distribution was not Normal and so a non-parametric test should be used, and these candidates could not be given any credit unless they had the correct hypotheses for a  $t$  test, which surprisingly some did. Some candidates had a 2-tailed alternative hypothesis and some defined the mean but omitted the word 'population' in their definition and either of these errors would cost a mark.

### Question 12 (a)

12 The continuous random variable  $X$  has cumulative distribution function given by

$$F(x) = \begin{cases} 0 & x < 0, \\ k(ax - 0.5x^2) & 0 \leq x \leq a, \\ 1 & x > a, \end{cases}$$

where  $a$  and  $k$  are positive constants.

- (a) Determine the median of  $X$  in terms of  $a$ . [7]

About half of candidates gained 6 or all 7 marks on this question. Those who did not score the final mark either did not realise that one of their two answers to their equation was not in the domain of the function or chose the wrong one of the two answers. In order to score any marks, candidates had to try to find the value of  $k$ . This was essential in order to find the median. Some candidates, having found  $k$  correctly, found the correct equation for the median but then either did not know how to solve it, or solved it incorrectly. Those who did try to solve it usually used the quadratic formula but some of the higher ability candidates correctly solved the equation using completing the square.

## Question 12 (b)

- (b) Given that  $a = 10$ , determine the probability that  $X$  is within one standard deviation of its mean. [7]

This question discriminated between candidates very well, with over two fifths gaining full marks and under a third scoring zero. The other marks were fairly evenly distributed. In order to gain any credit, candidates first had to find the equation of the probability density function. Of those candidates who did this but did not get all of the marks, most had correct integrals for the expectation of  $X$  and the expectation of  $X^2$  and often evaluated them correctly. In evaluating the integrals, many candidates used their calculator functions, but many others actually integrated the functions. In this case there was not much difference in the two methods, as the integrals and the limits were very simple, although had the integrals been more complicated, candidates would be advised to use the former method. Having evaluated the integrals, some candidates then forgot to subtract the square of the mean in order to find the variance. Some did not have the correct limits due to previous errors, or sometimes due to forgetting to square root the variance. A few had the correct limits, but then made an error in evaluating the final probability.

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