



Oxford Cambridge and RSA

GCE

Further Mathematics B (MEI)

Y413/01: MEI Modelling with algorithms

AS Level

Mark Scheme for June 2022

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

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Text Instructions

Annotations and abbreviations

Annotation in scoris	Meaning
✓ and ✖	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
E	Explanation mark 1
SC	Special case
^	Omission sign
MR	Misread
BP	Blank page
Highlighting	
Other abbreviations in mark scheme	Meaning
E1	Mark for explaining a result or establishing a given result
dep*	Mark dependent on a previous mark, indicated by *. The * may be omitted if only previous M mark.
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This indicates that the instruction In this question you must show detailed reasoning appears in the question.

2. Subject-specific Marking Instructions for AS Level Mathematics B (MEI)

- a Annotations must be used during your marking. For a response awarded zero (or full) marks a single appropriate annotation (cross, tick, M0 or ^) is sufficient, but not required.

For responses that are not awarded either 0 or full marks, you must make it clear how you have arrived at the mark you have awarded and all responses must have enough annotation for a reviewer to decide if the mark awarded is correct without having to mark it independently.

It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

Award NR (No Response)

- if there is nothing written at all in the answer space and no attempt elsewhere in the script
- OR if there is a comment which does not in any way relate to the question (e.g. 'can't do', 'don't know')
- OR if there is a mark (e.g. a dash, a question mark, a picture) which isn't an attempt at the question.

Note: Award 0 marks only for an attempt that earns no credit (including copying out the question).

If a candidate uses the answer space for one question to answer another, for example using the space for 8(b) to answer 8(a), then give benefit of doubt unless it is ambiguous for which part it is intended.

- b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not always be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner. If you are in any doubt whatsoever you should contact your Team Leader.

- c The following types of marks are available.

M

A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A method mark may usually be implied by a correct answer unless the question includes the DR statement, the command words “Determine” or “Show that”, or some other indication that the method must be given explicitly.

A

Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B

Mark for a correct result or statement independent of Method marks.

E

A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

- d When a part of a question has two or more ‘method’ steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation ‘dep*’ is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.
- e The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case, please escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
- Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be ‘follow through’. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is numerically correct and expressed either in SI or in the units of the question. (e.g. lengths will be assumed to be in metres unless in a particular question all the lengths are in km, when this would be assumed to be the unspecified unit.)

We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so.

- When a value is **given** in the paper only accept an answer correct to at least as many significant figures as the given value.
- When a value is **not given** in the paper accept any answer that agrees with the correct value to **2 s.f.** unless a different level of accuracy has been asked for in the question, or the mark scheme specifies an acceptable range.

NB for Specification A the rubric specifies 3 s.f. as standard, so this statement reads “3 s.f”

Follow through should be used so that only one mark in any question is lost for each distinct accuracy error.

Candidates using a value of 9.80, 9.81 or 10 for g should usually be penalised for any final accuracy marks which do not agree to the value found with 9.8 which is given in the rubric.

g Rules for replaced work and multiple attempts:

- If one attempt is clearly indicated as the one to mark, or only one is left uncrossed out, then mark that attempt and ignore the others.
- If more than one attempt is left not crossed out, then mark the last attempt unless it only repeats part of the first attempt or is substantially less complete.
- if a candidate crosses out all of their attempts, the assessor should attempt to mark the crossed out answer(s) as above and award marks appropriately.

h For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate’s data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A or B mark in the question. Marks designated as cao may be awarded as long as there are no other errors. If a candidate corrects the misread in a later part, do not continue to follow through. E marks are lost unless, by chance, the given results are established by equivalent working. Note that a miscopy of the candidate’s own working is not a misread but an accuracy error.

i If a calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers provided that there is nothing in the wording of the question specifying that analytical methods are required such as the bold “In this question you must show detailed reasoning”, or the command words “Show” and “Determine. Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

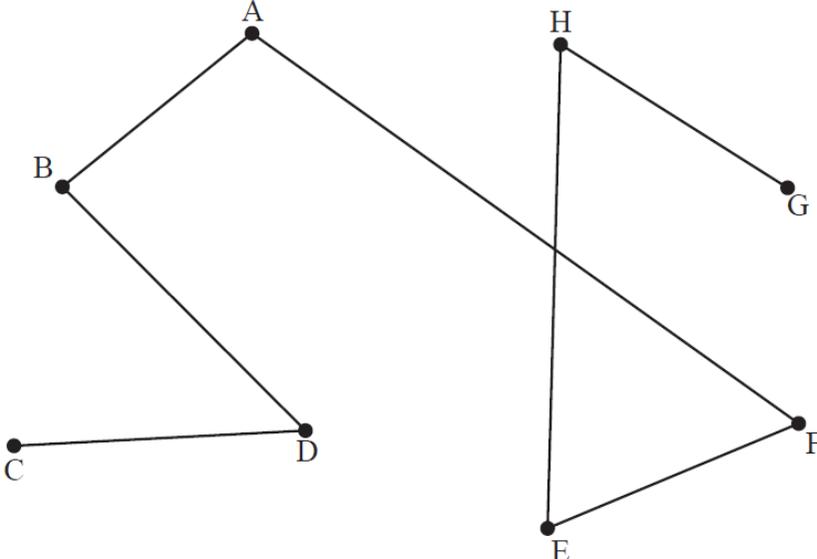
j If in any case the scheme operates with considerable unfairness consult your Team Leader.

Question			Answer	Marks	AO	Guidance
1	(a)	(i)	15	B1	1.1	cao

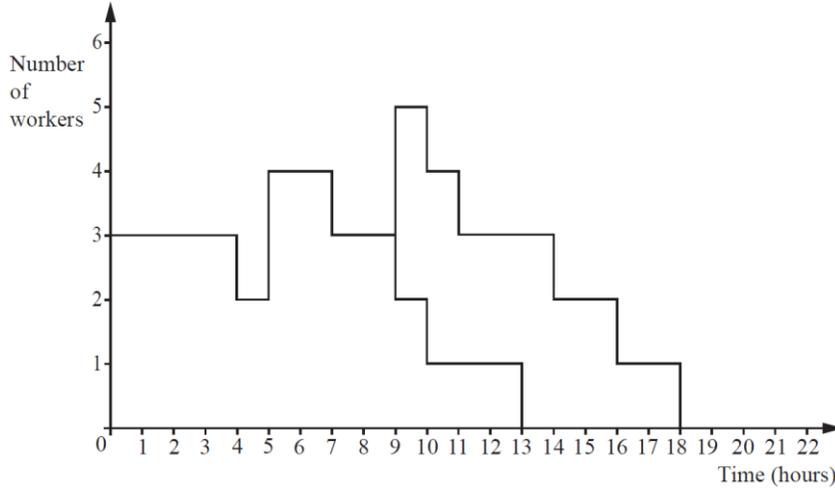
Question		Answer	Marks	AO	Guidance
			[1]		
	(ii)	5	B1	1.1	cao
			[1]		
(b)	(i)		B1	1.1	cao
			[1]		
	(ii)	8	B1	1.1	cao
			[1]		

Question		Answer	Marks	AO	Guidance																				
2	(a)	The flow chart contains a finite sequence of operations for solving the problem of finding the square root of N and so is an algorithm	B1	1.2	Correct definition of an algorithm – must include ‘finite sequence of operations’, ‘finite set of instruction’ oe but not just ‘finite’																				
			[1]																						
	(b)	<table border="1" style="margin-left: auto; margin-right: auto;"> <thead> <tr> <th>N</th> <th>A</th> <th>B</th> <th>Is $-10^{-6} < B - A < 10^{-6}$</th> </tr> </thead> <tbody> <tr> <td>73</td> <td>8</td> <td>8.5625 (or $\frac{137}{16}$)</td> <td>No</td> </tr> <tr> <td></td> <td>8.5625 (or $\frac{137}{16}$)</td> <td>8.544023723</td> <td>No</td> </tr> <tr> <td></td> <td>8.544023723</td> <td>8.544003745</td> <td>No</td> </tr> <tr> <td></td> <td>8.544003745</td> <td>8.544003745</td> <td>Yes</td> </tr> </tbody> </table> <p>Output = 8.5440037</p>	N	A	B	Is $-10^{-6} < B - A < 10^{-6}$	73	8	8.5625 (or $\frac{137}{16}$)	No		8.5625 (or $\frac{137}{16}$)	8.544023723	No		8.544023723	8.544003745	No		8.544003745	8.544003745	Yes	M1 A1 A1	3.4 1.1 1.1	Initial values for A and B correct and each updated at least once (with B to at least 7 d.p.) cao – table completely correct with values correct to at least 8 d.p. (where appropriate) oe. Ignore rows if completed after ‘Yes’ statement Correct output (to exactly 7 d.p.) – independent of first A mark
N	A	B	Is $-10^{-6} < B - A < 10^{-6}$																						
73	8	8.5625 (or $\frac{137}{16}$)	No																						
	8.5625 (or $\frac{137}{16}$)	8.544023723	No																						
	8.544023723	8.544003745	No																						
	8.544003745	8.544003745	Yes																						
			[3]																						
	(c)	Change A to $-A$ in the line ‘Let $B = \frac{\frac{N}{A} + A}{2}$ ’,	B1	3.5c	oe e.g. change to ‘(Let B =) $\frac{-\frac{N}{A} - A}{2}$ ’																				
			[1]																						
	(d)	The student’s claim is incorrect as if $A = 0$ then B will be undefined	B1	2.3	Mention that A cannot take the value of zero																				
			[1]																						

Question			Answer	Marks	AO	Guidance
3	(a)		C	B1	1.1	
				[1]		
3	(b)	(i)		M1	1.2	Correct working values at B
				A1	1.1a	Working values
				A1	1.1a	Labels
				A1	1.1	Order of labelling
						SC for starting at A or G only can score M1 A1 A0 A0 max. For example, starting at A, M1 for correct working values at D (31 27) and A1 for correct working values at all other nodes. Starting at any other node is M0
			Shortest path from A to C: ABDC Shortest path from C to G: CDEHG	B1	2.2a	Both correct but not for CDDBA
				[5]		
	(b)	(ii)	Weight is $41 + 49 = 90$	B1ft	3.1b	90 or follow through final values at A and F but no follow through if not starting at C in (b)(i)
				[1]		

Question	Answer	Marks	AO	Guidance
(c)	Prim's: AF, EF, EH, AB, BD, CD, GH	<p>M1</p> <p>A1</p>	<p>1.1</p> <p>1.1</p>	<p>First four arcs (or five nodes) chosen correctly in the correct order</p> <p>cao (must be stated as arcs in the correct order) – M1 only if starting at a different node (than A) – explicit rejection of arcs scores M1 only</p>
		B1	1.1	cao
		[3]		

Question	Answer	Marks	AO	Guidance																										
<p>4 (a)</p>	<table border="1"> <thead> <tr> <th data-bbox="383 252 573 290">Activity</th> <th data-bbox="573 252 1088 290">Immediate Predecessor(s)</th> </tr> </thead> <tbody> <tr><td data-bbox="383 290 573 328">A</td><td data-bbox="573 290 1088 328">-</td></tr> <tr><td data-bbox="383 328 573 367">B</td><td data-bbox="573 328 1088 367">-</td></tr> <tr><td data-bbox="383 367 573 405">C</td><td data-bbox="573 367 1088 405">-</td></tr> <tr><td data-bbox="383 405 573 443">D</td><td data-bbox="573 405 1088 443">A</td></tr> <tr><td data-bbox="383 443 573 481">E</td><td data-bbox="573 443 1088 481">A, B, G</td></tr> <tr><td data-bbox="383 481 573 520">F</td><td data-bbox="573 481 1088 520">A, B, G</td></tr> <tr><td data-bbox="383 520 573 558">G</td><td data-bbox="573 520 1088 558">C</td></tr> <tr><td data-bbox="383 558 573 596">H</td><td data-bbox="573 558 1088 596">C</td></tr> <tr><td data-bbox="383 596 573 635">I</td><td data-bbox="573 596 1088 635">D, E</td></tr> <tr><td data-bbox="383 635 573 673">J</td><td data-bbox="573 635 1088 673">D, E</td></tr> <tr><td data-bbox="383 673 573 711">K</td><td data-bbox="573 673 1088 711">D, E</td></tr> <tr><td data-bbox="383 711 573 750">L</td><td data-bbox="573 711 1088 750">F, H, I</td></tr> </tbody> </table>	Activity	Immediate Predecessor(s)	A	-	B	-	C	-	D	A	E	A, B, G	F	A, B, G	G	C	H	C	I	D, E	J	D, E	K	D, E	L	F, H, I	<p>B1</p> <p>B1</p>	<p>1.1</p> <p>1.1</p>	<p>Any 5 rows correct (not including rows A, B, C)</p> <p>cao</p>
Activity	Immediate Predecessor(s)																													
A	-																													
B	-																													
C	-																													
D	A																													
E	A, B, G																													
F	A, B, G																													
G	C																													
H	C																													
I	D, E																													
J	D, E																													
K	D, E																													
L	F, H, I																													
		<p>[2]</p>																												
<p>(b)</p>		<p>M1</p> <p>M1</p> <p>A1</p>	<p>1.1</p> <p>1.1</p> <p>1.1</p>	<p>Forward pass - numbers increasing from source to sink (allow one slip) – all complete (condone crossed out values not replaced especially at the event at the end of J)</p> <p>Backward pass – numbers decreasing from sink to source (allow one slip) – all complete – condone missing 0</p> <p>cao for backward and forward pass – bod if 9 + y seen and replaced with their value of y</p>																										
	<p>Minimum completion time = 18 (hours)</p>	<p>B1</p>	<p>1.1</p>	<p>cao</p>																										
	<p>Critical activities are A, F and L</p>	<p>B1</p>	<p>1.1</p>	<p>cao</p>																										
		<p>[5]</p>																												

Question	Answer	Marks	AO	Guidance
(c)	$18 - 9 - y = 4 \Rightarrow y = 5$	B1	3.1a	
		[1]		
(d)	$x = 3$	B1	2.2a	cao
		[1]		
(e)		M1 A1	1.1 1.1	Five workers in time interval 9 – 10 and correct number of workers from 14 – 18 cao
		[2]		

Question	Answer	Marks	AO	Guidance
(e)	There are two arcs, DT and ET, that flow into the sink T and so the flow through the pipes (according to the output) is $11 + 95 = 106$ (litres/min)	B1	2.4	Must mention that the 106 comes from DT (11) and ET (95)
		[1]		
(f)		M1 A1	1.1 1.1	<p>Consistent flow pattern (flow in = flow out at each node) – one number on each arc. SA = 45, SC = 35, BC = 0, BD = 31, DT = 11, ET = 95, FE = 70 and SB = 26 – condone lack of 0 on BC for M1 only</p> <p>Condone incorrect flow across one vertex for the M mark</p> <p>cao</p>
		[2]		
(g)	The (capacity of the) cut which partitions the vertices into the sets {S, A, C}, {B, D, E, F, T} (is $22 + 23 + 26 + 17 + 18 = 106$)	B1*	3.1b	Condone without S and/or T - allow listing of cut arcs (AD, AB, SB, BC, CF, CE)
	\therefore minimum cut ≤ 106 so by the maximum flow-minimum cut theorem the maximum flow is equal to the minimum cut and so therefore the maximum flow through the system is 106 (litres per minute)	B1dep*	2.1	Requires correct cut, 106 stated and mention of max flow-min cut theorem
		[2]		

Question		Answer	Marks	AO	Guidance																																																						
6	(a)	$y + z \leq 32 \Rightarrow y + z + s_1 = 32$ $y \geq 3z \Rightarrow -y + 3z \leq 0 \therefore -y + 3z + s_2 = 0$ $\frac{1}{10}(x + y + z) \leq z \Rightarrow x + y - 9z + s_3 = 0$ $0.55x + 0.5y + 0.4z \leq 50 \Rightarrow 11x + 10y + 8z + s_4 = 1000$	M1 A1 A1 A1	3.1b 1.1 1.1 1.1	Any one correct equation or two correct inequality constraints Any two correct equations Any three correct equations All four correct equations																																																						
		$P = x + y + z \Rightarrow P - x - y - z = 0$	B1	3.1a	Can be implied by correct corresponding row in tableau																																																						
		<table border="1"> <thead> <tr> <th>P</th> <th>x</th> <th>y</th> <th>z</th> <th>s_1</th> <th>s_2</th> <th>s_3</th> <th>s_4</th> <th>RHS</th> </tr> </thead> <tbody> <tr> <td>1</td> <td>-1</td> <td>-1</td> <td>-1</td> <td>0</td> <td>0</td> <td>0</td> <td>0</td> <td>0</td> </tr> <tr> <td>0</td> <td>0</td> <td>1</td> <td>1</td> <td>1</td> <td>0</td> <td>0</td> <td>0</td> <td>32</td> </tr> <tr> <td>0</td> <td>0</td> <td>-1</td> <td>3</td> <td>0</td> <td>1</td> <td>0</td> <td>0</td> <td>0</td> </tr> <tr> <td>0</td> <td>1</td> <td>1</td> <td>-9</td> <td>0</td> <td>0</td> <td>1</td> <td>0</td> <td>0</td> </tr> <tr> <td>0</td> <td>11</td> <td>10</td> <td>8</td> <td>0</td> <td>0</td> <td>0</td> <td>1</td> <td>1000</td> </tr> </tbody> </table>	P	x	y	z	s_1	s_2	s_3	s_4	RHS	1	-1	-1	-1	0	0	0	0	0	0	0	1	1	1	0	0	0	32	0	0	-1	3	0	1	0	0	0	0	1	1	-9	0	0	1	0	0	0	11	10	8	0	0	0	1	1000	M1 A1	3.3 1.1	Two constraint rows correct cao oe
P	x	y	z	s_1	s_2	s_3	s_4	RHS																																																			
1	-1	-1	-1	0	0	0	0	0																																																			
0	0	1	1	1	0	0	0	32																																																			
0	0	-1	3	0	1	0	0	0																																																			
0	1	1	-9	0	0	1	0	0																																																			
0	11	10	8	0	0	0	1	1000																																																			
			[7]																																																								

Question		Answer									Marks	AO	Guidance
(b)	(i)	<i>P</i>	<i>x</i>	<i>y</i>	<i>z</i>	<i>s</i> ₁	<i>s</i> ₂	<i>s</i> ₃	<i>s</i> ₄	RHS	M1	3.4	Pivot row and column correct (condone pivoting on the 104/3 for this mark)
		1	0	0	0	2.5	2.5	1	0	80			
		0	0	1	0	0.75	-0.25	0	0	24			
		0	0	0	1	0.25	0.25	0	0	8			
		0	1	0	0	1.5	2.5	1	0	48			
		0	0	0	0	-26	-27	-11	1	168			
										[3]			
(b)	(ii)	There are no negative values in the objective (top) row and so the solution obtained after the third iteration is optimal									B1	2.2a	Must mention objective <u>row</u> – B0 for ‘there are only positive values in the objective row’
										[1]			
(c)	(i)	Should order 48 litres of whole, 24 litres of semi-skimmed and 8 litres of skimmed milk									B1	3.2a	Must be in context (but litres not required)
										[1]			
(c)	(ii)	$50 - 0.55(48) - 0.5(24) - 0.4(8) = (\text{£})8.40$									B1	3.4	Condone 8.4
										[1]			
(d)		$y + z = 40 \Rightarrow y + z \leq 40$ and $y + z \geq 40$ $y + z + s_5 = 40, y + z - s_6 + a_1 = 40$									B1	3.1a	oe allow any alternative subscript (but the two <i>s</i> 's must have different subscripts) – must be using <i>s</i> and <i>a</i> (any other letter must be defined)
		$11x + 10y + 8z \geq 1000 \Rightarrow 11x + 10y + 8z - s_7 + a_2 = 1000$									B1	1.1	oe
		$Q = a_1 + a_2 \Rightarrow Q + 11x + 11y + 9z - s_6 - s_7 = 1040$									B1	3.5c	
		Where <i>Q</i> is the new objective to be minimised, <i>s</i> ₅ is a slack variable, <i>s</i> ₆ , <i>s</i> ₇ are surplus variables, and <i>a</i> ₁ , <i>a</i> ₂ are artificial variables (with no change to the constraints $x + y - 9z + s_3 = 0$, $-y + 3z + s_2 = 0$ or the objective $P - x - y - z = 0$)									B1	2.5	<i>s</i> _{<i>i</i>} , <i>a</i> _{<i>i</i>} must be consistent with earlier constraints – dependent on using $y + z = 40$ and their $11x + 10y + 8z \geq 1000$
										[4]			

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