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Examiners' report

FURTHER MATHEMATICS B (MEI)

H635

For first teaching in 2017

Y410/01 Summer 2023 series

Contents

Introduction

Our examiners' reports are produced to offer constructive feedback on candidates' performance in the examinations. They provide useful guidance for future candidates.

The reports will include a general commentary on candidates' performance, identify technical aspects examined in the questions and highlight good performance and where performance could be improved. A selection of candidate answers is also provided. The reports will also explain aspects which caused difficulty and why the difficulties arose, whether through a lack of knowledge, poor examination technique, or any other identifiable and explainable reason.

Where overall performance on a question/question part was considered good, with no particular areas to highlight, these questions have not been included in the report.

A full copy of the question paper and the mark scheme can be downloaded from OCR.

Paper Y410/01 series overview

This proved to be a relatively straightforward paper, with a mean mark of 43. The majority of candidates achieved over half marks, and a third gained 50 marks or more. There were very few unsuccessful scripts – virtually all candidates achieved 20 marks. There was no evidence of a lack of time to complete the paper, and many scripts were well presented.

Candidates who did well on this paper generally:	Candidates who did less well on this paper generally:
• showed good knowledge of matrices and their use in transformations and vector geometry • understood complex numbers and their use in the solution of polynomial equations • demonstrated accurate arithmetic and algebraic skills.	• showed limited understanding of matrices and their application to transformations and vector geometry • demonstrated some basic knowledge of complex number work but did not apply this to more demanding questions • made algebraic and arithmetic errors in their working.

Question 1 (a)

1 The transformation R of the plane is reflection in the line $x = 0$.

(a) Write down the matrix **M** associated with R. **[1]**

This was intended to be a straightforward introductory question. However, confusion between the line $x = 0$ and the x-axis meant that many candidates gave the matrix for reflection in $y = 0$. Another error which was seen occasionally was to give a 3x3 instead of a 2x2 reflection matrix.

Question 1 (b)

(b) Find $\mathbf{M}^2$. **[1]**

Most candidates gained this mark. We condoned the use of the matrix for reflection in $y = 0$ here, but not a 3x3 matrix.

Question 1 (c)

(c) Interpret the result of part **(b)** in terms of the transformation R. **[1]**

Again this was well answered, although occasionally the response did not relate the identity matrix to the effect of two reflections.

Question 2

2 **In this question you must show detailed reasoning.**

The equation $x^2 - kx + 2k = 0$, where k is a non-zero constant, has roots α and β.

Find $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$ in terms of k, simplifying your answer. **[4]**

There has not been a similar item to this in previous Y410 papers, so some candidates were not familiar with standard approach of considering $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$, and so did not use the sum and product of roots to produce the required result. Another common error was to take $\alpha\beta = 2$ instead of $2k$. Despite these problems, almost half of candidates achieved full marks.

Question 3

3 In this question you must show detailed reasoning.

The function $f(z)$ is given by $f(z) = 2z^3 - 7z^2 + 16z - 15$.

By first evaluating $f\left(\frac{3}{2}\right)$, find the roots of $f(z) = 0$. **[6]**

This question was extremely well done, with nearly all candidates getting 5 or 6 marks. Those who lost a mark usually did so because they did not show adequate method for solving the quadratic in *z*. Long division or its equivalent was the most common method for factoring the cubic, although some candidates used symmetric properties of the roots effectively.

Assessment for learning

Candidates should be made aware of the significance of '**In this question you must show detailed reasoning**' questions. For example, quadratics should be clearly solved by factorising, completion of the square or quadratic formula.

For the above question, simply showing the factors as $(z - 1 - 2i)\ (z - 1 + 2i)$ does not constitute detailed reasoning as candidates might be 'constructing' the factors from the roots $1 + 2i$ and $1 - 2i$ shown on their calculator. Those candidates that went to the calculator to find the roots need to first justify $z = 3/2$ by showing the substitution for the factor theorem and then use their $(2z - 3)$ to find the quadratic term $(z^2 - 2z + 5)$. There then needs to be a clear justification for why this subsequently gives the conjugate pair of roots.

Question 4

4 You are given that $\sum_{r=1}^{n}(ar+b) = n^2$ for all n, where a and b are constants.

By finding $\sum_{r=1}^{n}(ar+b)$ in terms of a, b and n, determine the values of a and b. **[6]**

This was a non-standard question of its type, and caused candidates some trouble. Most managed to split the sum into $a\ \Sigma r + \Sigma b$, and use the standard formula for Σr. However, $\Sigma b = b$ was quite a common error here. Thereafter, quite a few candidates struggled to determine the values of *a* and *b*. This was fundamentally because they treated the quadratic expressions for *n* as an equation rather than an identity, and thereby struggled to solve it for *a* and *b*. Nevertheless, over a third of candidates gained full marks.

Exemplar 1

4

$$\sum_{r=1}^{n}(ar+b) = n^2 \qquad \sum_{r=1}^{n} r = 1+2+3+\dots+n$$

$$= a\sum_{r=1}^{n} r + b\sum_{r=1}^{n} 1$$

$$\sum_{r=1}^{n}(ar+b) = n^2$$

$$a\sum_{r=1}^{n} r + b\sum_{r=1}^{n} 1 = n^2$$

$$a\left(\sum_{r=1}^{n} r\right) + b(n)$$

$$\sum_{r=1}^{n}(ar+b) = \tfrac{1}{2}an^2 + \tfrac{1}{2}an + bn$$

$$a\left(\tfrac{1}{2}n(n+1)\right) + bn$$

$$a\left(\tfrac{1}{2}n^2 + \tfrac{1}{2}n\right) + bn$$

$$\tfrac{1}{2}an^2 + \tfrac{1}{2}an + bn =$$

$$\tfrac{1}{2}an^2 + \tfrac{1}{2}an + bn = n^2$$

$$n^2\left(\tfrac{1}{2}a - 1\right) + n\left(\tfrac{1}{2}a\right) + n(b) = 0$$

for $n=1$ $\quad \tfrac{1}{2}a - 1 + \tfrac{1}{2}a + b = 0$

$$a + b - 1 = 0$$

$$bn = -\tfrac{1}{2}an^2 - \tfrac{1}{2}an$$

$$b = -\tfrac{1}{2}an - \tfrac{1}{2}a$$

$$= -\tfrac{1}{2}a(n+1)$$

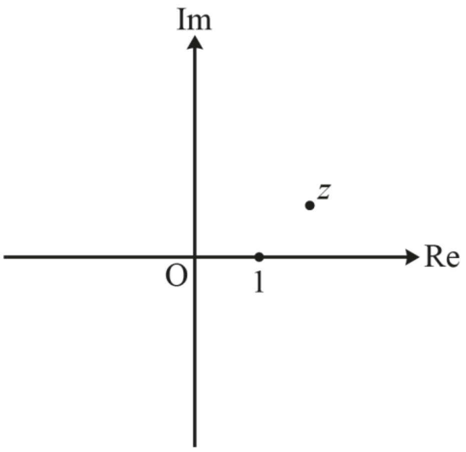
$$-\tfrac{1}{2}a(1+1) + a - 1 = 0$$

$$-a + a - 1 = 0$$

This exemplar illustrates the confusion between an equation and an identity. The candidate correctly finds the expression for the sum, but does not equate coefficients to find *a* and *b*. Although they substitute *n* = 1 (see alternative solution), this is not enough to gain an M1.

Question 5

5 The Argand diagram below shows the points representing 1 and z, where $|z| = 2$.



Mark the points representing the following complex numbers on the copy of the diagram in the Printed Answer Booklet, labelling them clearly.

- z^*
- $\dfrac{1}{z}$
- $1 + z$
- $\mathrm{i}z$ **[4]**

z^* and $1 + z$ were answered correctly by all. However, $1/z$ was often drawn with too large a modulus or argument, and iz often appeared to be the reflection of z in the Im axis, or had an incorrect modulus. The modal mark was 3 out of 4.

Question 6 (a)

6 The matrix **M** is $\begin{pmatrix} 2 & 1 \\ -1 & 0 \end{pmatrix}$.

(a) Calculate $\mathbf{M}^2$, $\mathbf{M}^3$ and $\mathbf{M}^4$. **[2]**

Almost all candidates got full marks here, although the purpose of this part is obviously to set up parts (b) and (c).

Question 6 (b)

(b) Hence make a conjecture about the matrix $\mathbf{M}^n$. **[1]**

The majority of candidates gave a correct formula for $\mathbf{M}^n$, although sometimes in an unsimplified form. Some attempted verbal descriptions, but we gave them the mark if subsequently seen as part of the solution to part (c).

Question 6 (c)

(c) Prove your conjecture. **[5]**

This is a very simple proof by induction and, hence, worth fewer marks than usual. Many candidates used a 'target' to show the result for $n = k + 1$. The final concluding statement was more successfully done than before. Overall, just under half the entry gained full marks.

Assessment for learning

A common mis-statement of the principle of induction is to say 'as the result is true for $n = 1$, k, **and** $k+1$ it is true for all n'. It is important to emphasise in teaching the importance of the **if** … **then** … in this final statement. So: '**As** it is true for $n = 1$, and **if** it is true for $n = k$, **then** it is true for $n = k + 1$, therefore it is true for all $n \geq 1$'.

Exemplar 2

6c

$$MM^k = \begin{pmatrix} 2 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} 1+k & k \\ -k & -k+1 \end{pmatrix}$$

$$= \begin{pmatrix} 2+2k-k & 2k-k+1 \\ -k-1 & -k \end{pmatrix}$$

$$= \begin{pmatrix} (k+1)+1 & k+1 \\ -(k+1) & -(k+1)+1 \end{pmatrix}$$

If true for n=1, as true for n=k and true for n= k+1 then true for all positive n

In this exemplar, the candidate, having previously checked $n = 1$, calculates $\mathbf{M}\,\mathbf{M}^k$ correctly, establishes this is $\mathbf{M}^n$ with $n = k+1$, but loses a mark for an inaccurate summary induction statement.

Question 7 (a)

7 In this question you must show detailed reasoning.

The complex number $\sqrt{3}+\mathrm{i}$ is denoted by z.

(a) By expanding $(\sqrt{3}+\mathrm{i})^5$, express z^5 in the form $a+b\mathrm{i}$ where a and b are real and exact. **[3]**

The majority of candidates did this correctly, either using the binomial expansion or by direct multiplication of brackets.

Question 7 (b) (i)

(b) (i) Express z in modulus-argument form. **[3]**

Again, this part was extremely well answered. Conversion to modulus-argument form was well understood by most candidates.

Question 7 (b) (ii)

(ii) Hence find z^5 in modulus-argument form. **[2]**

The majority of candidates used $z_1 z_2 = r_1 r_2 \cos(\theta_1 + \theta_2)$ to write this down by taking the 5th power of the modulus and multiplying the argument by 5. Alternatively, some candidates worked out the modulus and argument from their response to part (a).

Question 7 (b) (iii)

(iii) Use this result to verify your answers to part **(a)**. **[2]**

If candidates used the alternative approach to Question 7 (b) (ii), they were here required to verify the result using the 5th power of the modulus and multiplying the argument by 5. Half the candidates for the paper got full marks for the question, which suggests a good understanding of this complex number work, and also good use of the binomial theorem from AS Mathematics.

Question 8 (a)

8 The equations of three planes are

$$\begin{aligned} 2x + y + 3z &= 3, \\ 3x - y - 2z &= 2, \\ -4x + 3y + 7z &= k, \end{aligned}$$

where k is a constant.

(a) By considering a suitable determinant, show that the planes do **not** meet at a single point. **[2]**

Virtually all candidates knew about the matrix of coefficients and its zero determinant implying no unique solution to the equations.

Question 8 (b)

(b) Given that the planes form a sheaf, determine the value of k. **[4]**

There are a number of different approaches to this, as shown in the mark scheme. Of these, the neatest is to substitute any value, such as $x = 0$, into the three equations, solve for the other two variables, and then substitute these values into the third to find the value of k which secured consistency. However, the second method proved to be the most common, although this was sometimes marred by arithmetical errors.

Question 9

9 A transformation T of the plane is represented by the matrix $\mathbf{M} = \begin{pmatrix} k+1 & -1 \\ 1 & k \end{pmatrix}$, where k is a constant.

Show that, for all values of k, T has no invariant lines through the origin. **[6]**

Many candidates chose to ignore the 'through the origin' here and proceeded with using $y = mx + c$ instead of $y = mx$. This added to the difficulty, and often led to sign errors in their argument. It also begged the question of what to do with the resulting equation in the constants. We insisted on either seeing the discriminant of the quadratic in m or the complex roots to gain full marks.

Misconception

There was less evidence of confusion between 'invariant points' and 'invariant lines' this year, but it is still worth emphasising the difference in your teaching. Perhaps it is also worth pointing out the 'lines through the origin' stated in the question as this makes the working considerably easier.

Question 10

10 The plane P has normal vector $2\mathbf{i} + a\mathbf{j} - \mathbf{k}$, where a is a positive constant, and the point $(3, -1, 1)$ lies in P. The plane $x - z = 3$ makes an angle of 45° with P.

Find the cartesian equation of P. **[7]**

This question was more demanding, with about a third of candidates navigating the way through it. Crucial components of a correct response were knowledge of the relationship between plane equation and normal, angle between planes being the angle between the normal, and calculating the correct moduli. Some candidates seemed to calculate the angle between the wrong vectors, but still managed to pick up a few marks, for example for identifying the vector **i – k** as the normal to $x - z = 3$. Another error we saw was writing the plane equation as $a\mathbf{i} + b\mathbf{j} + c\mathbf{k} = d$, and some candidates misused the '**k**' component as a constant '*k*' in the scalar product.

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