GCE

Further Mathematics B (MEI)

Y421/01: Mechanics major

Advanced GCE

Mark Scheme for November 2020

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

Text Instructions

Annotations and abbreviations

Annotation in scoris	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
E	Explanation mark 1
SC	Special case
^	Omission sign
MR	Misread
BP	Blank page
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining a result or establishing a given result
dep*	Mark dependent on a previous mark, indicated by *. The * may be omitted if only previous M mark.
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This indicates that the instruction **In this question you must show detailed reasoning** appears in the question.

Subject-specific Marking Instructions for AS Level Mathematics B (MEI)

a Annotations must be used during your marking. For a response awarded zero (or full) marks a single appropriate annotation (cross, tick, M0 or ^) is sufficient, but not required.

For responses that are not awarded either 0 or full marks, you must make it clear how you have arrived at the mark you have awarded and all responses must have enough annotation for a reviewer to decide if the mark awarded is correct without having to mark it independently.

It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

Award NR (No Response)

- if there is nothing written at all in the answer space and no attempt elsewhere in the script
- OR if there is a comment which does not in any way relate to the question (e.g. 'can't do', 'don't know')
- OR if there is a mark (e.g. a dash, a question mark, a picture) which isn't an attempt at the question.

Note: Award 0 marks only for an attempt that earns no credit (including copying out the question).

If a candidate uses the answer space for one question to answer another, for example using the space for 8(b) to answer 8(a), then give benefit of doubt unless it is ambiguous for which part it is intended.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not always be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
If you are in any doubt whatsoever you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.
A method mark may usually be implied by a correct answer unless the question includes the DR statement, the command words "Determine" or "Show that", or some other indication that the method must be given explicitly.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep*' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case, please escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is numerically correct and expressed either in SI or in the units of the question. (e.g. lengths will be assumed to be in metres unless in a particular question all the lengths are in km, when this would be assumed to be the unspecified unit.)
We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so.

- When a value is **given** in the paper only accept an answer correct to at least as many significant figures as the given value.
- When a value is **not given** in the paper accept any answer that agrees with the correct value to **2 s.f.** unless a different level of accuracy has been asked for in the question, or the mark scheme specifies an acceptable range.
  NB for Specification A the rubric specifies 3 s.f. as standard, so this statement reads "3 s.f"

Follow through should be used so that only one mark in any question is lost for each distinct accuracy error.
Candidates using a value of 9.80, 9.81 or 10 for *g* should usually be penalised for any final accuracy marks which do not agree to the value found with 9.8 which is given in the rubric.

g Rules for replaced work and multiple attempts:

- If one attempt is clearly indicated as the one to mark, or only one is left uncrossed out, then mark that attempt and ignore the others.
- If more than one attempt is left not crossed out, then mark the last attempt unless it only repeats part of the first attempt or is substantially less complete.
- if a candidate crosses out all of their attempts, the assessor should attempt to mark the crossed out answer(s) as above and award marks appropriately.

h For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A or B mark in the question. Marks designated as cao may be awarded as long as there are no other errors. If a candidate corrects the misread in a later part, do not continue to follow through. E marks are lost unless, by chance, the given results are established by equivalent working. Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

i If a calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers provided that there is nothing in the wording of the question specifying that analytical methods are required such as the bold "In this question you must show detailed reasoning", or the command words "Show" and "Determine. Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

j If in any case the scheme operates with considerable unfairness consult your Team Leader.

Question			Answer	Marks	AOs	Guidance	
1			At lowest point, $\text{PE} = (\pm)0.5g(3+x)$	B1	1.1	Correct use of PE	x is the extension in the string
			$\text{EPE} = \frac{75x^2}{2(3)}$	B1	1.1	Correct use of $\frac{\lambda x^2}{2a}$	
			$\frac{75x^2}{6} - 0.5g(3+x) = 0$	M1	3.3	Use of conservation of energy (correct number of terms)	
			$75x^2 - 29.4x - 88.2 = 0$				
			$x = 1.29...$ (because $x \neq -0.90...$) as $x > 0$	A1	2.3	**BC** – as a minimum must state $x > 0$ if no explicit rejection of –0.906… seen	x = 1.298005445… or x = –0.906005…
			(Provided the string has not been stretched beyond its elastic limit the) length of string is 4.30 (m)	A1	2.2b	Awrt 4.30	
				[5]			
2			$[f] = \text{T}^{-1}$, $[C] = \text{MLT}^{-2}$ and $[\sigma] = \text{ML}^{-1}$	B2	1.2 1.2	**B1** for any one correct	
			$\text{T}^{-1} = \left(\text{MLT}^{-2}\right)^{\alpha} \text{L}^{\beta} \left(\text{ML}^{-1}\right)^{\gamma}$	M1	2.1	Using their $[f], [C]$ and $[\sigma]$ to obtain an equation in M, L and T	
			$-2\alpha = -1, \alpha + \beta - \gamma = 0, \alpha + \gamma = 0$	M1	1.1a	Setting up all three equations	
			$\alpha = \frac{1}{2}, \beta = -1, \gamma = -\frac{1}{2}$	A1	1.1		
				[5]			
3	(a)		$\mathbf{F} = a\mathbf{i} + b\mathbf{j}$				
			$a + 2 - 3 = 0$ and $2 + 1 + b = 0$	M1	1.1	Setting up equations for a and b	Allow sign errors only
			$\mathbf{F} = \mathbf{i} - 3\mathbf{j}$	A1	1.1		
			$\lvert\mathbf{F}\rvert = \sqrt{10}$	A1ft	1.1	oe e.g. 3.16 (ft their **F**)	3.16227766…
			Direction is 71.6° below the horizontal	A1	1.1	oe 18.4° to the downward vertical	In radians: 1.25 or 0.322
				[4]			

Question			Answer	Marks	AOs	Guidance	
3	(b)		e.g. taking moments about O: $3(2)+2(2)-2(1)-1(3)$	M1	1.1a	Taking moments about any point with correct number of terms	
			5	A1	1.1		
			anti-clockwise	A1	2.5	oe (could be seen on a diagram)	
				[3]			
4	(a)		$\mathbf{v}=20\mathbf{i}+(10-10t)\mathbf{j}$	M1*	1.1	Attempt at differentiation – at least one component correct	
			Setting $t=0$	M1dep*	1.1	Substituting $t=0$ into their $\mathbf{v}$	
			$\mathbf{v}=20\mathbf{i}+10\mathbf{j}$	A1	1.1		
				[3]			
4	(b)		$\begin{pmatrix}20\\10\end{pmatrix}\cdot\begin{pmatrix}20\\10-10T\end{pmatrix}=0$	M1*	3.4	Setting up scalar product with their initial $\mathbf{v}$ and $\mathbf{v}$ and equating to zero	
			$400+10(10-10T)=0$	M1dep*	1.1	Correct use of scalar product to form a linear equation in T	Allow in terms of t
			$T=5$	A1	1.1		
				[3]			
4	(c)		$t=0, \mathbf{r}=5\mathbf{i}+95\mathbf{j}$	B1	1.1		
			$t=5, \mathbf{r}=105\mathbf{i}+20\mathbf{j}$	B1ft	1.1	Follow through their value of T	
			Displacement vector $(105\mathbf{i}+20\mathbf{j})-(5\mathbf{i}+95\mathbf{j})=100\mathbf{i}-75\mathbf{j}$	M1	1.1	Difference in their $\mathbf{r}$ values	
			Distance is 125	A1	1.1		
				[4]			

5	(a)		Driving force $= \dfrac{60000}{v}$	B1	1.2		
			$\dfrac{60000}{v} - 1500 = 900\dfrac{\mathrm{d}v}{\mathrm{d}t}$	M1	3.3	Applying N,II with correct number of terms – allow a for $\mathrm{d}v/\mathrm{d}t$	
			$200 - 5v = 3v\dfrac{\mathrm{d}v}{\mathrm{d}t} \Rightarrow \dfrac{3v}{5}\dfrac{\mathrm{d}v}{\mathrm{d}t} = 40 - v$	A1	2.2a	**AG** – sufficient working must be shown as answer given	
				[3]			
5	(b)		$v = 0, t = 24\ln\left(\dfrac{40}{40}\right) - \dfrac{3}{5}(0) = 24\ln 1 = 0$	B1	1.1	Must explicitly verify initial conditions	
			$t = 24\ln\left(\dfrac{40}{40-v}\right) - \dfrac{3}{5}v \Rightarrow \dfrac{\mathrm{d}t}{\mathrm{d}v} = \ldots$	M1*	1.1	Attempt at differentiation – must see terms $\dfrac{1}{\left(\dfrac{40}{40-v}\right)}$ and $(40-v)^{-2}$	
			$\dfrac{\mathrm{d}t}{\mathrm{d}v} = 24\left(\dfrac{1}{\left(\dfrac{40}{40-v}\right)}\right)(-1)(-40)(40-v)^{-2} - \dfrac{3}{5}$	A1	1.1	Correct derivative (allow unsimplified) or $\dfrac{\mathrm{d}t}{\mathrm{d}v} = \dfrac{-24}{40-v}(-1) - \dfrac{3}{5}$ from $t = 24\ln 40 - 24\ln(40-v) - \frac{3}{5}v$	
			$\dfrac{\mathrm{d}t}{\mathrm{d}v} = \dfrac{24}{40-v} - \dfrac{3}{5} = \dfrac{3v}{5(40-v)} \Rightarrow \dfrac{\mathrm{d}v}{\mathrm{d}t} = \ldots$	M1dep*	1.1	Simplify to single fraction before taking reciprocal	
			$\dfrac{\mathrm{d}v}{\mathrm{d}t} = \dfrac{5(40-v)}{3v} \Rightarrow \dfrac{3v}{5}\dfrac{\mathrm{d}v}{\mathrm{d}t} = 40 - v$	A1	1.1	**AG** – sufficient working must be shown as answer given	
				[5]			

5	(b)		**ALT**				
			$\frac{3v}{5}\frac{\mathrm{d}v}{\mathrm{d}t}=40-v \Rightarrow \frac{5}{3}\int \mathrm{d}t=\int \frac{v}{40-v}\mathrm{d}v$	**B1**		Separate variables correctly	
			$\frac{5}{3}t=-\int 1-\frac{40}{40-v}\mathrm{d}v$	**M1***		Rewrite in an integratable form and attempt to integrate (must contain a log term of $40-v$)	
			$\frac{5}{3}t=-\left(v+40\ln\left(40-v\right)\right)(+c)$	**A1**			$+c$ not required for this mark
			$t=0, v=0 \Rightarrow c=40\ln 40$	**M1dep***		Use given initial conditions to find c	
			$\frac{5}{3}t=40\ln 40-40\ln\left(40-v\right)-v$ $\Rightarrow t=24\ln\left(\frac{40}{40-v}\right)-\frac{3}{5}v$	**A1**		**AG**	
				[5]			
6	(a)		$78.4=\frac{1}{2}(9.8)t_1^{\,2}\ (\Rightarrow t_1=4)$	**B1**	**3.3**	Correct equation for the time t_1 from release to first bounce	Or correct speed at the first impact for B1M1
			$v_1=gt_1$	**M1***	**3.4**	Use of $v=u+at$ with $u=0$ to find the speed v_1 at the first impact	oe suvat equation ($v_1=39.2$)
			$v_2=get_1$	**A1ft**	**1.1**	Correct expression for the speed v_2 after first impact in terms of their t_1	$v_2=39.2e$
			$0=\left(get_1\right)t_2-\frac{1}{2}gt_2^2$	**M1***	**3.4**	Use of $s=ut+0.5at^2$ with $s=0$ to find the time between first and second impact t_2	
			$t_2=8e$	**A1**	**1.1**		
			$4+8e+8e^2+\ldots=6$	**M1dep***	**2.1**	Setting up an infinite series (at least three terms on the lhs) equal to 6	
			$4+8e\left(1+e+\ldots\right)=6 \Rightarrow 4+8e\left(\frac{1}{1-e}\right)=6$	**M1**	**3.1a**	Correct use of the sum to infinity formula for a GP of the form $u_0+eu_1+e^2u_2+\ldots$	Dependent on all previous M marks
			$8e=2\left(1-e\right) \Rightarrow e=0.2$	**A1**	**2.2a**		
				[8]			

6	(b)		$23.52 = m\left(get_1 - \left(-gt_1\right)\right)$	M1	3.3	Use of impulse = change in momentum	Allow with no value of e substituted
			$m = \dfrac{23.52}{9.8(0.2)(4) + 9.8(4)} = 0.5$	A1	1.1		
				[2]			
7	(a)		Hooke's law: $T = \dfrac{3mge}{6a}$	B1	1.2		Where e is the extension of the string when P is in equilibrium
			$T = mg\sin 30$	M1	3.3	Resolving parallel to the plane with P in equilibrium	Allow sin/cos confusion
			$\dfrac{mge}{2a} = \dfrac{mg}{2} \Rightarrow e = a$	A1	2.2a		
			$mg\sin 30 - T = m\dfrac{\mathrm{d}^2x}{\mathrm{d}t^2}$	M1*	3.3	NII parallel to the plane (with correct number of terms)	
			$mg\sin 30 - \dfrac{3mg}{6a}(a + x) = m\dfrac{\mathrm{d}^2x}{\mathrm{d}t^2}$	M1dep*	3.4	Use of Hooke's law in NII with correct extension	
			$m\dfrac{\mathrm{d}^2x}{\mathrm{d}t^2} - \dfrac{mg}{2} + \dfrac{3mga}{6a} + \dfrac{3mgx}{6a} = 0 \Rightarrow \dfrac{\mathrm{d}^2x}{\mathrm{d}t^2} + \dfrac{gx}{2a} = 0$	A1	2.2a	**AG** – sufficient working must be shown as answer givn	
				[6]			

7	(b)		$x = A\cos\sqrt{\frac{g}{2a}}t + B\sin\sqrt{\frac{g}{2a}}t$	B1	1.2a	Or as the motion starts at the extreme point of the motion $x = A\cos\sqrt{\frac{g}{2a}}t$	A and B are arbitrary constants
			$t = 0, x = 2a \Rightarrow A = 2a$ $t = 0, \dot{x} = 0 \Rightarrow B = 0$	M1	3.4	Use initial conditions to find A and B (or just A)	
			$x = 2a\cos\sqrt{\frac{g}{2a}}t$	A1	1.1		
			Slack when $x = -a \Rightarrow \cos\left(t\sqrt{\frac{g}{2a}}\right) = -\frac{1}{2}$	M1	3.1b	Substituting $x = -a$ into their equation for x	
			$t\sqrt{\frac{g}{2a}} = \frac{2\pi}{3} \Rightarrow t = \frac{2\pi}{3}\sqrt{\frac{2a}{g}}$	A1	2.2a		
				[5]			
7	(c)		$v^2 = \frac{g}{2a}\left((2a)^2 - (-a)^2\right)$	M1	3.4	Use of $v^2 = \omega^2\left(A^2 - x^2\right)$ with their values or differentiation of their x	
			$v = \sqrt{\frac{3ga}{2}}$	A1	1.1		
				[2]			

8	**(a)**		$y=\frac{r}{h}x$	**B1**	**1.2**	oe e.g. a correct equation for a line that could be rotated about the y-axis	
			$V\bar{x}=\pi\int_0^h x\left(\frac{r}{h}x\right)^2\mathrm{d}x$	**M1**	**2.1**	Use of $V\bar{x}=\pi\int xy^2\mathrm{d}x$ with their y	Limits not required
			$\frac{1}{3}\pi r^2 h\bar{x}=\frac{\pi r^2}{h^2}\left[\frac{x^4}{4}\right]_0^h$	**M1**	**1.1**	Use of $V=\frac{1}{3}\pi r^2 h$ and attempt at integration	Limits not required
			$\frac{1}{3}\pi r^2 h\bar{x}=\frac{\pi r^2}{h^2}\left(\frac{h^4}{4}-0\right)\Rightarrow\bar{x}=...$	**M1**	**1.1**	Use of correct limits and attempt to solve for $\bar{x}$	
			$\bar{x}=\frac{3h}{4}$	**A1**	**1.1**	**AG** **SC B0M1M0M1A0** for those who use $y=x$	
				[5]			
8	**(b)**			**M1***	**2.1**	Table of values idea – correct number of terms (dimensionally consistent)	
			$\left(\frac{1}{3}\pi r^3+\pi r^2(2r)\right)x_G=\frac{1}{3}\pi r^3\left(\frac{3r}{4}\right)+2r\left(\pi r^2(2r)\right)$	**A1** **A1**	**1.1** **1.1**	Correct LHS Correct RHS	x_G is the centre of mass of the composite body from the vertex
			$x_G=\frac{51}{28}r$	**A1**	**1.1**	oe (e.g. $\frac{33}{28}r$ from the base)	
			$\cos 45=\frac{x}{\left(\frac{51r}{28}\right)}$ $\left(\Rightarrow x=\frac{51}{56}\sqrt{2}r\right)$	**M1dep***	**3.1b**	Use of correct angle and their x_G to find horizontal distance distance of centre of mass from vertex	Or $\tan\theta=\frac{\frac{51}{28}r-r}{r}$
			Slant height of cone is $r\sqrt{2}$	**B1**	**1.1**	Or comparison with $45°$ or $\tan\theta=1$ (dependent on first M mark)	
			$\frac{51}{56}r\sqrt{2}<r\sqrt{2}\Rightarrow$ does not topple	**A1**	**3.2a**		
				[7]			

8	(c)		Moment of frictional force about any point of contact with the horizontal floor is zero and so has no effect on the stability of the toy	B1	2.4	Or equivalent	
				[1]			
9	(a)			M1	3.1b	Taking moments about A for the rod – correct number of terms	
			$(8W)(a\cos\theta)=\ldots$	B1	1.1		
			$\ldots=(2a\cos\theta)(T\sin\theta)+(2a\sin\theta)(T\cos\theta)$	B1	1.1	oe e.g. $2aT\sin 2\theta$	
			$T=2W\operatorname{cosec}\theta$	A1	1.1	oe	
				[4]			
9	(b)			M1*	3.3	Resolving vertically and horizontally at the ring – correct number of terms. Allow this mark if only one direction stated correctly	R_C is the normal contact force between the ring and the rod
			$R_C=W+T\sin\theta$ $F_C=T\cos\theta$	A1	3.3		F_C is the frictional force between the ring and the rod
			$F_C\le\mu R_C\Rightarrow 2W\cot\theta\le\mu(3W)$	M1dep*	3.4	Use of $F\le\mu R$ with their F_C and R_C	Allow equals
			$\cot\theta\le\frac{3\mu}{2}\Rightarrow\tan\theta\ge\frac{2}{3\mu}$ least value of tan gives greatest distance of the ring from A	A1	3.1b	$\tan\theta=\frac{2}{3\mu}$	
			$2.4a=4a\cos\theta\Rightarrow\cos\theta=0.6$	B1	3.1a	oe e.g. stating the angle or $\sin\theta=0.8$	
			$\frac{2}{3\mu}=\frac{4}{3}\Rightarrow\mu=\frac{1}{2}$	A1	2.2a		
				[6]			

10	(a)		At highest point, PE $= mg(2a)$ and KE $= 0$ At angle θ, PE $= mga(1-\cos\theta)$, KE $= \frac{1}{2}mv^2$	B1	1.1	Correct mechanical energy at either A or θ - candidates may calculate initial speed of P as $2\sqrt{ga}$ for this mark	PE zero at A
			$2mga = mga(1-\cos\theta) + \frac{1}{2}mv^2$	M1*	3.3	Use of conservation of energy – correct number of terms	
			$v^2 = 2ga(1+\cos\theta)$	A1	1.1		
			$R - mg\cos\theta = \frac{mv^2}{a}$	M1*	3.3	NII radially – correct number of terms – condone a for acceleration	Allow r for radius
			$R = mg(2+3\cos\theta)$	M1dep*	3.4	Substitute their expression for v^2 to get expression for R in terms of m, g and θ	
			$mg(2+3\cos\theta) = \frac{7}{2}mg \Rightarrow \theta = ...$	M1	1.1	Equate R with $\frac{7}{2}mg$ and solve to find θ	**Dependent on all previous M marks**
			$\theta = \frac{\pi}{3}$	A1	1.1		Allow in degrees $\theta = 60^\circ$
				[7]			
10	(b)			M1*	3.4	Use of $v = r\omega$ with $r = a$ and their v	
			$a^2\omega^2 = 2ga(1+\cos\theta)$	A1	1.1	oe	
			$a^2\omega^2 = 4ga\cos^2\left(\frac{\theta}{2}\right)$	M1dep*	3.1a	Use of correct double-angle identity $\cos^2\theta = \frac{1}{2}(1+\cos 2\theta)$	
			$\omega^2 = \frac{4g}{a}\cos^2\left(\frac{\theta}{2}\right) \Rightarrow \omega = 2\sqrt{\frac{g}{a}}\cos\left(\frac{\theta}{2}\right)$	A1	2.2a	$k = 2$	
				[4]			

10	(c)		$\frac{d\omega}{dt} = -\frac{1}{2}\left(2\sqrt{\frac{g}{a}}\right)\sin\left(\frac{\theta}{2}\right)\omega$	M1*	2.1	Differentiate ω with respect to t	
			$\frac{d\omega}{dt} = -\sqrt{\frac{g}{a}}\sin\left(\frac{\pi}{6}\right)\left(2\sqrt{\frac{g}{a}}\cos\left(\frac{\pi}{6}\right)\right)$	M1dep*	3.4	Substitute their value of θ into their expression for angular acceleration	
			$\ddot{\theta} = -\frac{g\sqrt{3}}{2a}$	A1	2.2a		
				[3]			
11	(a)		$mu\cos\alpha = mv_1 + mv_2$	M1*	3.3	Use of conservation of linear momentum – correct number of terms	m is the mass of A and B, v_1 is the component of the velocity of A parallel to the line of centres after impact and v_2 is the equivalent component for B
			$v_1 - v_2 = -\frac{1}{3}u\cos\alpha$	M1*	3.3	Use of Newton's experimental law – correct number of terms and consistent with conservation of linear momentum	
			$v_1 = \frac{1}{3}u\cos\alpha$	A1	1.1		
			$\tan\beta = \frac{u\sin\alpha}{v_1}$	M1dep*	3.4	Use of tan ratio for β with their v_1	
			$\tan\beta = \frac{u\sin\alpha}{\frac{1}{3}u\cos\alpha} \Rightarrow \tan\beta = 3\tan\alpha$	A1	2.2a	**AG** – sufficient working must be shown as answer given	
				[5]			
11	(b)		The component of the velocity of A perpendicular to the line of centres does not change	B1	3.5b		
				[1]			

11	**(c)**		$\tan\gamma = \tan(\beta-\alpha) = \frac{3\tan\alpha - \tan\alpha}{1+(3\tan\alpha)(\tan\alpha)}$	**M1**	**3.1b**	Use of a correct compound-angle formula for $\tan(\beta \pm \alpha)$ and substitute given result from **(a)**	
			$\tan\gamma = \frac{2\tan\alpha}{1+3\tan^2\alpha}$	**A1**	**1.1**		
				[2]			
11	**(d)**			**M1***	**3.1b**	Attempt to differentiate using quotient rule	
			$\left(\sec^2\gamma\frac{\mathrm{d}\gamma}{\mathrm{d}\alpha}=\right)$ $\frac{(1+3\tan^2\alpha)(2\sec^2\alpha)-(2\tan\alpha)(6\tan\alpha\sec^2\alpha)}{(1+3\tan^2\alpha)}=0$	**A1**	**1.1**	Correct derivative equated to zero	
			$1+3\tan^2\alpha-6\tan^2\alpha=0 \Rightarrow \tan\alpha=\frac{1}{\sqrt{3}}$	**M1dep***	**1.1**	Find value of $\tan\alpha$ or $\tan^2\alpha$	$\alpha=\frac{\pi}{6}$
			$\tan\gamma=\frac{2\left(\frac{\sqrt{3}}{3}\right)}{1+3\left(\frac{\sqrt{3}}{3}\right)^2} \Rightarrow \gamma=\ldots$	**M1**	**1.1**	Substitute their value for α or $\tan\alpha$ into their expression for $\tan\gamma$ - dependent on both previous M marks	$\tan\gamma=\frac{\sqrt{3}}{3}$
			$\gamma=\frac{\pi}{6}$	**A1**	**1.1**		
				[5]			

12	(a)			$r^2 = r^2 + l^2 - 2rl\cos\alpha \Rightarrow l = 2r\cos\alpha$	B1	3.1b		l is the length of the string
				$\sin\alpha = \dfrac{x}{l} \Rightarrow x = 2r\cos\alpha\sin\alpha$	B1	3.1b	oe	x is the radius of the horizontal circle
					M1*	3.3	Resolving vertically for P – correct number of terms with components of R and T	Condone use of the same angle for this mark
				$R\sin\theta = T\cos\alpha + mg$	A1	1.1		T is the tension in the string, R is the normal contact force and θ is the angle between the horizontal and the normal contact force
					M1*	3.3	NII horizontally – correct number of terms – allow any form for radius	Condone use of the same angle
				$R\cos\theta + T\sin\alpha = m(2r\cos\alpha\sin\alpha)\omega^2$	A1	1.1	oe	
				$\cos\theta = \dfrac{2r\cos\alpha\sin\alpha}{r} = 2\cos\alpha\sin\alpha$ **or** $\sin\theta = \dfrac{r - 2r\cos^2\alpha}{r} = 1 - 2\cos^2\alpha$	B1	1.1	Either correct expression for $\cos\theta$ or $\sin\theta$ in terms of α or stating a correct relationship between θ and α e.g. $\theta = 2\alpha - \frac{1}{2}\pi$	
				$R(1 - 2\cos^2\alpha) = T\cos\alpha + mg$ $R(2\cos\alpha\sin\alpha) + T\sin\alpha = 2mr\omega^2\cos\alpha\sin\alpha$	M1dep*	3.4	Eliminate θ from both equations	Dependent on both previous M marks
				$T = 2mr\omega^2\cos\alpha - 2R\cos\alpha$ $\Rightarrow R(1 - 2\cos^2\alpha) = (2mr\omega^2\cos\alpha - 2R\cos\alpha)\cos\alpha + mg$ $R - 2R\cos^2\alpha = 2mr\omega^2\cos^2\alpha - 2R\cos^2\alpha + mg$ $R = mg + 2mr\omega^2\cos^2\alpha$	A1	2.2a	**AG**	
					[9]			

12	(b)		$T = 2m\cos\alpha\left(r\omega^2 - g - 2r\omega^2\cos^2\alpha\right)$	M1*	3.4	Use given expression for R to find expression for T in terms of r, m, g, α and ω	
			$T > 0 \Rightarrow r\omega^2 - g - 2r\omega^2\cos^2\alpha > 0$	M1dep*	3.4	Setting $T > 0$	
			$g < r\omega^2\left(1 - 2\cos^2\alpha\right)$	A1	2.2a	$k_1 = 1$ and $k_2 = -2$	
				[3]			

OCR (Oxford Cambridge and RSA Examinations)
The Triangle Building
Shaftesbury Road
Cambridge
CB2 8EA

OCR Customer Contact Centre

Education and Learning
Telephone: 01223 553998
Facsimile: 01223 552627
Email: general.qualifications@ocr.org.uk

www.ocr.org.uk