GCE

Further Mathematics B (MEI)

Y433/01: Modelling with algorithms

Advanced GCE

Mark Scheme for Autumn 2021

Oxford Cambridge and RSA Examinations

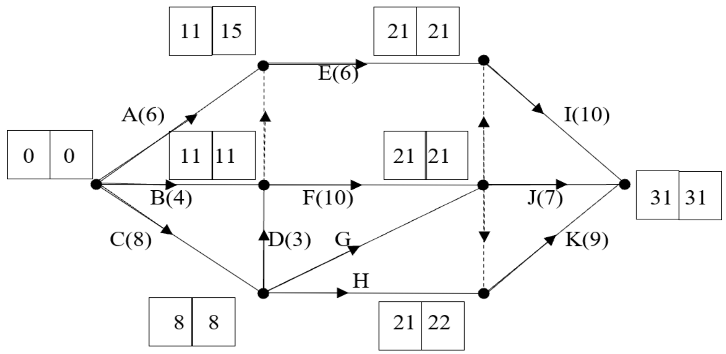
This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

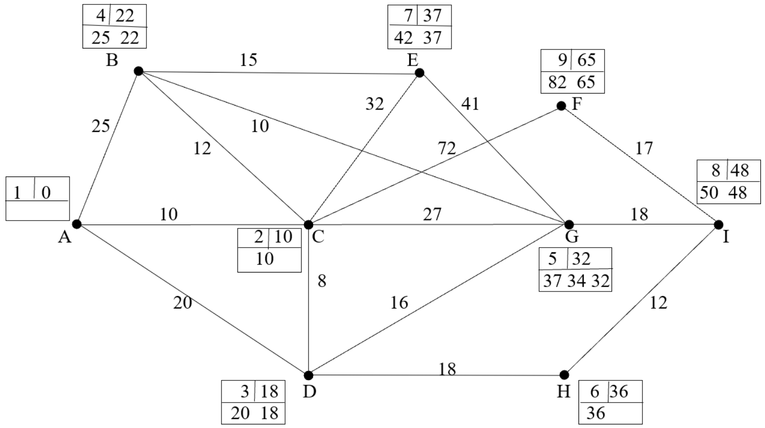
Mark schemes should be read in conjunction with the published question papers and the report on the examination.

Annotations and abbreviations

Annotation in scoris	**Meaning**
✓ and ×	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
E	Explanation mark 1
SC	Special case
^	Omission sign
MR	Misread
BP	Blank page
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining a result or establishing a given result
dep*	Mark dependent on a previous mark, indicated by *. The * may be omitted if only previous M mark.
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This indicates that the instruction **In this question you must show detailed reasoning** appears in the question.

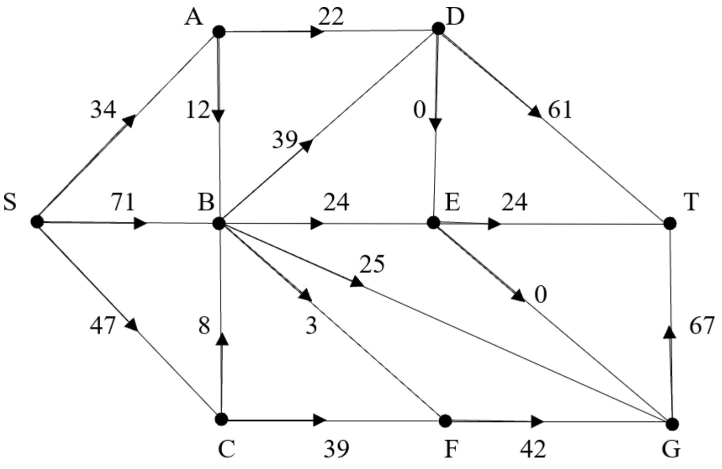
Question			Answer	Marks	AOs	Guidance
1	**(a)**		Bin 1: **5 16 12 10** Bin 2: **15 21** 5 3 Bin 3: 17 6 13 5 Bin 4: 24	**M1** **A1** **[2]**	**1.1** **1.1**	First six values placed correctly (the values in bold) – so the 10 in the correct bin cao
1	**(b)**		e.g. Bin 1: 24 21 Bin 2: 16 15 6 5 3 Bin 3: 17 13 10 5 Bin 4: 12 5	**M1** **A1** **[2]**	**1.1** **1.1**	At least two full bins (= 45) cao (three full bins with 17 units in the non-full bin)
2 **2**	**(a)** **(b)**			**M1** **A1** **A1** **[3]**	**3.1b** **3.1a** **1.1**	Activity on arc, single start vertex Precedences correct for A, B, C, D, G, H Directions may be implied Durations not necessary Single finish Precedences correct for E, F, I, J, K Directions may be implied Durations not necessary All three dummies correct and no extras All arcs directed
				M1 ft **M1 ft** **A1** **[3]**	**3.1b** **1.1** **1.1**	Network must have at least one burst and at least one merge, other than start and finish Forward pass, increasing, allow 1 blank Backward pass, decreasing, allow 1 blank Forward pass and backward pass correct

Question			Answer	Marks	AOs	Guidance
2	(c)		Minimum completion time is 31 (hours)	B1ft [1]	2.2a	Follow through their network
2	(d)		Interfering float for H is (22 – 8) – (21 – 8) = 1 (hour)	B1ft [1]	3.4	Follow through using their early and late event times at the beginning and end of H
2	(e)		Total float for E is 21 – 11 – 6 (= 4) and Total float for G is 21 – 8 – x (= 13 – x)	M1 *	1.1	Correct calculations of the total float for their E and G
			$13 - x \le 2\times4$ or $13 - x \le 8$	M1dep*	2.1	Using the given information to set up an inequality for x
			$5 \le x < 13$	A1 [3]	2.2a	cao
3	(a)	(i)	The sum of the vertex orders equals the number of arc endings Each arc has two ends so the sumber of arc endings is twice the number of arcs So the sum of the vertex orders is twice the number of arcs, which is even	B1	2.1	States or uses the result that the sum of the order of the vertices is equal to twice the number of arcs
			Alternative method Let a graph have e edges and n nodes (vertices), let d_i represent the order of the ith node so $\sum_{i=1}^{n} d_i = 2e$, which is even	B1		
				[1]		
3	(a)	(ii)	The sum of the orders of all the even vertices will be an even number so the sum of the order of the odd vertices must be an even number too Hence a graph must have an even number of vertices of odd order So no graph has an odd number of odd vertices	B1 [1]	2.2a	Correctly explains why a graph cannot have an odd number of vertices with odd order (or must have an even number of vertices with odd order) Must refer to even vertices as well as odd

<table>
<tr><th colspan="3">Question</th><th>Answer</th><th>Marks</th><th>AOs</th><th>Guidance</th></tr>
<tr><td>3</td><td>(b)</td><td></td><td>

Shortest path from A to F is ACDHIF</td><td>M1
A1
A1
A1
B1
[5]</td><td>1.2
1.1a
1.1a
1.1
1.1</td><td>Correct working values at D
Working values
Labels
Order of labelling
Allow one slip</td></tr>
<tr><td>3</td><td>(c)</td><td></td><td>STEP 1
<table>
<tr><th>Possible pairings of odd nodes</th><th>Corresponding shortest path</th><th>Weight of shortest path</th></tr>
<tr><td>AE</td><td>ACBE</td><td>37</td></tr>
<tr><td>AG</td><td>ACBG</td><td>32</td></tr>
<tr><td>AI</td><td>ACDHI</td><td>48</td></tr>
<tr><td>EG</td><td>EBG</td><td>25</td></tr>
<tr><td>EI</td><td>EBGI</td><td>43</td></tr>
<tr><td>GI</td><td>GI</td><td>18</td></tr>
<tr><td></td><td></td><td></td></tr>
<tr><td></td><td></td><td></td></tr>
</table></td><td>M1 *
M1 dep*
M1 dep*
A1
[4]</td><td>1.1
1.1
1.1
1.1</td><td>Any two rows correct
Any three rows correct
Any four rows correct
All correct</td></tr>
</table>

Question			Answer	Marks	AOs	Guidance
3	**(d)**		STEP 2 AE and GI STEP 3 353 + 37 + 18 = 408	**B1** **B1** **[2]**	**3.4** **1.1**	Both chosen, allow ACBE and GI cao
4	**(a)**	**(i)**	Cut $\alpha = 22 + 43 + 71 + 47 = 183$	**B1**	**1.1**	cao, need not show working
				[1]		
4	**(a)**	**(ii)**	Cut $\beta = 82 + 33 + 43 + 71 + 25 + 39 = 293$	**B1**	**1.1**	cao, need not show working
				[1]		
4	**(b)**		The maximum possible flow is (at most) 183 (litres per minute)	**B1 ft** **[1]**	**1.1**	min{their **(a)(i)**, their **(a)(ii)**}
4	**(c)**		The only arc leading into C is SC and the only arcs out of C are CB and CF and hence SC – CB – CF = 0	**B1** **[1]**	**2.4**	Flow in = flow out at C and stating that these are the only arcs that flow into C and out of C
4	**(d)**		Maximise DT + ET + GT SB + AB + CB – BD – BE – BG – BF = 0 BE + DE – EG – ET = 0 $DT \leq 82, ET \leq 24, GT \leq 67$	**B1** **B1** **B1** **[3]**	**3.1b** **3.3** **3.3**	Maximise and DT + ET + GT Flow in = flow out at B and at E represented using these equations Capacities for arcs into T represented using these inequalities

Question			Answer	Marks	AOs	Guidance
4	(e)			M1 A1 [2]	2.1 2.2a	Flow = 152. Consistent flow pattern (flow in = flow out at each node) – flow through every arc apart from DE and EG Condone incorrect or missing flow through one arc for the M mark A correct flow (flow ≤ capacity for each arc)
4	(f)		The capacity of the cut which partitions the vertices into the sets {S, A, B, C, E, F, G}, {D, T} is 22 + 39 + 24 + 67 = 152 [∴ minimum cut is ≤ 152] By the maximum flow-minimum cut theorem the maximum flow is equal to the minimum cut and so therefore the maximum flow through the system is 152 litres per minute	M1 A1 [2]	3.1b 2.1	{S, A, B, C, E, F, G}, {D, T} described in any way (but not implied) Max flow = min cut (o.e)
4	(g)		From the source there is only one non-saturated arc SA and into the sink there is only one non-saturated arc DT. Therefore the flow can be increased by the least of 82 – 61 = 21 and 62 – 34 = 28 giving a maximum flow of 152 + 21 = 173 (litres per minute) The corresponding value of x is 21 + 22 = 43	B1 B1 [2]	3.4 2.2a	173 43

Question			Answer	Marks	AOs	Guidance
5	(a)		$x+y+z=50 \Rightarrow x+y+z \le 50$ and $x+y+z \ge 50$	M1	3.1a	Dealing with equality constraint as two inequalities or implied from two correct equations (with slack, surplus and artificial variables)
			$x+y+z+s_1=50$ and $x+y+z-s_2+a_1=50$	A1	1.1	Or **SC B1** for one correct equation (if previous mark not earned)
			$x \le 25 \Rightarrow x+s_3=25$ $-y+3z \le 0 \Rightarrow -y+3z+s_4=0$	M1	1.1	Adding a slack variable appropriately to any of these three
			$x+4y+12z \le 210 \Rightarrow x+4y+12z+s_5=210$	A1	3.1b	All three correct in this form Allow $x-y-z \le 0$ o.e. for $x \le 25$ Or equivalent with surplus and artificial variables in one of these equations
			$P=2x+5y+20z \Rightarrow P-2x-5y-20z=0$	B1	3.1a	cao
			$Q=a_1$ so $Q+x+y+z-s_2=50$	M1	2.1	Attempt to substitute expression for a_1 (artificial variable for equality constraint)
				A1	2.2a	cao

Q	P	x	y	z	s_1	s_2	s_3	s_4	s_5	a_1	RHS
1	0	1	1	1	0	–1	0	0	0	0	50
0	1	–2	–5	–20	0	0	0	0	0	0	0
0	0	1	1	1	1	0	0	0	0	0	50
0	0	1	1	1	0	–1	0	0	0	1	50
0	0	1	0	0	0	0	1	0	0	0	25
0	0	0	–1	3	0	0	0	1	0	0	0
0	0	1	4	12	0	0	0	0	1	0	210

Question			Answer	Marks	AOs	Guidance
				M1	3.3	Any three rows correct
				A1	1.1	cao (rows in any order, with slack variables used appropriately)
				[9]		

Question			Answer	Marks	AOs	Guidance
5	(b)		$x \le 25$, $-y+3(50-x-y) \le 0$ and $x+4y+12(50-x-y) \le 210$	M1 *	3.1a	Substitute $x+y+z=50$ to form expressions in x and y only
				M1 dep*	1.1	Any two of these correct
			$x \le 25, 3x+4y \ge 150$ and $11x+8y \ge 390$	A1	1.1	All correct, need not be simplified
				M1	1.1	Two of their lines drawn correctly (may need to check constraints in (a) as well)
				A1	1.1	All three lines correct with correct feasible region, from shading or labelled
				[5]		

Question			Answer	Marks	AOs	Guidance
5	**(c)**		$P = 2x + 5y + 20(50 - x - y) \Rightarrow P = (1000) - 18x - 15y$ So maximising the negative expression $-3(6x + 5y)$ is equivalent to minimising the equivalent positive expression $3(6x + 5y)$ and the optimal values of x and y can be found by just considering $6x + 5y$	**M1** **A1** **[2]**	**3.4** **2.4**	Substitute $x + y + z = 50$ into P and simplify
5	**(d)**	**(i)**	Leo should answer 18 algebra questions, 24 trigonometry questions and 8 calculus questions	**B1** **[1]**	**3.2a**	In context
5	**(d)**	**(ii)**	Leo will score 316 points	**B1** **[1]**	**1.1**	
5	**(e)**		There is no guarantee that Leo will get the answers to the questions correct	**B1** **[1]**	**3.5b**	oe correct reason

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