OCR
Oxford Cambridge and RSA

Monday 4 October 2021 – Afternoon

AS Level Further Mathematics B (MEI)

Y410/01 Core Pure

Time allowed: 1 hour 15 minutes

9048235259

You must have:
- the Printed Answer Booklet
- the Formulae Booklet for Further Mathematics B (MEI)
- a scientific or graphical calculator

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give your final answers to a degree of accuracy that is appropriate to the context.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION
- The total mark for this paper is **60**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **4** pages.

ADVICE
- Read each question carefully before you start your answer.

Answer **all** the questions.

1 Using standard summation formulae, find $\sum_{r=1}^{n}(r^2-3r)$, giving your answer in fully factorised form. **[3]**

2 The equation $3x^2-4x+2=0$ has roots α and β.

Find an equation with integer coefficients whose roots are $3-2\alpha$ and $3-2\beta$. **[3]**

3 Three planes have the following equations.

$$\begin{aligned} 2x-3y+z&=-3,\\ x-4y+2z&=1,\\ -3x-2y+3z&=14. \end{aligned}$$

(a) **(i)** Write the system of equations in matrix form. **[1]**

(ii) Hence find the point of intersection of the planes. **[2]**

(b) **In this question you must show detailed reasoning.**

Find the acute angle between the planes $2x-3y+z=-3$ and $x-4y+2z=1$. **[4]**

4 Anika thinks that, for two square matrices $\mathbf{A}$ and $\mathbf{B}$, the inverse of $\mathbf{AB}$ is $\mathbf{A}^{-1}\mathbf{B}^{-1}$. Her attempted proof of this is as follows.

$$\begin{aligned} (\mathbf{AB})(\mathbf{A}^{-1}\mathbf{B}^{-1}) &= \mathbf{A}(\mathbf{BA}^{-1})\mathbf{B}^{-1}\\ &= \mathbf{A}(\mathbf{A}^{-1}\mathbf{B})\mathbf{B}^{-1}\\ &= (\mathbf{AA}^{-1})(\mathbf{BB}^{-1})\\ &= \mathbf{I}\times\mathbf{I}\\ &= \mathbf{I} \end{aligned}$$

Hence $(\mathbf{AB})^{-1}=\mathbf{A}^{-1}\mathbf{B}^{-1}$

(a) Explain the error in Anika's working. **[2]**

(b) State the correct inverse of the matrix $\mathbf{AB}$ and amend Anika's working to prove this. **[3]**

5 Prove by induction that $\sum_{r=1}^{n} r \times 2^{r-1} = 1 + (n-1)2^n$ for all positive integers n. **[5]**

6 A transformation T of the plane has associated matrix $\mathbf{M} = \begin{pmatrix} 1 & \lambda+1 \\ \lambda-1 & -1 \end{pmatrix}$, where λ is a non-zero constant.

(a) **(i)** Show that T reverses orientation. **[3]**

(ii) State, in terms of λ, the area scale factor of T. **[1]**

(b) **(i)** Show that $\mathbf{M}^2 - \lambda^2\mathbf{I} = \mathbf{0}$. **[2]**

(ii) Hence specify the transformation equivalent to two applications of T. **[1]**

(c) In the case where $\lambda = 1$, T is equivalent to a transformation S followed by a reflection in the x-axis.

(i) Determine the matrix associated with S. **[3]**

(ii) Hence describe the transformation S. **[2]**

7 **(a)** **(i)** Find the modulus and argument of z_1, where $z_1 = 1 + \mathrm{i}$. **[2]**

(ii) Given that $|z_2| = 2$ and $\arg(z_2) = \frac{1}{6}\pi$, express z_2 in $a + b\mathrm{i}$ form, where a and b are exact real numbers. **[2]**

(b) Using these results, find the exact value of $\sin\frac{5}{12}\pi$, giving the answer in the form $\frac{\sqrt{m}+\sqrt{n}}{p}$, where m, n and p are integers. **[5]**

8 **In this question you must show detailed reasoning.**

The equation $x^3 + kx^2 + 15x - 25 = 0$ has roots α, β and $\frac{\alpha}{\beta}$. Given that $\alpha > 0$, find, in any order,

- the roots of the equation,
- the value of k. **[7]**

9 **(a)** On a single Argand diagram, sketch the loci defined by

- $\arg(z-2) = \frac{3}{4}\pi$,
- $|z| = |z+2-\mathrm{i}|$. **[4]**

(b) **In this question you must show detailed reasoning.**

The point of intersection of the two loci in part **(a)** represents the complex number w.

Find w, giving your answer in exact form. **[5]**

END OF QUESTION PAPER

