Oxford Cambridge and RSA

Practice Paper – Set 1 (March 2018)

A Level Further Mathematics B (MEI)

Y420 Core Pure

MARK SCHEME

Duration: 2 hours 40 minutes

MAXIMUM MARK 144

Text Instructions

1. Annotations and abbreviations

Annotation in scoris	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining a result or establishing a given result
dep*	Mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This indicates that the instruction **In this question you must show detailed reasoning** appears in the question.

2. Subject-specific Marking Instructions for A Level Further Mathematics B (MEI)

a Annotations should be used whenever appropriate during your marking. The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded. For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
If you are in any doubt whatsoever you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep*' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case, please escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is numerically correct and expressed either in SI or in the units of the question. (e.g. lengths will be assumed to be in metres unless in a particular question all the lengths are in km, when this would be assumed to be the unspecified unit.) We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so. When a value is given in the paper only accept an answer correct to at least as many significant figures as the given value. This rule should be applied to each case. When a value is not given in the paper accept any answer that agrees with the correct value to 2 s.f. Follow through should be used so that only one mark is lost for each distinct accuracy error, except for errors due to premature approximation which should be penalised only once in the examination. There is no penalty for using a wrong value for *g*. E marks will be lost except when results agree to the accuracy required in the question.

g Rules for replaced work: if a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests; if there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others. NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question. Marks designated as cao may be awarded as long as there are no other errors. E marks are lost unless, by chance, the given results are established by equivalent working. 'Fresh starts' will not affect an earlier decision about a misread. Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

i If a calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers (provided, of course, that there is nothing in the wording of the question specifying that analytical methods are required). Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

j If in any case the scheme operates with considerable unfairness consult your Team Leader.

Question			Answer	Marks	AOs	Guidance	
1			$\sum_{r=1}^{n} r(r+2) = \sum_{r=1}^{n}(r^2+2r)$	M1	1.1a	Splitting into separate terms	
			$= \frac{1}{6}n(n+1)(2n+1)+n(n+1)$	M1	1.1	Use of both standard formulae	
				A1	1.1	Correct expression (unsimplified)	
			$= \frac{1}{6}n(n+1)(2n+7)$	M1	1.1	Taking out common factors	
				A1	1.1	cao	
				[5]			

2	(i)		shear	M1	1.2		
			x-axis fixed, $(0, 1) \to (k, 1)$	A1	2.5		
				[2]			
2	(ii)		$\begin{pmatrix}1 & k\\0 & 1\end{pmatrix}\begin{pmatrix}2\\3\end{pmatrix}=\begin{pmatrix}2+3k\\3\end{pmatrix}$	B1	1.1		
			so $2 + 3k = 0$	M1	2.2a		
			$k = -\frac{2}{3}$	A1	1.1		
				[3]			

Question			Answer	Marks	AOs	Guidance	
3			**DR**				
			$2^3 + k \times 2^2 + 7 \times 2 - 6 = 0$	**M1**	**3.1a**	Substitution to find k	
			$k = -4$	**A1**	**1.1**		
			$(z-2)(z^2 - 2z + 3) = 0$	**M1**	**1.1a**	Use of factor theorem	
				A1	**1.1**	Correct factorisation	
			$z = \dfrac{2 \pm \sqrt{2^2 - 4 \times 3}}{2}$	**M1**	**1.1**	Solution of quadratic	
			$= 1 \pm \sqrt{2}\mathrm{i}$	**A1**	**1.1**		
				[6]			

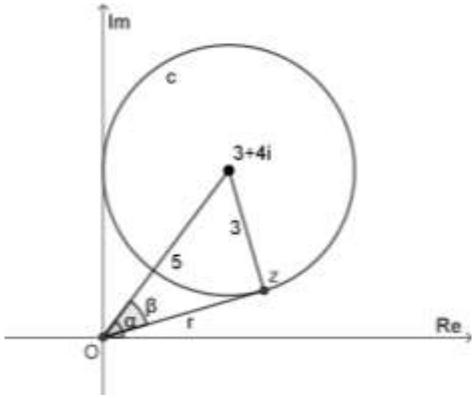
Question			Answer	Marks	AOs	Guidance	
4			**DR**				
			$V = \int_0^1 \pi \dfrac{1}{3 + x^2}\, \mathrm{d}x$	**M1**	**1.1a**		
			$= \pi \left[\dfrac{1}{\sqrt{3}} \arctan \dfrac{x}{\sqrt{3}} \right]_0^1$	**B1**	**1.1**	For indefinite integral $\dfrac{1}{\sqrt{3}} \arctan \dfrac{x}{\sqrt{3}}$	
			$= \pi \times \dfrac{1}{\sqrt{3}} \times \dfrac{\pi}{6}$	**M1**	**1.1a**	Use of $\arctan \dfrac{1}{\sqrt{3}} = \dfrac{\pi}{6}$	
			$= \dfrac{\sqrt{3}\pi^2}{18}$	**A1**	**1.1**	Or equivalent exact form	
				[4]			

Question			Answer	Marks	AOs	Guidance	
5			**DR**				
			$\int_0^1 \sinh 2x\,\mathrm{d}x = \left[\tfrac{1}{2}\cosh 2x\right]_0^1$	**B1**	**1.1a**	For indefinite integral $\tfrac{1}{2}\cosh 2x$	
			$= \tfrac{1}{2}\left(\dfrac{\mathrm{e}^2+\mathrm{e}^{-2}}{2} - 1\right)$	**M1**	**1.1**	Substitution for cosh in terms of e	
			$= \tfrac{1}{4}\left(\mathrm{e}^2 - 2 + \mathrm{e}^{-2}\right) = \tfrac{1}{4}\left(\mathrm{e} - \dfrac{1}{\mathrm{e}}\right)^2$	**E1**	**2.1**	**AG** so sufficient working is required	
				[3]			

Question			Answer	Marks	AOs	Guidance	
6			$2+2\lambda=-6-3\mu,\quad -3+3\lambda=-4+\mu,\quad 2\lambda=6+4\mu$	**M1**	**3.1a**	Forming 3 equations in λ and μ	
			eg $\lambda=3+2\mu$, so $-3+9+6\mu=-4+\mu$	**M1**	**1.1**	Solving any pair of the equations	
			$\mu=-2$	**A1**	**1.1**		
			$\lambda=-1$	**A1**	**1.1**		
			eg $2+2\times(-1)=0=-6-3\times(-2)$	**M1**	**2.1**	Checking values in the third equation	
			lines meet at (0, –6, –2)	**B1**	**2.2a**	Correct point of intersection stated	
				[6]			

Question			Answer	Marks	AOs	Guidance	
7			**DR**				
			$\int_0^\infty \dfrac{1}{(1+2x)^2}\,\mathrm{d}x = \left[-\tfrac{1}{2}(1+2x)^{-1}\right]_0^\infty$	**M1**	**1.1**	For $k(1+2x)^{-1}$ or ku^{-1}	
				A1	**1.1**	Correct value $-\tfrac{1}{2}$ for k in indefinite int	
			$\lim_{x\to\infty}(1+2x)^{-1}=0$	**A1**	**1.1**		
			so integral is $0-\left(-\tfrac{1}{2}\right)=\tfrac{1}{2}$	**A1**	**2.4**	Correct answer $\tfrac{1}{2}$	
				[4]			

Question			Answer	Marks	AOs	Guidance	
8			$\dfrac{1}{r^2+r}=\dfrac{1}{r(r+1)}$	**M1**	**3.1a**	Factorise and attempt partial fractions	
			$=\dfrac{1}{r}-\dfrac{1}{r+1}$	**A1**	**3.1a**		
			$\displaystyle\sum_{r=10}^{100}\frac{1}{r^r+r}=\sum_{r=10}^{100}\left(\frac{1}{r}-\frac{1}{r+1}\right)$	**M1**	**1.1**		
			$=\dfrac{1}{10}-\dfrac{1}{11}+\dfrac{1}{11}-\dfrac{1}{12}+\text{K}+\dfrac{1}{100}-\dfrac{1}{101}$	**M1**	**2.5**	With cancelling terms indicated	
			$=\dfrac{1}{10}-\dfrac{1}{101}=\dfrac{91}{1010}$	**A1**	**2.1**		
				[5]			

Question			Answer	Marks	AOs	Guidance	
9	(i)		Locus is a circle of radius 3 with centre at 3 + 4i	**M1** **A1** **[2]**	**1.1a** **1.1**	For sketch of a circle (located anywhere) In 1st quadrant, touching Im axis	
9	(ii)						
			z is point of contact of tangent from origin	**M1**	**3.1a**	For locating z correctly, e.g. on diagram	
			$r = \sqrt{25-9} = 4$	**B1**	**1.1**	Correct modulus of z	
			$\alpha = \arctan\left(\frac{4}{3}\right)$	**B1**	**3.1a**		
			$\beta = \arcsin\left(\frac{3}{5}\right)$	**B1**	**2.1**		
			$\alpha - \beta = 0.2838$	**B1**	**2.1**		0.2837941…
			so $z = 4(\cos 0.2838 + \mathrm{i}\sin 0.2838)$	**M1**	**1.1**	Correct use of modulus-argument form	
			$= 3.84 + 1.12\mathrm{i}$	**A1** **[7]**	**3.2a**		

Question			Answer	Marks	AOs	Guidance	
10			eg $\overrightarrow{BA} = 3\mathbf{i} + 6\mathbf{j}$	B1	3.1a	Any one vector in the plane	
			$\overrightarrow{AC} = -\mathbf{i} + 6\mathbf{j} + \mathbf{k}$	B1	1.1	Another vector in the plane	
			$\mathbf{n} = (\mathbf{i} + 2\mathbf{j}) \times (-\mathbf{i} + 6\mathbf{j} + \mathbf{k})$	M1	3.1a	Use of vector product to find a normal	
			normal vector is $2\mathbf{i} - \mathbf{j} + 8\mathbf{k}$	A1	2.1	Correct normal (can be any multiple)	
			equation of ABC is $\begin{pmatrix} x \\ y \\ z \end{pmatrix} . \begin{pmatrix} 2 \\ -1 \\ 8 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} . \begin{pmatrix} 2 \\ -1 \\ 8 \end{pmatrix}$	M1	2.5	Use of **r.n** = **a.n**, oe	
			$2x - y + 8z = 18$	A1	1.1	Correct equation of plane ABC	
			$8 - \lambda + 8 = 18$	M1	3.2a	Use of coordinates of D to find λ	
			$\lambda = -2$	A1	2.2a		
				[8]			

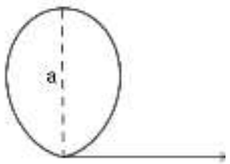
Question			Answer	Marks	AOs	Guidance	
11	**(i)**		$\det \mathbf{M} = c^2 + 1$ so c = i or –i	**B1** **B1** **[2]**	**1.1a** **1.1**		
11	**(ii)**		When $n = 1$, $\mathbf{M}^1 = 2^0\mathbf{M} = \mathbf{M}$, as required Assume true for $n = k$, so $\mathbf{M}^k = 2^{k-1}\mathbf{M}$ If c = i, then $\mathbf{M}^{k+1} = 2^{k-1}\begin{pmatrix} 1 & \mathrm{i} \\ -\mathrm{i} & 1 \end{pmatrix}\begin{pmatrix} 1 & \mathrm{i} \\ -\mathrm{i} & 1 \end{pmatrix}$ $= 2^{k-1}\begin{pmatrix} 2 & 2\mathrm{i} \\ -2\mathrm{i} & 2 \end{pmatrix} = 2^k\begin{pmatrix} 1 & \mathrm{i} \\ -\mathrm{i} & 1 \end{pmatrix} = 2^k\mathbf{M}$ If c = –i, then $\mathbf{M}^{k+1} = 2^{k-1}\begin{pmatrix} 1 & -\mathrm{i} \\ \mathrm{i} & 1 \end{pmatrix}\begin{pmatrix} 1 & -\mathrm{i} \\ \mathrm{i} & 1 \end{pmatrix}$ $= 2^{k-1}\begin{pmatrix} 2 & -2\mathrm{i} \\ 2\mathrm{i} & 2 \end{pmatrix} = 2^k\begin{pmatrix} 1 & -\mathrm{i} \\ \mathrm{i} & 1 \end{pmatrix} = 2^k\mathbf{M}$ so if true for $n = k$ then also true for $n = k + 1$ therefore true for all positive integers n	**B1** **M1** **M1** **A1*** **B1** **B1dep*** **B1dep*** **[7]**	**2.1** **2.1** **2.1** **2.2a** **2.1** **2.2a** **2.5**	Check of initial case Statement of induction hypothesis Or using c = –i Or using c = i	May use matrix with c and $c^2 = -1$ to cover both cases for M1A1B1

Question			Answer	Marks	AOs	Guidance	
12	(i)		$f(x) = \arcsin\left(\frac{1}{2}x\right)$				
			$f'(x) = \frac{1}{\sqrt{1-\frac{1}{4}x^2}} \times \frac{1}{2} = \frac{1}{\sqrt{4-x^2}}$	B1	2.1		
			$f''(x) = -\frac{1}{2}(4-x^2)^{-\frac{3}{2}} \times (-2x) = x(4-x^2)^{-\frac{3}{2}}$	M1	2.1		
				A1	1.1	Correct 2nd derivative	
			$f'''(x) = (4-x^2)^{-\frac{3}{2}} + 3x^2(4-x^2)^{-\frac{5}{2}}$	M1	2.1		
				A1	1.1	Correct 3rd derivative	
			so $f(0) = 0,\ f'(0) = \frac{1}{2},\ f''(0)=0,\ f'''(0) = \frac{1}{8}$				
			giving $\arcsin\left(\frac{1}{2}x\right) = \frac{1}{2}x + \frac{1}{48}x^3 + \text{K}$	A1	2.2a		
				[6]			
12	(ii)		let $x = 1$ so that $\arcsin\left(\frac{1}{2}\right) = \frac{1}{6}\pi$	B1	3.1a		
			so $\frac{1}{6}\pi \approx \frac{1}{2} + \frac{1}{48} = \frac{25}{48}$	M1	1.1		
			giving $\pi \approx \frac{25}{8}$	A1	1.1		
				[3]			

Question			Answer	Marks	AOs	Guidance	
13	(i)		$1+2\sinh^2 x = 1+2\left(\frac{e^x - e^{-x}}{2}\right)^2$	M1	1.2		
			$= 1+\frac{e^{2x} - 2 + e^{-2x}}{2}$	M1	1.1		
			$= \frac{e^{2x} + e^{-2x}}{2} = \cosh 2x$	E1	2.1	AG	
				[3]			
13	(ii)		**DR**				
			$2(1+2\sinh^2 x) - \sinh x = 5$	M1	1.1	Use of the identity	
			$4\sinh^2 x - \sinh x - 3 = 0$	A1	1.1		
			$\sinh x = 1$ or $-\frac{3}{4}$	A1	1.1	**BC**	
			so $x = \ln\left(1+\sqrt{2}\right)$	M1	1.2	Use of $\operatorname{arsinh} x = \ln\left(x+\sqrt{1+x^2}\right)$	For either case
				A1	1.1	Answer $\ln\left(1+\sqrt{2}\right)$	
			or $x = \ln\left(-\frac{3}{4}+\frac{5}{4}\right) = \ln\frac{1}{2}$	A1	1.1	Answer $\ln\frac{1}{2}$ or $-\ln 2$	
				[6]			

Question			Answer	Marks	AOs	Guidance	
14	(i)		$\begin{vmatrix} \lambda & 1 & 1 \\ 1 & \lambda & 2 \\ 2 & -1 & 1 \end{vmatrix} = \lambda(\lambda+2) - 1(-3) + 1(-1-2\lambda)$	M1	3.1a		
			$= \lambda^2 + 2$	A1	1.1		
			$\lambda^2 + 2 > 0$	M1	3.1a		
			so determinant is non-zero and matrix has an inverse	M1	2.2a		
			hence equations have a unique solution, ie planes meet at a point	E1	2.2b		
				[5]			
14	(ii)		$\mathbf{M}^{-1} = \frac{1}{\lambda^2+2}\begin{pmatrix} \lambda+2 & -2 & 2-\lambda \\ 3 & \lambda-2 & 1-2\lambda \\ -1-2\lambda & \lambda+2 & \lambda^2-1 \end{pmatrix}$	M1 A1 A1	3.1a 1.1 1.1	Attempt to calculate the inverse matrix At most 2 errors All correct	
			$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{\lambda^2+2}\begin{pmatrix} \lambda+2 & -2 & 2-\lambda \\ 3 & \lambda-2 & 1-2\lambda \\ -1-2\lambda & \lambda+2 & \lambda^2-1 \end{pmatrix}\begin{pmatrix} \lambda^2+2 \\ 0 \\ 0 \end{pmatrix}$	M1	2.1	Use of inverse matrix to solve equations	
			$= \begin{pmatrix} \lambda+2 \\ 3 \\ -1-2\lambda \end{pmatrix}$	A1ft	2.1		
			$= \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix} + \lambda\begin{pmatrix} 1 \\ 0 \\ -2 \end{pmatrix}$	M1	1.1		
			So point always lies on the line $\mathbf{r} = \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix} + \lambda\begin{pmatrix} 1 \\ 0 \\ -2 \end{pmatrix}$	E1	2.2a	oe, eg cartesian form of equation	
				[7]			

Question			Answer	Marks	AOs	Guidance	
15	(i)		$\dot{x} = x - y + \sin t \Rightarrow \ddot{x} = \dot{x} - \dot{y} + \cos t$	M1	1.1a	Differentiation of given equation	
			$\ddot{x} = \dot{x} - (6x - 4y + \cos t) + \cos t = \dot{x} - 6x + 4y$	M1	1.1	Substitute for $\dot{y}$	
			$\ddot{x} = \dot{x} - 6x + 4(x + \sin t - \dot{x})$	M1	2.1	Substitute for y	
			$\ddot{x} + 3\dot{x} + 2x = 4\sin t$	E1	2.2a	**AG**	
				[4]			
15	(ii)		Auxiliary equation: $\lambda^2 + 3\lambda + 2 = 0 \Rightarrow \lambda = -2,\ -1$	M1	1.2		
			Complementary function is $x = Ae^{-2t} + Be^{-t}$	A1	1.1		
			Particular integral is $x = C\cos t + D\sin t$	B1	1.2		
			$\dot{x} = -C\sin t + D\cos t, \quad \ddot{x} = -C\cos t - D\sin t$	M1	1.1		
			$C + 3D = 0, \quad D - 3C = 4$	M1	2.1	Substituting PI and equating coefficients	
			$\Rightarrow C = -1.2, \quad D = 0.4$	A1	1.1		
			General solution is $x = Ae^{-2t} + Be^{-t} - 1.2\cos t + 0.4\sin t$	A1	2.2a		
				[7]			
15	(iii)		$t = 0, x = 0 \Rightarrow A + B - 1.2 = 0$	B1	3.1a		
			$t = 0, \dot{x} = 0 \Rightarrow -2A - B + 0.4 = 0$	B1	3.1a		
			Hence $A = -0.8,\ B = 2$	M1	1.1		
			Particular soln is $x = 2e^{-t} - 0.8e^{-2t} - 1.2\cos t + 0.4\sin t$	A1	2.2a		
				[4]			
15	(iv)		For large t, $x \approx -1.2\cos t + 0.4\sin t$	M1	3.2a	Neglecting exponential terms in solution	
			amplitude is $\sqrt{(-1.2)^2 + 0.4^2} = 1.26$	A1	3.2a		
				[2]			

Question			Answer	Marks	AOs	Guidance	
16	**(i)**		$z^n + \frac{1}{z^n} = \cos n\theta + \mathrm{i}\sin n\theta + \cos(-n\theta) + \mathrm{i}\sin(-n\theta)$	**M1**	**1.1a**	Use of de Moivre's theorem	
			$= 2\cos n\theta$	**A1**	**1.1**		
			$z^n - \frac{1}{z^n} = \cos n\theta + \mathrm{i}\sin n\theta - \cos n\theta + \mathrm{i}\sin n\theta = 2\mathrm{i}\sin n\theta$	**A1**	**1.1**		
				[3]			
16	**(ii)**		$\left(z - \frac{1}{z}\right)^4 = z^4 - 4z^2 + 6 - \frac{4}{z^2} + \frac{1}{z^4}$	**M1** **A1**	**3.1a** **1.1**		
			$= z^4 + \frac{1}{z^4} - 4\left(z^2 + \frac{1}{z^2}\right) + 6$	**M1**	**2.1**	Grouping terms appropriately	
			$(2\mathrm{i}\sin\theta)^4 = 2\cos 4\theta - 8\cos 2\theta + 6$	**A1**	**2.1**		
			$\sin^4\theta = \frac{1}{8}\cos 4\theta - \frac{1}{2}\cos 2\theta + \frac{3}{8}$	**E1**	**2.2a**	**AG**	
				[5]			
16	**(iii)**	*(A)*		**M1** **A1**	**1.1** **1.1**	Loop from pole with correct symmetry $r = a$ at $\theta = \frac{1}{2}\pi$ soi	
				[2]			
16	**(iii)**	*(B)*	Area is $\int_0^{\pi} \frac{1}{2}a^2 \sin^4\theta \, \mathrm{d}\theta$	**M1**	**1.2**		
			$= \frac{1}{2}a^2 \int_0^{\pi} \left(\frac{1}{8}\cos 4\theta - \frac{1}{2}\cos 2\theta + \frac{3}{8}\right) \mathrm{d}\theta$	**M1**	**3.1a**	Substituting for $\sin^4\theta$	
			$= \frac{1}{2}a^2 \left[\frac{1}{32}\sin 4\theta - \frac{1}{4}\sin 2\theta + \frac{3}{8}\theta\right]_0^{\pi}$	**A1**	**1.1**	Correct indefinite integration	
			$= \frac{3}{16}\pi a^2$	**A1**	**1.1**		
				[4]			

Question			Answer	Marks	AOs	Guidance	
17	(i)		Upwards force acting is $P - kv - mg$, and acceleration is $\frac{dv}{dt}$, so Newton II gives $m\frac{dv}{dt} = P - kv - mg$	E1 [1]	3.3	AG	
17	(ii)	(A)	$\frac{dv}{dt} = 12 - 0.1v - 10 = 2 - 0.1v$	B1	1.1		
			$\int \frac{1}{2-0.1v}\,dv = \int dt$	M1	1.1a	Separation of variables	
			$-10\ln(2-0.1v) = t + c$	A1	1.1		
			$v = 0$ when $t = 0$ so $c = -10\ln 2$	B1	3.3		
			$\ln\left(\frac{2-0.1v}{2}\right) = -0.1t \Rightarrow 1 - 0.05v = e^{-0.1t}$				
			$\Rightarrow v = 20(1 - e^{-0.1t})$	A1	2.1		
			Alternative solution				
			$\frac{dv}{dt} + 0.1v = 2$	B1			
			Integrating factor is $e^{\int 0.1\,dt} = e^{0.1t}$	M1			
			$ve^{0.1t} = \int 2e^{0.1t}\,dt = 20e^{0.1t} + c$	A1			
			$v = 0$ when $t = 0$ so $c = -20$	B1			
			$\Rightarrow v = 20(1 - e^{-0.1t})$	E1			
				[5]			
17	(ii)	(B)	Terminal velocity is 20 ($m\,s^{-1}$)	B1 [1]	3.5a		
17	(iii)		The value of m (the mass of the rocket) will decrease with time as the propellant in the firework burns up	B1 [1]	3.5c		

Question			Answer	Marks	AOs	Guidance	
17	(iv)		$m\frac{dv}{dt}=-u\frac{dm}{dt}-kv-mg$ and $m=1-t$ $\Rightarrow (1-t)\frac{dv}{dt}=u-0.1v-10(1-t)$ $\Rightarrow \frac{dv}{dt}+\frac{0.1}{1-t}v=\frac{u}{1-t}-10$	**B1** **[1]**	**1.1**	**AG**	
17	(v)		Integrating factor is $e^{\int 0.1(1-t)^{-1}\,dt}$ $=e^{-0.1\ln(1-t)}=(1-t)^{-0.1}$	**M1** **E1** **[2]**	**1.1a** **2.1**	 **AG**	
17	(vi)		$\frac{d}{dt}\left\{v(1-t)^{-0.1}\right\}=u(1-t)^{-1.1}-10(1-t)^{-0.1}$ $v(1-t)^{-0.1}=u\frac{(1-t)^{-0.1}}{0.1}+10\frac{(1-t)^{0.9}}{0.9}+c$ $v=10u+\frac{100}{9}(1-t)+c(1-t)^{0.1}$ $v=0$ when $t=0$ so $c=-10u-\frac{100}{9}$ $v=10u+\frac{100}{9}(1-t)-10u(1-t)^{-0.1}-\frac{100}{9}(1-t)^{-0.1}$ $=10u\{1-(1-t)^{-0.1}\}-\frac{100}{9}\{(1-t)^{-0.1}+t-1\}$	**M1** **A1** **B1** **B1** **E1** **[5]**	**2.1** **2.1** **2.1** **3.3** **2.2a**	 **AG**	
17	(vii)	(*A*)	$v\approx 10u\{1-(1-0.1t)\}-\frac{100}{9}\{(1-0.1t)+t-1\}$ $=10u\times 0.1t-\frac{100}{9}\times 0.9t=(u-10)t$	**M1** **E1** **[2]**	**3.4** **2.1**	Binomial approximation **AG**	
17	(vii)	(*B*)	Rocket fails to get off the ground, as $u > 10$ is required for the velocity to be positive for small t	**B1** **[1]**	**3.5a**		