OCR
Oxford Cambridge and RSA

A Level Further Mathematics B (MEI)

Y420/01 Core Pure

Practice Paper – Set 1

Time allowed: 2 hours 40 minutes

You must have:
- Printed Answer Booklet
- Formulae Further Mathematics B (MEI)

You may use:
- a scientific or graphical calculator

INSTRUCTIONS
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION
- The total number of marks for this paper is **144**.
- The marks for each question are shown in brackets **[ ]**.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is used. You should communicate your method with correct reasoning.
- The Printed Answer Booklet consists of **24** pages. The Question Paper consists of **8** pages.

Section A (33 marks)

Answer **all** the questions.

1 Using standard summation formulae, find $\sum_{r=1}^{n} r(r+2)$, giving your answer in a fully factorised form. **[5]**

2 **(i)** Describe the transformation of the plane represented by the matrix $\mathbf{M} = \begin{pmatrix} 1 & k \\ 0 & 1 \end{pmatrix}$, where k is a non-zero constant. **[2]**

(ii) The image of the point (2, 3) under the transformation in part **(i)** lies on the y-axis. Find k. **[3]**

3 **In this question you must show detailed reasoning.**

Given that $z = 2$ is a root of the equation $z^3 + kz^2 + 7z - 6 = 0$, find, in exact form, the other two roots. **[6]**

4 **In this question you must show detailed reasoning.**

Fig. 4 shows the region bounded by the curve $y = \frac{1}{\sqrt{3+x^2}}$, the x-axis, the y-axis and the line $x = 1$.

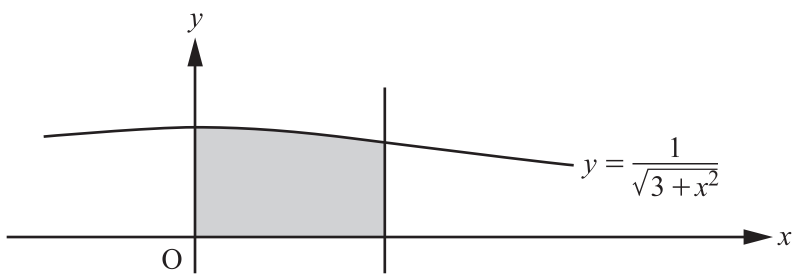

Fig. 4

This region is rotated through 2π radians about the x-axis. Find, in an exact form, the volume of the solid of revolution generated. **[4]**

5 **In this question you must show detailed reasoning.**

Show that $\int_0^1 \sinh 2x \, dx = \frac{1}{4}\left(e - \frac{1}{e}\right)^2$. **[3]**

6 Verify that the lines $\frac{x-2}{2} = \frac{y+3}{3} = \frac{z}{2}$ and $\frac{x+6}{-3} = \frac{y+4}{1} = \frac{z-6}{4}$ intersect, stating clearly the coordinates of the point of intersection. **[6]**

7 **In this question you must show detailed reasoning.**

Evaluate $\int_0^{\infty} \frac{1}{(1+2x)^2} \, dx$. **[4]**

Section B (111 marks)

Answer **all** the questions.

8 **In this question you must show detailed reasoning.**

Find $\sum_{r=10}^{100} \frac{1}{r^2+r}$, expressing your answer as an exact fraction. **[5]**

9 **(i)** On an Argand diagram, draw the locus of points defined by $|z-3-4\text{i}| = 3$. **[2]**

(ii) Find the complex number z which lies on the locus in part **(i)** and has the smallest possible argument. Give your answer in the form $a + b\text{i}$. **[7]**

10 Given that the points A (1, 0, 2), B (–2, –6, 2), C (0, 6, 3) and D (4, λ, 1) are coplanar, find λ. **[8]**

11 Matrix $\mathbf{M}$ is given by $\mathbf{M} = \begin{pmatrix} 1 & c \\ -c & 1 \end{pmatrix}$.

(i) Given that $\det \mathbf{M} = 0$, find the possible values of c. **[2]**

(ii) Prove by induction that $\mathbf{M}^n = 2^{n-1}\mathbf{M}$, for all positive integers n. **[7]**

12 **(i)** Determine, from first principles, the Maclaurin series up to the x^3 term for the function $\arcsin \frac{1}{2}x$. **[6]**

(ii) Use the series in part **(i)** to find a rational approximation for π. **[3]**

13 **(i)** Prove, using exponential functions, that $\cosh 2x = 1 + 2\sinh^2 x$. **[3]**

(ii) **In this question you must show detailed reasoning.**

Use the result in part **(i)** to solve the equation

$$2\cosh 2x - \sinh x = 5,$$

giving the answers in exact logarithmic form. **[6]**

14 **(i)** Show that the planes with equations

$$\begin{aligned} \lambda x + y + z &= a \\ x + \lambda y + 2z &= b \\ 2x - y + z &= c \end{aligned}$$

where λ, a, b and c are real constants, always meet at a point. **[5]**

(ii) You are now given that $a = \lambda^2 + 2$, $b = 0$ and $c = 0$. Show that, for different values of λ, the point of intersection of the planes always lies on a fixed line. **[7]**

15 You are given that x and y satisfy the simultaneous differential equations

$$\frac{\mathrm{d}x}{\mathrm{d}t} = x - y + \sin t \quad \text{and} \quad \frac{\mathrm{d}y}{\mathrm{d}t} = 6x - 4y + \cos t.$$

(i) Show that $\frac{\mathrm{d}^2x}{\mathrm{d}t^2} + 3\frac{\mathrm{d}x}{\mathrm{d}t} + 2x = 4\sin t$. **[4]**

(ii) Find the general solution of the differential equation in part **(i)**. **[7]**

(iii) Find the particular solution of this differential equation, given that, when $t = 0$, $x = 0$ and $\frac{\mathrm{d}x}{\mathrm{d}t} = 0$. **[4]**

(iv) For the particular solution found in part **(iii)**, find the limit of the amplitude of x as $t \to \infty$. **[2]**

16 **(i)** Given that $z = \cos\theta + \mathrm{i}\sin\theta$, express $z^n + \frac{1}{z^n}$ and $z^n - \frac{1}{z^n}$ in terms of trigonometric functions of $n\theta$. **[3]**

(ii) By considering $\left(z - \frac{1}{z}\right)^4$, show that $\sin^4\theta = \frac{1}{8}\cos 4\theta - \frac{1}{2}\cos 2\theta + \frac{3}{8}$. **[5]**

(iii) (A) Sketch the curve with polar equation $r = a\sin^2\theta$, for $0 \leqslant \theta \leqslant \pi$, where a is a positive constant. **[2]**

(B) **In this question you must show detailed reasoning.**

Find, in terms of a and π, the area of the region enclosed by the curve. **[4]**

17 A firework rocket of mass m kg rises vertically from rest at ground level. The velocity of the rocket at time t s is $v\,\mathrm{m\,s^{-1}}$, and the thrust is P N. The resistance to motion is proportional to the velocity.

(i) Show that $m\dfrac{\mathrm{d}v}{\mathrm{d}t} = P - kv - mg$, where k is a constant. **[1]**

(ii) Using the values $k = 0.1$, $m = 1$ and $P = 12$, and taking $g = 10\,\mathrm{m\,s^{-2}}$,

(A) solve the differential equation, expressing v in terms of t, **[5]**

(B) find the terminal velocity of the rocket. **[1]**

[The terminal velocity is the limit of v as $t \to \infty$.]

Assume now that, in the differential equation in part **(i)**, $k = 0.1$, $g = 10$, $m = 1 - \lambda t$, where λ is a constant, and $P = -u\dfrac{\mathrm{d}m}{\mathrm{d}t}$, where $u\,\mathrm{m\,s^{-1}}$ is a constant representing the relative velocity of fuel ejected from the rocket.

(iii) Give a reason why using $m = 1 - \lambda t$ might be expected to be more realistic than the value $m = 1$ used in part **(ii)**. **[1]**

(iv) Taking $\lambda = 1$, show that, for $t < 1$,

$$\frac{\mathrm{d}v}{\mathrm{d}t} + \frac{0.1}{1-t}v = \frac{u}{1-t} - 10.$$ **[1]**

(v) Show that the integrating factor for the differential equation in part **(iv)** is $(1-t)^{-0.1}$. **[2]**

(vi) Hence show that

$$v = 10u\left\{1 - (1-t)^{0.1}\right\} - \tfrac{100}{9}\left\{(1-t)^{0.1} + t - 1\right\}.$$ **[5]**

(vii) (A) Show that, for small t, $v \approx (u-10)t$. **[2]**

(B) What can you conclude if $u < 10$? **[1]**

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