OCR
Oxford Cambridge and RSA

A Level Further Mathematics B (MEI)

Y420/01 Core Pure

Practice Paper – Set 2

Time allowed: 2 hours 40 minutes

You must have:
- Printed Answer Booklet
- Formulae Further Mathematics B (MEI)

You may use:
- a scientific or graphical calculator

INSTRUCTIONS
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION
- The total mark for this paper is **144**.
- The marks for each question are shown in brackets **[ ]**.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is used. You should communicate your method with correct reasoning.
- The Printed Answer Booklet consists of **24** pages. The Question Paper consists of **8** pages.

Section A (35 marks)

Answer **all** the questions.

1 The triangular numbers $u_1, u_2, u_3, \ldots$ are defined by $u_r = \frac{1}{2}r(r+1),\ r \in \mathbb{N}$.

Using standard formulae, find the sum of the first n triangular numbers, expressing your answer in fully factorised form. **[4]**

2 **In this question you must show detailed reasoning.**

Find the mean value of the function $\dfrac{1}{4+x^2}$ between $x = 0$ and $x = 2$. **[3]**

3 Solve the equation $k^2x^2 - kx + 1 = 0$, where the real constant k is positive. Give the answers in modulus-argument form, in terms of k and π. **[4]**

4 **In this question you must show detailed reasoning.**

The equation $x^3 + 2x^2 - 5 = 0$ has roots α, β and γ. Find an equation with integer coefficients whose roots are $1+2\alpha$, $1+2\beta$ and $1+2\gamma$. **[4]**

5 Fig. 5 shows two complex numbers z_1 and z_2 represented by points P and Q on an Argand diagram. The angle POQ is $\frac{1}{4}\pi$ radians, and OQ = 2OP.

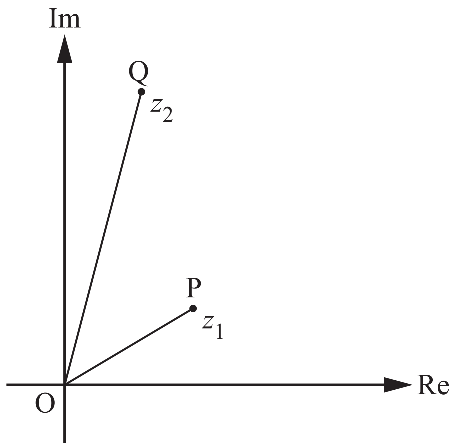

Fig. 5

Find $\dfrac{z_2}{z_1}$ in the form $a + \mathrm{i}b$, where a and b are exact real numbers. **[3]**

6 The complex number a has cube roots β, $\beta\omega$ and $\beta\omega^2$, where $\omega = e^{\frac{2}{3}\pi i}$.

(i) Prove that $\beta + \beta\omega + \beta\omega^2 = 0$. **[3]**

In the case where $a = 2 + 2i$.

(ii) Find the three cube roots, expressing them in the form $re^{i\theta}$. **[4]**

(iii) Represent the three cube roots on an Argand diagram. **[2]**

7 Suppose **M** and **N** are any two non-singular square matrices of equal size.

(i) Using the case where $\mathbf{M} = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ and $\mathbf{N} = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$, disprove the statement $(\mathbf{MN})^{-1} = \mathbf{M}^{-1}\mathbf{N}^{-1}$. **[4]**

(ii) Amend the statement to express the inverse of **MN** in terms of the inverses of **M** and **N**. **[1]**

(iii) Prove your amended statement from part **(ii)** in the general case. [Associativity for matrix multiplication may be assumed.] **[3]**

Section B (109 marks)

Answer **all** the questions.

8 **(i)** Show using exponentials that $\dfrac{2\tanh x}{1+\tanh^2 x} = \tanh 2x$. **[4]**

(ii) Hence solve the equation $5\tanh 2x - 8\tanh x = 0$, giving the answers, where appropriate, in exact logarithmic form. **[6]**

9 **In this question you must show detailed reasoning.**

A curve has polar equation $r = a\sin\theta$, for $0 \leqslant \theta \leqslant \pi$, where a is a positive constant.

(i) Find the area of the region enclosed by the curve. **[4]**

(ii) Show that the curve is a circle, stating its centre and radius. **[4]**

10 Two lines have equations $\mathbf{r} = \begin{pmatrix} 4 \\ 0 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$ and $\mathbf{r} = \begin{pmatrix} 0 \\ 1 \\ a \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}$, where a is a constant.

(i) Given that the lines meet.

(a) Find the angle between the lines. **[3]**

(b) Find the value of a. **[3]**

(ii) Given instead that the shortest distance between the lines is 1 unit, find the possible values of a. **[6]**

11 De Moivre's theorem states that, for all positive integers n, $(\cos\theta + \mathrm{i}\sin\theta)^n = \cos n\theta + \mathrm{i}\sin n\theta$.

(i) Prove this result by induction. **[5]**

(ii) By considering $\dfrac{1}{(\cos\theta + \mathrm{i}\sin\theta)^n}$, extend the proof to negative integers. **[4]**

12 **(i)** Write down the 3×3 matrix $\mathbf{M}$ which represents a rotation of $90°$ anticlockwise about the y-axis. **[1]**

The matrix $\mathbf{R}$ is given by $\mathbf{R}=\begin{pmatrix} a & 0 & -b \\ 0 & 1 & 0 \\ b & 0 & a \end{pmatrix}$, where a and b are positive constants.

(ii) Find $\det \mathbf{R}$ in terms of a and b. **[1]**

You are given that $\mathbf{R}^3=\mathbf{M}$.

(iii) (A) Find $\mathbf{R}^2$ and $\mathbf{R}^3$ in terms of a and b. **[4]**

(B) Hence or otherwise find the exact values of a and b. **[3]**

(C) Calculate $\det \mathbf{R}$. **[1]**

The matrix $\mathbf{R}$ represents a rotation of $30°$ anticlockwise about the y-axis.

(iv) Explain the significance of the value of $\det \mathbf{R}$ in relation to this transformation. **[2]**

13 **In this question you must show detailed reasoning.**

The plane Π contains the point (2, 0, 5) and the line $\frac{x-10}{3}=\frac{y-3}{2}=\frac{z-7}{2}$. Find the distance between the origin and Π, giving your answer correct to 3 significant figures. **[8]**

14 **(i)** (A) Using a standard Maclaurin series, justify the statement

$$1-\tfrac{1}{2}+\tfrac{1}{3}-\tfrac{1}{4}+\ldots=\ln 2.$$ **[2]**

(B) Find the error in approximation in using the first 10 terms of the series in part **(i)**(A) to approximate $\ln 2$. **[2]**

(ii) Find the Maclaurin series up to the x^3 term for $\operatorname{arsinh} x$. **[5]**

(iii) (A) **In this question you must show detailed reasoning.**

Given that $\operatorname{arsinh} x=\ln 2$, find x. **[2]**

(B) Hence use the result in part **(ii)** to find a rational approximation for $\ln 2$. **[1]**

(C) Find the error in approximation. **[1]**

(iv) Explain the significance of the results of parts **(i)**(B) and **(iii)**(C) for the two series used to approximate $\ln 2$. **[1]**

15 You are given that x and y satisfy the differential equation $x\frac{\mathrm{d}y}{\mathrm{d}x}+3y=\sqrt{1+x^3}$, and $y=1$ when $x=2$.

Find y in terms of x. **[8]**

16 Fig. 16 represents the profile of a suspension bridge. With respect to coordinate axes as shown, the cable BCD hangs in a curve modelled by the equation $y = a\cosh\left(\frac{x}{a}\right) - b$, where a and b are positive constants and $-50 \leqslant x \leqslant 50$. The points A and E are vertically below B and D respectively. Units for x and y are metres.

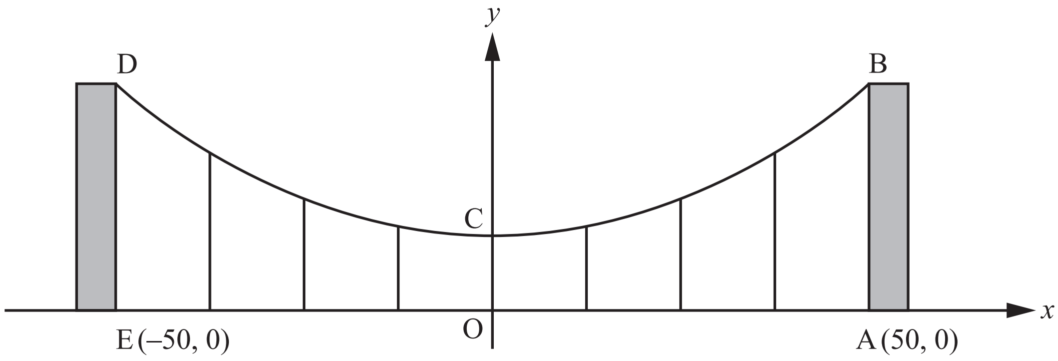

Fig. 16

You are given that OC = 10 m and AB = 30 m.

(i) (*A*) Show that $a\cosh\left(\frac{50}{a}\right) - a = 20$. **[3]**

(*B*) Verify that $a = 65.59$, correct to 2 decimal places. **[2]**

The length s of a curve $y = \mathrm{f}(x)$ from $x = c$ to $x = d$, where c and d are constants, is given by the formula $s = \int_c^d \sqrt{1 + (\mathrm{f}'(x))^2}\,\mathrm{d}x$.

(ii) In this question you must show detailed reasoning.

Use this formula to find the length of the cable, giving your answer to the nearest metre. **[4]**

17 [The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$, when a numerical value is needed, use $g = 9.8$.]
In order to perform a bungee jump, a jumper is attached to one end of an elastic cord of unstretched length 10 metres, the other end of which is anchored. She then falls vertically from rest. Throughout the motion which follows, the air resistance force is assumed to be negligible.

(i) (*A*) Show that at the instant when the cord begins to stretch, the speed of the jumper is $14\,\mathrm{m\,s^{-1}}$. **[2]**

(*B*) State another modelling assumption (besides zero air resistance) made in this calculation. **[1]**

For the remainder of the jump, the extension of the cord, y m, at time t seconds after the cord begins to stretch, is modelled by the differential equation

$$\frac{\mathrm{d}^2y}{\mathrm{d}t^2} + 2\frac{\mathrm{d}y}{\mathrm{d}t} + 2y = g.$$

(ii) Solve this differential equation. **[7]**

The jumper starts the jump from a bridge 20 metres above the ground.

(iii) (*A*) Show that the jumper does not hit the ground during the jump. **[5]**

(*B*) Find the final distance above the ground where the jumper comes to rest. **[2]**

The model used in the question so far has assumed the air resistance force is negligible.

(iv) Discuss briefly the effect on the answers in part **(iii)** of adding an air resistance force to the model. **[2]**

END OF QUESTION PAPER