Practice Paper – Set 3

A Level Further Mathematics B (MEI)

Y420/01 Core Pure

MARK SCHEME

Duration: 2 hours 40 minutes

MAXIMUM MARK 144

This document consists of 20 pages

Text Instructions

1. Annotations and abbreviations

Annotation in scoris	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining a result or establishing a given result
dep*	Mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This indicates that the instruction **In this question you must show detailed reasoning** appears in the question.

2. Subject-specific Marking Instructions for A Level Further Mathematics B (MEI)

a Annotations should be used whenever appropriate during your marking. The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded. For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
If you are in any doubt whatsoever you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more ‘method’ steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation ‘dep*’ is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case, please escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be ‘follow through’. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is numerically correct and expressed either in SI or in the units of the question. (e.g. lengths will be assumed to be in metres unless in a particular question all the lengths are in km, when this would be assumed to be the unspecified unit.) We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so. When a value is given in the paper only accept an answer correct to at least as many significant figures as the given value. This rule should be applied to each case. When a value is not given in the paper accept any answer that agrees with the correct value to 2 s.f. Follow through should be used so that only one mark is lost for each distinct accuracy error, except for errors due to premature approximation which should be penalised only once in the examination. There is no penalty for using a wrong value for g. E marks will be lost except when results agree to the accuracy required in the question.

g Rules for replaced work: if a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests; if there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others. NB Follow these maths-specific instructions rather than those in the assessor handbook.

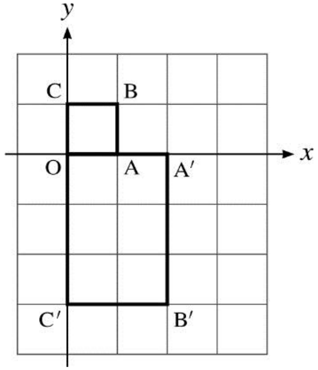
h For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate’s data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question. Marks designated as cao may be awarded as long as there are no other errors. E marks are lost unless, by chance, the given results are established by equivalent working. ‘Fresh starts’ will not affect an earlier decision about a misread. Note that a miscopy of the candidate’s own working is not a misread but an accuracy error.

i If a calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers (provided, of course, that there is nothing in the wording of the question specifying that analytical methods are required). Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

j If in any case the scheme operates with considerable unfairness consult your Team Leader.

Question			Answer	Marks	AOs	Guidance	
1			$\frac{1}{r^2}-\frac{1}{(r+1)^2}=\frac{(r+1)^2-r^2}{r^2(r+1)^2}=\frac{2r+1}{r^2(r+1)^2}$	M1 A1	1.1 1.1	Attempt to combine fractions Correct simplification	
			$\sum_{r=1}^{n}\frac{2r+1}{r^2(r+1)^2}=\sum_{r=1}^{n}\left(\frac{1}{r^2}-\frac{1}{(r+1)^2}\right)$	M1	1.1		
			$=\frac{1}{1^2}-\frac{1}{2^2}+\frac{1}{2^2}-...+\frac{1}{(n-1)^2}-\frac{1}{n^2}+\frac{1}{n^2}-\frac{1}{(n+1)^2}$	M1	1.1	Substituting, use of difference method	
			$=1-\frac{1}{(n+1)^2}$	A1	1.1		
				[5]			

2			**DR**				
			$-8=8e^{\pi i}$ or $-8=8(\cos\pi+i\sin\pi)$ oe	M1	1.1	soi by subsequent working	
			Roots are $2e^{\frac{1}{3}\pi i}$, $2e^{\pi i}$, $2e^{\frac{5}{3}\pi i}$ oe	M1	1.1	Use of de Moivre's theorem	
				A1	1.1	All three correct (any form)	
			giving $1+i\sqrt{3}$, $1-i\sqrt{3}$ and -2	A1	1.1	All three correct in cartesian form	
				[4]			

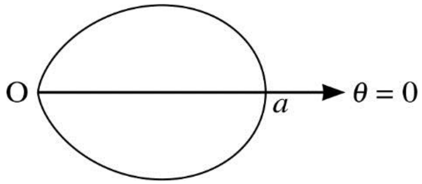
Question			Answer	Marks	AOs	Guidance	
3	(a)			M1 A1 [2]	1.1 1.1	$\begin{pmatrix}2 & 0\\0 & -3\end{pmatrix}\begin{pmatrix}0 & 1 & 1 & 0\\0 & 0 & 1 & 1\end{pmatrix}=\begin{pmatrix}0 & 2 & 2 & 0\\0 & 0 & -3 & -3\end{pmatrix}$ Correct image shown	soi from diagram
3	(b)	(i)	$\det \mathbf{M} = -6$	B1 [1]	1.1		
3	(b)	(ii)	Area multiplied by 6 Orientation reversed	B1 B1 [2]	1.1 1.1		

Question			Answer	Marks	AOs	Guidance	
4	(a)		**DR**				
			$z_1 = 2\left(\tfrac{1}{2}\sqrt{3}+\tfrac{1}{2}\mathrm{i}\right) = 2\left(\cos\tfrac{1}{6}\pi + \mathrm{i}\sin\tfrac{1}{6}\pi\right)$	**B1**	**1.1**		
			$z_2 = 2\left(\tfrac{1}{2}\sqrt{2}+\tfrac{1}{2}\sqrt{2}\mathrm{i}\right) = 2\left(\cos\tfrac{1}{4}\pi + \mathrm{i}\sin\tfrac{1}{4}\pi\right)$	**B1**	**1.1**		
				[2]			
4	(b)		**DR**				
			$\dfrac{z_2}{z_1} = \dfrac{\sqrt{2}+\sqrt{2}\,\mathrm{i}}{\sqrt{3}+\mathrm{i}} = \dfrac{\left(\sqrt{2}+\sqrt{2}\,\mathrm{i}\right)\left(\sqrt{3}-\mathrm{i}\right)}{\left(\sqrt{3}+\mathrm{i}\right)\left(\sqrt{3}-\mathrm{i}\right)}$	**M1**	**2.1**	Realizing denominator	
			$= \dfrac{\sqrt{6}+\sqrt{2}+\left(\sqrt{6}-\sqrt{2}\right)\mathrm{i}}{4}$	**A1**	**1.1**		
			$\dfrac{z_2}{z_1} = \dfrac{2\left(\cos\tfrac{1}{4}\pi+\mathrm{i}\sin\tfrac{1}{4}\pi\right)}{2\left(\cos\tfrac{1}{6}\pi+\mathrm{i}\sin\tfrac{1}{6}\pi\right)} = \cos\tfrac{1}{12}\pi + \mathrm{i}\sin\tfrac{1}{12}\pi$	**M1** **A1**	**2.1** **1.1**	Division of modulus-argument forms $\cos\tfrac{1}{12}\pi + \mathrm{i}\sin\tfrac{1}{12}\pi$	
			so $\cos\tfrac{1}{12}\pi = \mathrm{Re}\left(\dfrac{z_2}{z_1}\right) = \dfrac{\sqrt{6}+\sqrt{2}}{4}$	**E1**	**2.2a**	**AG** Sufficient working must be shown	
				[5]			

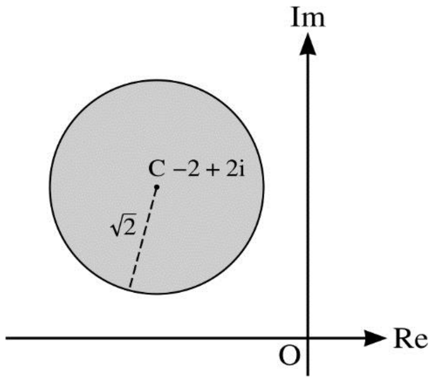
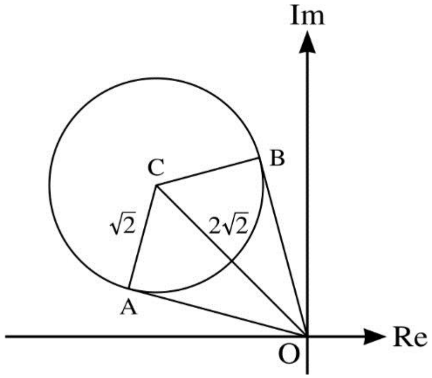
Question			Answer	Marks	AOs	Guidance	
5	(a)		$2\cosh^2 x - 1 = 2\dfrac{(e^x + e^{-x})^2}{4} - 1$	M1	2.1	Converting to exponentials	
			$= \dfrac{e^{2x} + 2 + e^{-2x} - 2}{2}$	M1	1.1	$(e^x)^2 = e^{2x}$ seen	
			$= \dfrac{e^{2x} + e^{-2x}}{2} = \cosh 2x$	E1	2.2a	**AG** fully correct working required	
				[3]			
5	(b)		$V = \int_0^{\frac{1}{2}} \pi \cosh^2 x \, dx$	B1	1.1a		
			$= \frac{1}{2}\pi \int_0^{\frac{1}{2}} (1 + \cosh 2x) \, dx$	M1	2.1	Use of 'double angle' formula	
			$= \frac{1}{2}\pi \left[x + \frac{1}{2}\sinh 2x \right]_0^{\frac{1}{2}}$	A1	1.1	Indefinite integral $x + \frac{1}{2}\sinh 2x$	
			$= \frac{1}{4}\pi \left(1 + \dfrac{e - e^{-1}}{2} \right)$ oe	A1	1.1	Accept any equivalent exact form	
				[4]			

Question			Answer	Marks	AOs	Guidance	
6			Another root is 2 − i	B1	1.1	Seen at any stage	
			Third root is α where $2+i+2-i+\alpha=\frac{5}{2}$	M1	2.1	Use of sum of roots	Condone sum = 5 for M1
			$\Rightarrow \alpha=-\frac{3}{2}$	A1	2.2a		
			$\frac{a}{2}=(2+i)(2-i)+\left(-\frac{3}{2}\right)(2+i)+\left(-\frac{3}{2}\right)(2-i)$	M1	2.1		Condone a for $\frac{a}{2}$ here
			$\Rightarrow a=-2$	A1	2.2a	**cao**	
			$-\frac{b}{2}=(2+i)(2-i)\left(-\frac{3}{2}\right)$	M1	2.1		Condone $-b$ for $-\frac{b}{2}$ here
			$\Rightarrow b=15$	A1	2.2a	**cao**	
			Alternative solution				
			$2(2+i)^3-5(2+i)^2+a(2+i)+b=0$	M1		Substituting $x=2+i$ and expanding	
			$\Rightarrow 4+22i-15-20i+2a+ai+b=0$	A1		Correct expansion of brackets	
			so $2i+ai=0$ and $2a+b-11=0$	M1		Equating real and imaginary parts to 0	
			$\Rightarrow a=-2,\ b=15$	A1		Both correct	
			Another root is 2 − i	B1		Seen at any stage	
			$(x-2-i)(x-2+i)=x^2-4x+5$ is a factor of the cubic	M1		Finding a quadratic factor	
			cubic is $(x^2-4x+5)(2x+3)$ so third root is $-\frac{3}{2}$	A1		Correct real root	
				[7]			

Question			Answer	Marks	AOs	Guidance	
7	(a)		Period is $4\pi = \frac{2\pi}{\omega} \Rightarrow \omega = \frac{1}{2}$	M1	3.1a	Finding ω, soi	
			$\frac{d^2x}{dt^2} = -\frac{1}{4}x$	A1	3.1a		
				[2]			
7	(b)		General solution: $x = A\cos\frac{1}{2}t + B\sin\frac{1}{2}t$	B1	1.2		
			$t = 0,\ x = 4 \Rightarrow A = 4$	B1	3.1a	Correct value $A = 4$	
			$\frac{dx}{dt} = -\frac{1}{2}A\sin\frac{1}{2}t + \frac{1}{2}B\cos\frac{1}{2}t$	B1	1.1		
			$t = 0,\ \frac{dx}{dt} = 1.5 \Rightarrow \frac{1}{2}B = 1.5 \Rightarrow B = 3$	B1	3.1a	Correct value $B = 3$	
			so $x = 4\cos\frac{1}{2}t + 3\sin\frac{1}{2}t$				
				[4]			
7	(c)		Amplitude is $\sqrt{4^2 + 3^2} = 5$ m	B1	3.4		
				[1]			

Question			Answer	Marks	AOs	Guidance	
8	(a)			M1 A1 **[2]**	1.1 2.5	Single loop Correctly positioned, *a* indicated	
8	(b)		**DR** Area is $\int_{-\frac{1}{2}\pi}^{\frac{1}{2}\pi} \frac{1}{2} a^2 \cos^4 \theta \, d\theta$ $= \frac{1}{2} a^2 \int_{-\frac{1}{2}\pi}^{\frac{1}{2}\pi} \left[\frac{1}{2}(1+\cos 2\theta)^2 \right] d\theta$ $= \frac{1}{8} a^2 \int_{-\frac{1}{2}\pi}^{\frac{1}{2}\pi} (1+2\cos 2\theta + \cos^2 2\theta) \, d\theta$ $= \frac{1}{8} a^2 \int_{-\frac{1}{2}\pi}^{\frac{1}{2}\pi} \left[1+2\cos 2\theta + \frac{1}{2}(1+\cos 4\theta)\right] d\theta$ $= \frac{1}{16} a^2 \int_{-\frac{1}{2}\pi}^{\frac{1}{2}\pi} (3+4\cos 2\theta + \cos 4\theta) \, d\theta$ $= \frac{1}{16} a^2 \left[3\theta + 2\sin 2\theta + \frac{1}{4}\sin 4\theta\right]_{-\frac{1}{2}\pi}^{\frac{1}{2}\pi}$ $= \frac{3}{16}\pi a^2$	B1 M1 A1 M1 A1 A1FT A1 **[7]**	1.1a 3.1a 1.1 3.1a 1.1 1.1 3.2a	Correct integral and limits Use of double angle formula Substituting for $\cos^2 2\theta$ Correctly integrated **cao**	

Question			Answer	Marks	AOs	Guidance	
9	(a)		$n=1:\ 4=2a+2b \Rightarrow a+b=2$	B1	2.1	Substituting any value of n	
			$n=2:\ 16=4a+8b \Rightarrow a+2b=4$	B1	1.1	Substituting any second value of n	
			so $a=0$ and $b=2$	B1	1.1	**AG** Working must be clear	
				[3]			
9	(b)		When n = 1, result is true from part **(a)**	B1	2.1	Or direct check if n = 1 not used in **(a)**	
			Assume true for $n = k$, so $\sum_{r=1}^{k}(r+1)2^r = k\times 2^{k+1}$	M1	2.1		
			Then $\sum_{r=1}^{k+1}(r+1)2^r = k\times 2^{k+1}+(k+2)\times 2^{k+1}$	M1	2.1		
			$=(2k+2)\times 2^{k+1}=(k+1)\times 2^{(k+1)+1}$, so true for $n = k + 1$	E1	2.4	Correct algebra and statement	
			So if true for $n = k$, true for $n = k + 1$; and as true for n = 1, true for all positive n	E1	2.2a	Proof correctly concluded	
				[5]			

Question			Answer	Marks	AOs	Guidance
10	(a)			M1 A1 A1 [3]	1.1 1.1 2.5	Circle Centre $-2+2\text{i}$ and radius $\sqrt{2}$ Interior indicated
10	(b)		$\arg(-2+2\text{i})=\frac{3}{4}\pi$ oe $\text{OC}=\|-2+2\text{i}\|=2\sqrt{2}$ $\sin\text{AOC}=\frac{1}{2}$ $\Rightarrow \text{AOC}=\frac{1}{6}\pi$ At A, $\arg z=\frac{3}{4}\pi+\frac{1}{6}\pi$ and at B, $\arg z=\frac{3}{4}\pi-\frac{1}{6}\pi$ so greatest value of $\arg z=\frac{11}{12}\pi$ and least value of $\arg z=\frac{7}{12}\pi$	B1 B1 M1 A1 M1 A1 A1 [7]	2.1 2.1 2.1 2.2a 2.1 2.2a 2.2a	Or using BOC For either of these 

Question			Answer	Marks	AOs	Guidance	
11	(a)		e > 1 so Maclaurin series for ln(1 + *x*) is not convergent when *x* = e, and claim is not valid	B1 B1 [2]	2.3 2.2a		
11	(b)		$\ln\left(1+\frac{1}{e}\right)=\frac{1}{e}-\frac{1}{2e^2}+\frac{1}{3e^3}-...$ $\ln\left(1+\frac{1}{e}\right)=\ln\left(\frac{1+e}{e}\right)=\ln(1+e)-\ln e=\ln(1+e)-1$ so $\ln(1+e)=1+\frac{1}{e}-\frac{1}{2e^2}+\frac{1}{3e^3}-...$	B1 M1 E1 [3]	1.1 3.1a 2.2a	AG	
11	(c)	(i)	1.3168	B1 [1]	1.1	Accept 1.32 or better	
11	(c)	(ii)	$\frac{1.316807...-1.31326...}{1.31326...}\times 100$ = 0.27%	M1 A1 [2]	1.1 1.1	Accept 0.3%	

Question			Answer	Marks	AOs	Guidance	
12	**(a)**	**(i)**	$\overrightarrow{AB} = \begin{pmatrix} -1 \\ 1 \\ -2 \end{pmatrix}$ and $\overrightarrow{AC} = \begin{pmatrix} -2 \\ -2 \\ 1 \end{pmatrix}$	**B1**	**1.1**	Both vectors correct (soi)	
			$\overrightarrow{AB} \times \overrightarrow{AC} = \begin{pmatrix} -3 \\ 5 \\ 4 \end{pmatrix}$	**B1** **[2]**	**1.1**	Correct answer, may be **BC**	
12	**(a)**	**(ii)**	$\lvert \overrightarrow{AB} \times \overrightarrow{AC} \rvert = \sqrt{(-3)^2 + 5^2 + 4^2} = 5\sqrt{2}$ oe	**B1**	**1.1**	**FT** their answer to **(a)**(*A*)	
			Area of triangle = $\frac{1}{2} \lvert \overrightarrow{AB} \times \overrightarrow{AC} \rvert$	**M1**	**3.1a**	Use of $\lvert \overrightarrow{AB} \times \overrightarrow{AC} \rvert = \text{AB.AC.sin BAC}$	
			$= \frac{5}{2}\sqrt{2}$ oe	**A1** **[3]**	**3.2a**	Accept 3.54 or better	
12	**(b)**	**(i)**	Equation of plane ABC is $-3x + 5y + 4z = d$	**M1**	**1.1**	oe, e.g. use of **r.n** = **a.n**	
			where $-3 \times 0 + 5 \times 2 + 4 \times 4 = d \Rightarrow -3x + 5y + 4z = 26$	**A1**	**1.1**	oe working, using coords of B or C	
			Distance from D to plane is $\dfrac{\lvert -3 \times 2 + 5 \times 4 + 4 \times 0 - 26 \rvert}{\sqrt{(-3)^2 + 5^2 + 4^2}}$	**M1**	**1.1**	Use of perpendicular distance formula	
			$= \frac{6}{5}\sqrt{2}$ oe	**A1** **[4]**	**1.1**	Accept 1.70 or better	
12	**(b)**	**(ii)**	Volume is $\frac{1}{3} \times \frac{5}{2}\sqrt{2} \times \frac{6}{5}\sqrt{2} = 2$	**B1** **[1]**	**3.2a**	**FT** their area and height	

Question			Answer	Marks	AOs	Guidance	
12	(c)		AB: $\mathbf{r}=\begin{pmatrix}0\\2\\4\end{pmatrix}+\lambda\begin{pmatrix}-1\\1\\-2\end{pmatrix}$ and CD: $\mathbf{r}=\begin{pmatrix}-2\\0\\5\end{pmatrix}+\lambda\begin{pmatrix}4\\4\\-5\end{pmatrix}$	M1	1.1	Equations of both lines	
			$\begin{pmatrix}-1\\1\\-2\end{pmatrix}\times\begin{pmatrix}4\\4\\-5\end{pmatrix}=\begin{pmatrix}3\\-13\\-8\end{pmatrix}$	M1 A1	1.1 1.1	Correct cross product calculated Correct result	
			$\text{Distance}=\left\lvert\frac{(3\mathbf{i}-13\mathbf{j}-8\mathbf{k}).(-2\mathbf{i}-2\mathbf{j}+\mathbf{k})}{\sqrt{3^2+(-13)^2+(-8)^2}}\right\rvert$	M1	1.1	Use of skew lines distance formula	
			$=\frac{6}{11}\sqrt{2}$	A1	1.1	Accept 0.77 or better	
				[5]			

13			$C+\mathrm{i}S=\mathrm{e}^{\mathrm{i}\theta}+\mathrm{e}^{\mathrm{i}\left(\theta+\frac{2\pi}{n}\right)}+\mathrm{e}^{\mathrm{i}\left(\theta+\frac{4\pi}{n}\right)}+\ldots+\mathrm{e}^{\mathrm{i}\left(\theta+\frac{(2n-2)\pi}{n}\right)}$	M1 A1	2.1 1.1	Forming C + iS All correct in exponential form, oe soi, e.g. by use of sum of GP formula	
			This is a GP with $a=\mathrm{e}^{\mathrm{i}\theta}$ and $r=\mathrm{e}^{\frac{2\pi}{n}\mathrm{i}}$	M1	2.5		
			so $C+\mathrm{i}S=\frac{\mathrm{e}^{\mathrm{i}\theta}\left[1-\left(\mathrm{e}^{\frac{2\pi}{n}\mathrm{i}}\right)^n\right]}{1-\mathrm{e}^{\frac{2\pi}{n}\mathrm{i}}}$	A1	2.1		
			$=\frac{\mathrm{e}^{\mathrm{i}\theta}\left(1-\mathrm{e}^{2\pi\mathrm{i}}\right)}{1-\mathrm{e}^{\frac{2\pi}{n}\mathrm{i}}}$	A1	2.1		
			Since $\mathrm{e}^{2\pi\mathrm{i}}=\cos 2\pi+\mathrm{i}\sin 2\pi=1$	E1	2.1		
			$C+\mathrm{i}S=0\Rightarrow C=0$ and $S=0$	E1	2.2a	**AG** results must be fully justified	
				[7]			

Question			Answer	Marks	AOs	Guidance	
14	(a)		$\mathbf{M} = \begin{pmatrix} \lambda & 1 & 1 \\ 3 & \lambda & -1 \\ -1 & 2 & 1 \end{pmatrix}$	**M1**	**3.1a**	Forming matrix of coefficients	
			$\det \mathbf{M} = \lambda(\lambda+2) - 2 + 6 + \lambda$ (oe)	**M1**	**3.1a**	Finding the determinant	
			$= \lambda^2 + 3\lambda + 4$	**A1**	**1.1**		
			Discriminant is $3^2 - 4\times1\times4 = -7 < 0$	**M1**	**3.1a**	Use of discriminant (oe)	$\left(\lambda+\frac{3}{2}\right)^2 + \frac{7}{4} > 0$
			Hence $\det \mathbf{M} \neq 0$ for any λ, so $\mathbf{M}^{-1}$ always exists and the three planes always meet at a point	**E1** **[5]**	**3.2a**	**AG**	
14	(b)	(i)	$\mathbf{M}^{-1} = \frac{1}{\lambda^2+3\lambda+4}\begin{pmatrix} \lambda+2 & 1 & -\lambda-1 \\ -2 & \lambda+1 & \lambda+3 \\ \lambda+6 & -2\lambda-1 & \lambda^2-3 \end{pmatrix}$	**M1** **A3** **A1FT**	**3.1a** **1.1, 1.1** **1.1** **1.1**	Attempt to invert the matrix For all matrix entries correct; allow **A2** if just one wrong entry, or **A1** if exactly two wrong entries Division by their determinant	
			$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \mathbf{M}^{-1}\begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix} = \frac{1}{\lambda^2+3\lambda+4}\begin{pmatrix} -2\lambda-1 \\ 3\lambda+7 \\ 2\lambda^2-2\lambda-7 \end{pmatrix}$	**M1**	**3.1a**	Use of inverse matrix	
			$x = \frac{-2\lambda-1}{\lambda^2+3\lambda+4},\ y = \frac{3\lambda+7}{\lambda^2+3\lambda+4},\ z = \frac{2\lambda^2-2\lambda-7}{\lambda^2+3\lambda+4}$	**A2** **[8]**	**1.1, 1.1**	Allow **A1** for any two correct	
14	(b)	(ii)	$x = 0 \Rightarrow \lambda = -\frac{1}{2}$	**M1**	**3.1a**	Equating x to 0 and finding λ	
				A1FT	**1.1**	Correct λ from their x	
			So $x = 0,\ y = 2,\ z = -2$	**A1** **[3]**	**1.1**	**cao** Correct values for y and z	

Question			Answer	Marks	AOs	Guidance	
15			$\int_0^{\frac{1}{2}} \frac{1}{\sqrt{5-4x+4x^2}}\,dx = \int_0^{\frac{1}{2}} \frac{1}{\sqrt{4+(2x-1)^2}}\,dx$	**M1** **A1**	**2.1** **1.1**	Completing the square $4+(2x-1)^2$	
			Let $u=2x-1 \Rightarrow du = 2dx$	**M1**	**2.1**	Use of substitution	Or $\left[k \operatorname{arsinh}\frac{2x-1}{2}\right]_0^{\frac{1}{2}}$
			giving $\int_{-1}^{0} \frac{1}{\sqrt{4+u^2}} \times \frac{1}{2}\,du$	**A1**	**1.1**		Or $\left[\frac{1}{2} \operatorname{arsinh}\frac{2x-1}{2}\right]_0^{\frac{1}{2}}$
			$=\frac{1}{2}\left[\operatorname{arsinh}\frac{u}{2}\right]_{-1}^{0} = -\frac{1}{2}\operatorname{arsinh}\left(-\frac{1}{2}\right)$	**A1**	**1.1**	$-\frac{1}{2}\operatorname{arsinh}\left(-\frac{1}{2}\right)$	
			$=-\frac{1}{2}\ln\left(-\frac{1}{2}+\sqrt{\frac{5}{4}}\right)$	**M1**	**2.1**	Use of logarithmic form of arsinh	Or $\frac{1}{2}\operatorname{arsinh}\frac{1}{2}$ (symmetry)
			$=\frac{1}{2}\ln\frac{2}{\sqrt{5}-1} = \frac{1}{2}\ln\frac{\sqrt{5}+1}{(\sqrt{5}-1)(\sqrt{5}+1)}$	**M1**	**1.1**	Using $-\ln u = \ln\left(\frac{1}{u}\right)$ and rationalizing	Or $\frac{1}{2}\ln\left(\frac{1}{2}+\sqrt{\frac{5}{4}}\right)$
			$=\frac{1}{2}\ln\left(\frac{2(\sqrt{5}+1)}{4}\right) = \frac{1}{2}\ln\left(\frac{\sqrt{5}+1}{2}\right)$	**E1**	**2.2a**	**AG**	
				[8]			

Question			Answer	Marks	AOs	Guidance	
16	(a)		$m\dfrac{\mathrm{d}v}{\mathrm{d}t} = mg - 2mv$	**B1** **[1]**	3.3		
16	(b)		$\displaystyle\int \frac{1}{g-2v}\,\mathrm{d}v = \int \mathrm{d}t$ $\Rightarrow -\tfrac{1}{2}\ln(g-2v) = t + c$ When $t = 0,\ v = 0 \Rightarrow c = -\tfrac{1}{2}\ln g$ $\ln(g-2v) - \ln g = -2t \Rightarrow \ln\left(\dfrac{g-2v}{g}\right) = -2t$ $\Rightarrow 1 - \dfrac{2v}{g} = \mathrm{e}^{-2t}$ $\Rightarrow v = \tfrac{1}{2}g(1-\mathrm{e}^{-2t})$	**M1** **A1** **B1** **M1** **A1** **E1** **[6]**	1.1 1.1 3.3 1.1 1.1 2.2a	Separating the variables in their DE **FT** if equivalent work **FT** evaluating their constant Combining logs **AG** full and correct working required	
16	(c)		As $t \to \infty,\ v \to \tfrac{1}{2}g$ $\dfrac{\mathrm{d}v}{\mathrm{d}t} \to 0$	**B1** **B1** **[2]**	3.4 3.4		

Question			Answer	Marks	AOs	Guidance	
16	(d)	(i)	Resistance force = $\frac{2mv}{1+t}$ N	B1 [1]	3.4	Must include factor *m*	Condone missing units
16	(d)	(ii)	Integrating factor is $e^{\int \frac{2}{1+t}\,dt}$ $= e^{2\ln(1+t)} = e^{\ln(1+t)^2} = (1+t)^2$ $\Rightarrow \frac{d}{dt}\left((1+t)^2 v\right) = g(1+t)^2$ $\Rightarrow (1+t)^2 v = \int g(1+t)^2\,dt = \frac{1}{3}g(1+t)^3 + c$ When $t = 0,\ v = 0 \Rightarrow c = -\frac{1}{3}g$ $v = \frac{1}{3}g\left[1+t-(1+t)^{-2}\right]$ oe	M1 A1 A1 A1 B1 A1 [6]	1.1 1.1 1.1 1.1 3.3 1.1	 $(1+t)^2$ **FT** $t = 0$, $v = 0$ in their equation	
16	(e)		As $t \to \infty,\ v \to \infty$ $\frac{dv}{dt} = \frac{1}{3}g\left[1+2(1+t)^{-3}\right]$ As $t \to \infty,\ \frac{dv}{dt} \to \frac{1}{3}g$	B1 M1 A1 [3]	3.4 1.1 3.4	Dependent on correct *v* Differentiating their *v* **cao**	
16	(f)		The long term velocity and acceleration are both greater in the second model This is consistent with the resistance force being less in the second model	B1 B1 [2]	3.5a 3.5a		