Oxford Cambridge and RSA

A Level Further Mathematics B (MEI)

Y420/01 Core Pure

Practice Paper – Set 3

Time allowed: 2 hours 40 minutes

You must have:
- Printed Answer Booklet
- Formulae Further Mathematics B (MEI)

You may use:
- a scientific or graphical calculator

INSTRUCTIONS
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION
- The total number of marks for this paper is **144**.
- The marks for each question are shown in brackets **[ ]**.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is used. You should communicate your method with correct reasoning.
- The Printed Answer Booklet consists of **24** pages. The Question Paper consists of **8** pages.

Section A (35 marks)

Answer **all** the questions

1 By considering $\frac{1}{r^2}-\frac{1}{(r+1)^2}$, find $\sum_{r=1}^{n}\frac{2r+1}{r^2(r+1)^2}$. **[5]**

2 **In this question you must show detailed reasoning.**

Find the cube roots of -8, expressing them in the form $a+b\mathrm{i}$, where a and b are real. **[4]**

3 The transformation T of the plane has associated matrix $\mathbf{M}=\begin{pmatrix}2 & 0\\ 0 & -3\end{pmatrix}$.

(a) On the grid in the Printed Answer Booklet, plot the image OA′B′C′ under the transformation T of the unit square with vertices $\mathrm{O}(0,0)$, $\mathrm{A}(1,0)$, $\mathrm{B}(1,1)$ and $\mathrm{C}(0,1)$. **[2]**

(b) **(i)** Find $\det\mathbf{M}$. **[1]**

(ii) Explain the significance of the value of $\det\mathbf{M}$ for the image of the unit square in part **(i)**. **[2]**

4 **In this question you must show detailed reasoning.**

The complex numbers z_1 and z_2 are given by $z_1=\sqrt{3}+\mathrm{i}$ and $z_2=\sqrt{2}+\sqrt{2}\,\mathrm{i}$.

(a) Express each of z_1 and z_2 in modulus-argument form. **[2]**

(b) By considering $\frac{z_2}{z_1}$, show that $\cos\frac{1}{12}\pi=\frac{\sqrt{6}+\sqrt{2}}{4}$. **[5]**

5 **(a)** Using exponentials, show that $2\cosh^2 x-1=\cosh 2x$. **[3]**

(b) The region enclosed by the x-axis, the y-axis, the curve $y=\cosh x$ and the line $x=\frac{1}{2}$ is rotated through 360° about the x-axis.
Find, in terms of e and π, the exact volume of the solid generated. **[4]**

6 You are given that $2+\mathrm{i}$ is a root of the equation $2x^3-5x^2+ax+b=0$, where a and b are real constants.

Find

- the other roots of the equation,
- the values of a and b. **[7]**

Section B (109 marks)

Answer **all** the questions.

7 A particle moves in a straight line with simple harmonic motion of period 4π s about a fixed point O. The displacement from O of the particle at time t s is x m.

(a) Express the relationship between x and t as a differential equation. **[2]**

(b) Find x in terms of t, given that when $t = 0$, $x = 4$ and $\dfrac{dx}{dt} = 1.5$. **[4]**

(c) Find the amplitude of the motion. **[1]**

8 **(a)** Sketch the curve $r = a\cos^2\theta$ for $-\frac{1}{2}\pi \leqslant \theta < \frac{1}{2}\pi$, where a is a positive constant. **[2]**

(b) **In this question you must show detailed reasoning.**

Find, in terms of a and π, the area of the region enclosed by the curve. **[7]**

9 It is conjectured that $\sum_{r=1}^{n}(r+1)2^r = (a+bn)2^n$, where a and b are constants.

(a) If the conjecture is true, verify that $a = 0$ and $b = 2$. **[3]**

(b) Prove the conjecture. **[5]**

10 A set S of complex numbers is defined by $S = \{z : |z+2-2\mathrm{i}| \leqslant \sqrt{2}\}$.

(a) Show on an Argand diagram the set of points that represents S. **[3]**

(b) Find, in terms of π, the greatest and least values of $\arg z$, given that $z \in S$. **[7]**

11 **(a)** Beth uses a Maclaurin series to claim that $\ln(1+\mathrm{e}) \approx \mathrm{e} - \dfrac{\mathrm{e}^2}{2} + \dfrac{\mathrm{e}^3}{3} - \dots$.

Explain whether Beth is correct. **[2]**

(b) Using the series for $\ln\left(1+\dfrac{1}{\mathrm{e}}\right)$, show that $\ln(1+\mathrm{e}) = 1 + \dfrac{1}{\mathrm{e}} - \dfrac{1}{2\mathrm{e}^2} + \dfrac{1}{3\mathrm{e}^3} - \dots$. **[3]**

(c) **(i)** Use the first 4 terms of this series to find an approximation for $\ln(1+\mathrm{e})$. **[1]**

(ii) Find, correct to 2 decimal places, the percentage error in this approximation. **[2]**

12 A pyramid has vertices $\mathrm{A}(0, 2, 4)$, $\mathrm{B}(-1, 3, 2)$, $\mathrm{C}(-2, 0, 5)$ and $\mathrm{D}(2, 4, 0)$.

(a) **(i)** Find $\overrightarrow{\mathrm{AB}} \times \overrightarrow{\mathrm{AC}}$. **[2]**

(ii) Hence find the area of triangle ABC. **[3]**

(b) **(i)** Find the distance between the point D and the plane ABC. **[4]**

(ii) Hence find the volume of the pyramid ABCD.

[The volume of a pyramid is $\frac{1}{3} \times \text{area of base} \times \text{height}$.] **[1]**

(c) Find the shortest distance between the lines AB and CD. **[5]**

13 You are given that

$$C = \cos\theta + \cos\left(\theta + \frac{2\pi}{n}\right) + \cos\left(\theta + \frac{4\pi}{n}\right) + \ldots + \cos\left(\theta + \frac{(2n-2)\pi}{n}\right),$$

$$S = \sin\theta + \sin\left(\theta + \frac{2\pi}{n}\right) + \sin\left(\theta + \frac{4\pi}{n}\right) + \ldots + \sin\left(\theta + \frac{(2n-2)\pi}{n}\right),$$

where n is an integer greater than 1.

By considering $C + \mathrm{i}S$, show that $C = 0$ and $S = 0$. **[7]**

14 Three planes have equations

$$\begin{aligned} \lambda x + y + z &= 0, \\ 3x + \lambda y - z &= 1, \\ -x + 2y + z &= 2, \end{aligned}$$

where λ is a constant.

(a) Show that, for all values of λ, these planes meet at a point. **[5]**

(b) **(i)** Find, in terms of λ, the point of intersection of the planes. **[8]**

(ii) Find the coordinates of the point of intersection given that it lies in the y-z plane. **[3]**

15 Show that $\displaystyle\int_0^{\frac{1}{2}} \frac{1}{\sqrt{5 - 4x + 4x^2}}\,\mathrm{d}x = \frac{1}{2}\ln\left(\frac{1+\sqrt{5}}{2}\right)$. **[8]**

16 A particle of mass m kg falls vertically from rest under gravity. After time t s, its velocity is $v\,\text{m s}^{-1}$.

In an initial model, a force of magnitude $2mv$ N resisting the motion is assumed to be acting.

(a) Formulate a differential equation in v and t. **[1]**

(b) Hence show that $v = \frac{1}{2}g(1 - e^{-2t})$. **[6]**

(c) Describe the behaviour as t tends to infinity of

- the velocity,
- the acceleration. **[2]**

An alternative model produces the differential equation $(1+t)\frac{dv}{dt} + 2v = g(1+t)$.

(d) **(i)** State the resistance force on the particle used by this model. **[1]**

(ii) Solve the differential equation. **[6]**

(e) For this alternative model, describe the behaviour as t tends to infinity of

- the velocity,
- the acceleration. **[3]**

(f) Comment on how the different results in parts **(c)** and **(e)** relate to the different resistance forces used in each model. **[2]**

END OF QUESTION PAPER

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