OCR
Oxford Cambridge and RSA

A Level Further Mathematics B (MEI)

Y435/01 Extra Pure

Practice Paper – Set 1

Time allowed: 1 hour 15 minutes

You must have:
- Printed Answer Booklet
- Formulae Further Mathematics B (MEI)

You may use:
- a scientific or graphical calculator

INSTRUCTIONS
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION
- The total number of marks for this paper is **60**.
- The marks for each question are shown in brackets **[ ]**.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is used. You should communicate your method with correct reasoning.
- The Printed Answer Booklet consists of **16** pages. The Question Paper consists of **4** pages.

Answer **all** the questions.

1 The table below is the composition table for a group, G.

	e	b	a	a^2	a^3	ab	a^2b	a^3b
e	e	b	a	a^2	a^3	ab	a^2b	a^3b
b	b	e	a^3b	a^2b	ab	a^3	a^2	a
a	a	ab	a^2	a^3	e	a^2b	a^3b	b
a^2	a^2	a^2b	a^3	e	a	a^3b	b	ab
a^3	a^3	a^3b	e	a	a^2	b	ab	a^2b
ab	ab	a	b	a^3b	a^2b	e	a^3	a^2
a^2b	a^2b	a^2	ab	b	a^3b	a	e	a^3
a^3b	a^3b	a^3	a^2b	ab	b	a^2	a	e

(i) (A) Find the order of each element. **[3]**

(B) Deduce that G is not cyclic. **[1]**

(ii) Show that G is not abelian. **[2]**

(iii) Write down the subgroup generated by the element ab. **[1]**

(iv) Write down the subgroup generated by the element a^3. **[1]**

2 **(i)** Write down the product of the matrices $\begin{pmatrix} a & 0 \\ a & 0 \end{pmatrix}$ and $\begin{pmatrix} b & 0 \\ b & 0 \end{pmatrix}$. **[1]**

A is the set of matrices $\left\{ \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} -1 & 0 \\ -1 & 0 \end{pmatrix}, \begin{pmatrix} \mathrm{i} & 0 \\ \mathrm{i} & 0 \end{pmatrix}, \begin{pmatrix} -\mathrm{i} & 0 \\ -\mathrm{i} & 0 \end{pmatrix} \right\}$.

(ii) Determine whether A forms a group under matrix multiplication. **[6]**

3 **(i)** Find the eigenvalues and associated eigenvectors of the matrix $\mathbf{A}$ where $\mathbf{A} = \begin{pmatrix} 3 & 1 \\ 1 & 3 \end{pmatrix}$. **[5]**

(ii) Prove that if k is some non-zero constant and $\mathbf{e}$ is an eigenvector of matrix $\mathbf{M}$, with associated eigenvalue λ, then $\mathbf{e}$ is also an eigenvector of $(k\mathbf{M})$ but with associated eigenvalue $(k\lambda)$. **[2]**

(iii) Hence write down the eigenvalues and associated eigenvectors of the matrix $\mathbf{B}$ where $\mathbf{B} = \begin{pmatrix} \frac{3}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{3}{4} \end{pmatrix}$. **[1]**

(iv) Write down a matrix $\mathbf{E}$ and a diagonal matrix $\mathbf{D}$ such that $\mathbf{B} = \mathbf{EDE}^{-1}$. **[2]**

(v) **In this question you must show detailed reasoning.**

Hence show that $\lim\limits_{n \to \infty} \mathbf{B}^n = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$. **[4]**

4 **(i)** Find the solution of $4a_{n+2} - 4a_{n+1} + a_n = 0$, $n \in \mathbb{Z}$, $n \geqslant 0$, for which $a_0 = a_1 = 1$. **[6]**

(ii) For this solution, find $\lim_{n\to\infty} \dfrac{4^n \times a_{2n+3}}{n}$. **[3]**

5 A surface S is defined by $z = xy + 2x + 2y - 5$.

(i) Find the coordinates of the stationary point on S. **[4]**

The points (1, 2, 3) and (3, 4, 21) lie on S.

(ii) Find the coordinates of the point where the normal to S at (3, 4, 21) intersects the tangent plane of S at (1, 2, 3). **[9]**

6 In this question you may use the fact that if p and q are distinct prime numbers then $\sqrt{pq}$ is an irrational number.

(i) Consider the following conjecture.

If a and b are irrational numbers then $a + b$ is also an irrational number.

Use a counterexample to show that this conjecture is not true. **[1]**

(ii) Prove that if $x \in \mathbb{Q}$ then $x^2 \in \mathbb{Q}$. **[2]**

(iii) Prove by contradiction that if m, $n \in \mathbb{Z}$ (with $n \neq 0$) and $r \notin \mathbb{Q}$ then $m + nr \notin \mathbb{Q}$. **[3]**

(iv) Prove that $\sqrt{2} + \sqrt{3}$ is an irrational number. **[3]**

END OF QUESTION PAPER