



Oxford Cambridge and RSA

A Level Further Mathematics B (MEI)

Y435/01 Extra Pure

Practice Paper – Set 1

Time allowed: 1 hour 15 minutes

You must have:

- Printed Answer Booklet
- Formulae Further Mathematics B (MEI)

You may use:

- a scientific or graphical calculator

INSTRUCTIONS

- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION

- The total number of marks for this paper is **60**.
- The marks for each question are shown in brackets [].
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is used. You should communicate your method with correct reasoning.
- The Printed Answer Booklet consists of **16** pages. The Question Paper consists of **4** pages.

Answer **all** the questions.

- 1 The table below is the composition table for a group, G .

	e	b	a	a^2	a^3	ab	a^2b	a^3b
e	e	b	a	a^2	a^3	ab	a^2b	a^3b
b	b	e	a^3b	a^2b	ab	a^3	a^2	a
a	a	ab	a^2	a^3	e	a^2b	a^3b	b
a^2	a^2	a^2b	a^3	e	a	a^3b	b	ab
a^3	a^3	a^3b	e	a	a^2	b	ab	a^2b
ab	ab	a	b	a^3b	a^2b	e	a^3	a^2
a^2b	a^2b	a^2	ab	b	a^3b	a	e	a^3
a^3b	a^3b	a^3	a^2b	ab	b	a^2	a	e

- (i) (A) Find the order of each element. [3]

- (B) Deduce that G is not cyclic. [1]

- (ii) Show that G is not abelian. [2]

- (iii) Write down the subgroup generated by the element ab . [1]

- (iv) Write down the subgroup generated by the element a^3 . [1]

- 2 (i) Write down the product of the matrices $\begin{pmatrix} a & 0 \\ a & 0 \end{pmatrix}$ and $\begin{pmatrix} b & 0 \\ b & 0 \end{pmatrix}$. [1]

A is the set of matrices $\left\{ \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} -1 & 0 \\ -1 & 0 \end{pmatrix}, \begin{pmatrix} i & 0 \\ i & 0 \end{pmatrix}, \begin{pmatrix} -i & 0 \\ -i & 0 \end{pmatrix} \right\}$.

- (ii) Determine whether A forms a group under matrix multiplication. [6]

- 3 (i) Find the eigenvalues and associated eigenvectors of the matrix $\mathbf{A}$ where $\mathbf{A} = \begin{pmatrix} 3 & 1 \\ 1 & 3 \end{pmatrix}$. [5]

- (ii) Prove that if k is some non-zero constant and $\mathbf{e}$ is an eigenvector of matrix $\mathbf{M}$, with associated eigenvalue λ , then $\mathbf{e}$ is also an eigenvector of $(k\mathbf{M})$ but with associated eigenvalue $(k\lambda)$. [2]

- (iii) Hence write down the eigenvalues and associated eigenvectors of the matrix $\mathbf{B}$ where $\mathbf{B} = \begin{pmatrix} \frac{3}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{3}{4} \end{pmatrix}$. [1]

- (iv) Write down a matrix $\mathbf{E}$ and a diagonal matrix $\mathbf{D}$ such that $\mathbf{B} = \mathbf{EDE}^{-1}$. [2]

- (v) In this question you must show detailed reasoning.

Hence show that $\lim_{n \rightarrow \infty} \mathbf{B}^n = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$. [4]

- 4 (i) Find the solution of $4a_{n+2} - 4a_{n+1} + a_n = 0$, $n \in \mathbb{Z}$, $n \geq 0$, for which $a_0 = a_1 = 1$. [6]
- (ii) For this solution, find $\lim_{n \rightarrow \infty} \frac{4^n \times a_{2n+3}}{n}$. [3]
- 5 A surface S is defined by $z = xy + 2x + 2y - 5$.
- (i) Find the coordinates of the stationary point on S . [4]
- The points $(1, 2, 3)$ and $(3, 4, 21)$ lie on S .
- (ii) Find the coordinates of the point where the normal to S at $(3, 4, 21)$ intersects the tangent plane of S at $(1, 2, 3)$. [9]
- 6 In this question you may use the fact that if p and q are distinct prime numbers then $\sqrt{pq}$ is an irrational number.
- (i) Consider the following conjecture.
- If a and b are irrational numbers then $a + b$ is also an irrational number.
- Use a counterexample to show that this conjecture is not true. [1]
- (ii) Prove that if $x \in \mathbb{Q}$ then $x^2 \in \mathbb{Q}$. [2]
- (iii) Prove by contradiction that if $m, n \in \mathbb{Z}$ (with $n \neq 0$) and $r \notin \mathbb{Q}$ then $m + nr \notin \mathbb{Q}$. [3]
- (iv) Prove that $\sqrt{2} + \sqrt{3}$ is an irrational number. [3]

END OF QUESTION PAPER

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