Oxford Cambridge and RSA

Practice Paper – Set 3

A Level Further Mathematics B (MEI)

Y435/01 Extra Pure

MARK SCHEME

Duration: 1 hour 15 minutes

MAXIMUM MARK 60

This document consists of 12 pages

Text Instructions

1. Annotations and abbreviations

Annotation in scoris	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining a result or establishing a given result
dep*	Mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This indicates that the instruction **In this question you must show detailed reasoning** appears in the question.

2. Subject-specific Marking Instructions for A Level Further Mathematics B (MEI)

a Annotations should be used whenever appropriate during your marking. The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded. For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
If you are in any doubt whatsoever you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more ‘method’ steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation ‘dep*’ is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case, please escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be ‘follow through’. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is numerically correct and expressed either in SI or in the units of the question. (e.g. lengths will be assumed to be in metres unless in a particular question all the lengths are in km, when this would be assumed to be the unspecified unit.) We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so. When a value is given in the paper only accept an answer correct to at least as many significant figures as the given value. This rule should be applied to each case. When a value is not given in the paper accept any answer that agrees with the correct value to 2 s.f. Follow through should be used so that only one mark is lost for each distinct accuracy error, except for errors due to premature approximation which should be penalised only once in the examination. There is no penalty for using a wrong value for *g*. E marks will be lost except when results agree to the accuracy required in the question.

g Rules for replaced work: if a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests; if there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others. NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate’s data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question. Marks designated as cao may be awarded as long as there are no other errors. E marks are lost unless, by chance, the given results are established by equivalent working. ‘Fresh starts’ will not affect an earlier decision about a misread. Note that a miscopy of the candidate’s own working is not a misread but an accuracy error.

i If a calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers (provided, of course, that there is nothing in the wording of the question specifying that analytical methods are required). Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

j If in any case the scheme operates with considerable unfairness consult your Team Leader.

Question			Answer	Marks	AOs	Guidance	
1			$t_{n+2} - 4t_{n+1} + 3t_n = 0$ and $t_n = Ap^n \Rightarrow p^2 - 4p + 3 = 0$	**M1**	**1.1a**	Auxiliary equation	
			$t_n = \alpha + \beta \times 3^n$	**A1**	**1.1**		
			Try $t_n = k \times 4^n$	**M1**	**1.1**	Correct form for a particular solution	Condone missing *k* for this mark
			$k \times 4^{n+2} - 4k \times 4^{n+1} + 3k \times 4^n = 6 \times 4^n$	**M1**	**1.1**	Substituting their form correctly into recurrence relation	Must include *k* for this mark
			$k = 2$	**A1**	**1.1**		
			General solution is $t_n = \alpha + \beta \times 3^n + 2 \times 4^n$	**M1**	**1.1**		
			$t_0 = 10 \Rightarrow \alpha + \beta + 2 = 10$ $t_1 = 30 \Rightarrow \alpha + 3\beta + 2 \times 4 = 30$	**M1**	**1.1**	Correctly substituting either $n = 0$ or $n = 1$ into their general solution and equating to the correct value	
			$t_n = 1 + 7 \times 3^n + 2 \times 4^n$	**A1** **[8]**	**1.1**	Or $\alpha = 1$ and $\beta = 7$	

Question			Answer	Marks	AOs	Guidance	
2	(a)		$\begin{vmatrix} -\lambda & 1 \\ 6 & -1-\lambda \end{vmatrix} = -\lambda(-1-\lambda)-6$	M1	1.1a	Consideration of the determinant of the correct matrix and attempt to expand	
			$\lambda^2+\lambda-6=0$	A1	1.1		
			Eigenvalues –3 and 2	B1FT	1.1a	Both	
			$\begin{pmatrix} 0 & 1 \\ 6 & -1 \end{pmatrix}\begin{pmatrix} 1 \\ y \end{pmatrix} = \begin{pmatrix} y \\ 6-y \end{pmatrix} = -3\begin{pmatrix} 1 \\ y \end{pmatrix}$ or $2\begin{pmatrix} 1 \\ y \end{pmatrix}$	M1	1.1	Correct matrix equation for either eigenvalue with correct form for eigenvector	
			$\lambda=-3 \Rightarrow \mathbf{e}_1 = \begin{pmatrix} 1 \\ -3 \end{pmatrix}, \quad \lambda=2 \Rightarrow \mathbf{e}_2 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$	A1 [5]	1.1	Eigenvectors must be named or associated with their eigenvalues	Accept eigenvalues/vectors written in the same order
2	(b)		**DR** $\mathbf{A} = \begin{pmatrix} 1 & 1 \\ -3 & 2 \end{pmatrix}\begin{pmatrix} -3 & 0 \\ 0 & 2 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ -3 & 2 \end{pmatrix}^{-1}$	B1FT	1.1	$\mathbf{A}=\mathbf{PDP}^{-1}$ where **P** is formed from their eigenvectors and **D** has their eigenvalues in consistent order in the leading diagonal, 0 everywhere else	
			$\mathbf{A}^{10} = \begin{pmatrix} 1 & 1 \\ -3 & 2 \end{pmatrix}\begin{pmatrix} (-3)^{10} & 0 \\ 0 & 2^{10} \end{pmatrix} \times \frac{1}{5}\begin{pmatrix} 2 & -1 \\ 3 & 1 \end{pmatrix}$	M1	1.1a	$\mathbf{A}^n = \mathbf{PD}^n\mathbf{P}^{-1}$ and reasonable attempt at $\mathbf{P}^{-1}$ and correct form for $\mathbf{D}^n$	Accept 3^{10}; condone -3^{10} only if subsequent calculation shows correct intent
			$\mathbf{A}^{10} = \begin{pmatrix} 24\,234 & -11\,605 \\ -69\,630 & 35\,839 \end{pmatrix}$	A1 [3]	1.1	cao	

Question			Answer	Marks	AOs	Guidance	
2	(c)	(i)	$\begin{pmatrix} x \\ y \end{pmatrix} = a\begin{pmatrix} 1 \\ -3 \end{pmatrix} + b\begin{pmatrix} 1 \\ 2 \end{pmatrix} \Rightarrow x = a + b,\ y = -3a + 2b$ $a = \dfrac{2x - y}{5}$ $b = \dfrac{3x + y}{5}$	M1 A1 A1 **[3]**	2.2a 1.1a 1.1	Using identity to derive equations for x and y Accept embedded answers, i.e. $\begin{pmatrix} x \\ y \end{pmatrix} = \dfrac{2x - y}{5}\begin{pmatrix} 1 \\ -3 \end{pmatrix} + \dfrac{3x + y}{5}\begin{pmatrix} 1 \\ 2 \end{pmatrix}$ for both marks	Or $\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ -3 & 2 \end{pmatrix}^{-1}\begin{pmatrix} x \\ y \end{pmatrix}$
2	(c)	(ii)	$\mathbf{Ar} = a\lambda_1\mathbf{e}_1 + b\lambda_2\mathbf{e}_2 = -\dfrac{3(2x - y)}{5}\mathbf{e}_1 + \dfrac{2(3x + y)}{5}\mathbf{e}_2$	B1 **[1]**	2.2a	**AG** so derivation must be clear	
2	(d)		$\mathbf{A}^2 + \mathbf{A} - 6\mathbf{I} = \mathbf{O}$ $\mathbf{A}^2 + \mathbf{A} = 6\mathbf{I} \Rightarrow 6\mathbf{A}^{-1} = \mathbf{A} + \mathbf{I}$ $p = q = \frac{1}{6}$	M1 M1 A1 **[3]**	1.1a 1.1 1.1	Substitution of **A** into their characteristic equation Multiplying by $\mathbf{A}^{-1}$ leading to equation with $\mathbf{A}^{-1}$, **A** and **I** terms Accept $\mathbf{A}^{-1} = \frac{1}{6}\mathbf{A} + \frac{1}{6}\mathbf{I}$	Condone missing **I** (if later recovered) and/or use of 0 for **O**

Question			Answer	Marks	AOs	Guidance	
3	(a)		T_1: Not a group because not closed	E1	2.2a		
			T_2: Not a group because no identity	E1	2.2a	Accept complete valid alternatives, eg not associative with an example or clearly showing difference between given table and that for each of the known groups of order 4	'no inverse' without reference to identity is insufficient
			T_3: Group	E1	2.2a		
			Explanation that all 4 axioms/conditions are satisfied	E1	2.4	Or by demonstrating isomorphism to the known (cyclic) group of order 3	
			T_4: Not a group because d does not have an inverse	E1	2.2a	Or properly and fully explained correct alternative (eg it's not a latin square or if group then $a \circ d = b \circ d \Rightarrow a = b$)	
			T_5: eg $(b \circ c) \circ d = e \circ d = b \neq b \circ (c \circ d) = b \circ e = c$	M1	2.5	eg $(b \circ c) \circ d \neq b \circ (c \circ d)$ is insufficient	Or **M1** for complete and fully explained reason (eg all groups of order 6 have an element of order 3 but all non-identity elements here have order 2)
			Not a group because not associative	A1 [7]	2.2a		**A1** for correct conclusion

Question			Answer	Marks	AOs	Guidance	
3	(b)	(i)	Not abelian; table is not symmetrical about leading diagonal	B1 [1]	2.2a	Or eg $s*r = r^4 * s \neq r*s$ but not just $s*r \neq r*s$	
3	(b)	(ii)	5 Order is 1, 2, 5 or 10, r is not the identity (so not 1); r^2 is not p (so not 2); r^5 (eg) is not a distinct element (so not 10)	B1 E1 [2]	1.1 2.1	Or other valid complete argument but 1, 2 and 10 must be eliminated or $r^5 = p$ and r^2, r^3 and r^4 dealt with (eg by stating that they are given as being different from p)	ie argument must encapsulate both that $r^5 = p$ and also that 5 is the smallest positive integer for which this is the case
3	(b)	(iii)	Subgroups must have order 2, 5 or 10 So there are no non-trivial cyclic subgroups of non-prime order since G itself is not cyclic since it has no element of order 10	M1 E1 [2]	2.1 2.1	Ignore consideration of 1 here If {1} counted then **E0**	Must consider G itself as a subgroup; accept any valid reason for G not being cyclic
3	(b)	(iv)	$\{p, s\}, \{p, rs\}, \{p, r^2s\}, \{p, r^3s\}, \{p, r^4s\}$, $\{p, r, r^2, r^3, r^4\}$	B1 [1]	3.1a	Condone repeats of subgroups of order 2	Condone also inclusion of {1} and/or G
3	(b)	(v)	G does contain isomorphic subgroups... ...since all groups of order 2 are isomorphic to each other	M1 E1 [2]	3.2a 1.2	Or other valid explanation	

Question			Answer	Marks	AOs	Guidance	
4	(a)		$\frac{\partial g}{\partial x} = -2x\mathrm{e}^{-(x^2+y^2)}$ or $\frac{\partial g}{\partial y} = -2y\mathrm{e}^{-(x^2+y^2)}$	M1	3.1a	Attempt to find the partial derivative of g wrt x or y	
			$\nabla g = \begin{pmatrix} -2x\mathrm{e}^{-(x^2+y^2)} \\ -2y\mathrm{e}^{-(x^2+y^2)} \\ -1 \end{pmatrix}$	A1	3.1a	Or $\begin{pmatrix} -2a\mathrm{e}^{-(a^2+b^2)} \\ -2b\mathrm{e}^{-(a^2+b^2)} \\ -1 \end{pmatrix}$	
			So equation of L is $\mathbf{r} = \begin{pmatrix} a \\ b \\ \mathrm{e}^{-(a^2+b^2)} \end{pmatrix} + \lambda \begin{pmatrix} -2a\mathrm{e}^{-(a^2+b^2)} \\ -2b\mathrm{e}^{-(a^2+b^2)} \\ -1 \end{pmatrix}$	M1	3.1a		
			x (or y) $= 0 \Rightarrow \lambda = \frac{1}{2}\mathrm{e}^{a^2+b^2}$	M1	1.1		
			$\Rightarrow y$ (or x) $= 0$ so L intersects z-axis	A1 [5]	3.2a	AG	

Question			Answer	Marks	AOs	Guidance	
4	**(b)**	**(i)**	z intercept = $\mathrm{e}^{-(a^2+b^2)} - \frac{1}{2}\mathrm{e}^{a^2+b^2} = 0$	**M1**	**3.1a**		
			$\mathrm{e}^{2(a^2+b^2)} = 2$	**M1**	**1.1a**	Combining the terms using laws of indices	
			$a^2 + b^2 = \frac{1}{2}\ln 2$ oe	**A1**	**1.1**		
			$\mathbf{r.n} = \begin{pmatrix} a \\ b \\ \mathrm{e}^{-(a^2+b^2)} \end{pmatrix} . \begin{pmatrix} -2a\mathrm{e}^{-(a^2+b^2)} \\ -2b\mathrm{e}^{-(a^2+b^2)} \\ -1 \end{pmatrix}$	**M1**	**3.1a**	Using ∇g at P to form $\mathbf{r.n} = p$	Must include an attempt to expand the scalar product
			$\mathbf{r.n} = -(2a^2 + 2b^2 + 1)\mathrm{e}^{-(a^2+b^2)}$, oe	**A1**	**1.1**	All correct	
			$x = 0,\ y = 0 \Rightarrow -z = -\left(2(a^2+b^2)+1\right)\mathrm{e}^{-(a^2+b^2)}$	**M1**	**2.2a**		
			$OA = (\ln 2 + 1)\mathrm{e}^{-\frac{1}{2}\ln 2}$	**M1**	**1.1**	Substituting in $a^2 + b^2 = \frac{1}{2}\ln 2$	
			$OA = (\ln 2 + 1)\mathrm{e}^{\ln 2^{-\frac{1}{2}}} = 2^{-\frac{1}{2}}(\ln 2 + 1) = \frac{\ln 2 + 1}{\sqrt{2}}$	**A1**	**3.2a**	**AG** so an intermediate step must be seen	
				[8]			
4	**(b)**	**(ii)**	When $a, b \to 0$, the z coordinate of z intercept tends to $\mathrm{e}^0 - \frac{1}{2}\mathrm{e}^0 = 1 - \frac{1}{2} = \frac{1}{2}$ so the point is $\left(0, 0, \frac{1}{2}\right)$	**B1**	**3.2a**		
				[1]			

Question			Answer	Marks	AOs	Guidance	
5	(a)		$h_3 = \frac{1}{2}+\frac{1}{3}+\frac{1}{4}$ $h_3 > \frac{1}{2}+\frac{1}{4}+\frac{1}{4} = \frac{1}{2}+\frac{1}{2} = 2\times\frac{1}{2}$	**M1** **A1** **[2]**	1.1 2.2a	 **AG** so reasoning must be clear	
5	(b)		$h_7 > \frac{1}{2}+\left(\frac{1}{4}+\frac{1}{4}\right)+\left(\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}\right) = \frac{1}{2}+\frac{1}{2}+\frac{1}{2} \Rightarrow h_7 > 3\times\frac{1}{2}$	**B1** **[1]**	2.2a	**AG** so reasoning must be clear	
5	(c)		$h_{15} > 4\times\frac{1}{2}$	**B1** **[1]**	2.2b		
5	(d)		Use of $h_{2^k-1} > k\times\frac{1}{2}$ (for $k > 1$) soi $n = 2^{20}-1 = 1\,048\,575$	**M1** **A1** **[2]**	2.2b 1.1		
5	(e)		$\lim_{n\to\infty} h_n = \infty$ $h_{2^{2N}-1} > N$ for any (large) N	**E1** **E1** **[2]**	2.2a 2.4	oe (eg verbal statement) ie the idea that it can be made as large as desired by taking sufficiently many terms	Accept eg 'No limit exists'