OCR
Oxford Cambridge and RSA

A Level Further Mathematics B (MEI)

Y435/01 Extra Pure

Practice Paper – Set 3

Time allowed: 1 hour 15 minutes

You must have:
- Printed Answer Booklet
- Formulae Further Mathematics B (MEI)

You may use:
- a scientific or graphical calculator

INSTRUCTIONS
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** If additional space is required, you should use the lined page(s) at the end of the Printed Answer Booklet. The question number(s) must be clearly shown.
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION
- The total number of marks for this paper is **60**.
- The marks for each question are shown in brackets **[ ]**.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is used. You should communicate your method with correct reasoning.
- The Printed Answer Booklet consists of **12** pages. The Question Paper consists of **4** pages.

Answer **all** the questions

1 Solve the recurrence relation $t_{n+2} - 4t_{n+1} + 3t_n = 6 \times 4^n$, for $n \geqslant 0$, given that $t_0 = 10$ and $t_1 = 30$. **[8]**

2 $\mathbf{e}_1$ and $\mathbf{e}_2$ are two distinct eigenvectors of the matrix $\mathbf{A} = \begin{pmatrix} 0 & 1 \\ 6 & -1 \end{pmatrix}$. Both $\mathbf{e}_1$ and $\mathbf{e}_2$ are of the form $\begin{pmatrix} 1 \\ k \end{pmatrix}$. The eigenvalue associated with $\mathbf{e}_1$ is negative and the eigenvalue associated with $\mathbf{e}_2$ is positive.

(a) Find $\mathbf{e}_1$ and $\mathbf{e}_2$. **[5]**

(b) **In this question you must show detailed reasoning.**

Using reduction to diagonal form, calculate $\mathbf{A}^{10}$. **[3]**

(c) Scalars a and b are such that the vector $\mathbf{r} = \begin{pmatrix} x \\ y \end{pmatrix}$ can be written as $\mathbf{r} = a\mathbf{e}_1 + b\mathbf{e}_2$.

(i) Find a and b in terms of x and y. **[3]**

(ii) Hence show that $\mathbf{Ar} = -\frac{3(2x-y)}{5}\mathbf{e}_1 + \frac{2(3x+y)}{5}\mathbf{e}_2$. **[1]**

(d) Use the Cayley-Hamilton theorem to determine the values of p and q such that $\mathbf{A}^{-1} = p\mathbf{A} + q\mathbf{I}$, where $\mathbf{I}$ is the 2×2 identity matrix. **[3]**

3 **(a)** Tables T_1 to T_5 are composition tables for a given set, S, under a binary operation $\circ$. In each table, different letters represent different elements. For each of tables T_1 to T_5, determine whether or not S is a group under $\circ$, briefly explaining your answer in each case. **[7]**

T_1 S = $\{a, b, c, d, e\}$

$\circ$	a	b	c	d	e
a	a	b	c	d	e
b	b	c	d	f	a
c	c	d	f	a	b
d	d	f	a	b	c
e	e	a	b	c	d

T_2 S = $\{a, b, c, d\}$

$\circ$	a	b	c	d
a	a	d	b	c
b	d	b	c	a
c	b	c	a	d
d	c	a	d	b

T_3 S = $\{a, b, c\}$

$\circ$	a	b	c
a	c	a	b
b	a	b	c
c	b	c	a

T_4 S = $\{a, b, c, d\}$

$\circ$	a	b	c	d
a	b	c	a	d
b	c	a	b	d
c	a	b	c	d
d	d	d	d	d

T_5 $\qquad$ $S = \{a, b, c, d, e, f\}$

$\circ$	a	b	c	d	e	f
a	a	b	c	d	e	f
b	b	a	e	f	c	d
c	c	f	a	e	d	b
d	d	c	b	a	f	e
e	e	d	f	b	a	c
f	f	e	d	c	b	a

(b) The group table for a group (G, $*$) of order 10 is given below.

$*$	p	r	r^2	r^3	r^4	s	rs	r^2s	r^3s	r^4s
p	p	r	r^2	r^3	r^4	s	rs	r^2s	r^3s	r^4s
r	r	r^2	r^3	r^4	p	rs	r^2s	r^3s	r^4s	s
r^2	r^2	r^3	r^4	p	r	r^2s	r^3s	r^4s	s	rs
r^3	r^3	r^4	p	r	r^2	r^3s	r^4s	s	rs	r^2s
r^4	r^4	p	r	r^2	r^3	r^4s	s	rs	r^2s	r^3s
s	s	r^4s	r^3s	r^2s	rs	p	r^4	r^3	r^2	r
rs	rs	s	r^4s	r^3s	r^2s	r	p	r^4	r^3	r^2
r^2s	r^2s	rs	s	r^4s	r^3s	r^2	r	p	r^4	r^3
r^3s	r^3s	r^2s	rs	s	r^4s	r^3	r^2	r	p	r^4
r^4s	r^4s	r^3s	r^2s	rs	s	r^4	r^3	r^2	r	p

(i) State, with justification, whether (G, $*$) is abelian. **[1]**

(ii) Find the order of element r, justifying your answer. **[2]**

(iii) Determine whether or not there are any non-trivial cyclic subgroups of G which are not of prime order, justifying your answer. **[2]**

(iv) Find all the proper non-trivial subgroups of G. **[1]**

(v) Hence determine whether G contains any subgroups which are isomorphic to each other, justifying your answer. **[2]**

4 A surface S is defined by $g(x, y, z) = e^{-(x^2+y^2)} - z = 0$. P is a point on S with $x = a$ and $y = b$, where neither a nor b is equal to 0. The tangent plane to S at P intersects the z-axis at A. The normal to S at P is denoted by L.

(a) Using ∇g, show that L intersects the z-axis. **[5]**

(b) **(i)** Given that L passes through the origin, show that $OA = \frac{\ln 2 + 1}{\sqrt{2}}$. **[8]**

(ii) Given instead that P approaches the point (0, 0, 1), show that the z-intercept of L approaches a specific point whose coordinates should be found. **[1]**

5 The sequence h_n is defined by the following recurrence relation:

$$h_{n+1} = h_n + \frac{1}{n+2}, \quad h_1 = \tfrac{1}{2}.$$

(a) By writing h_3 as the sum of three fractions and noting that $\frac{1}{3} > \frac{1}{4}$, show that $h_3 > 2 \times \frac{1}{2}$. **[2]**

(b) By considering h_7 in the form $\left(\frac{1}{2}\right) + \left(\frac{1}{3} + \frac{1}{4}\right) + \left(\frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8}\right)$, show that $h_7 > 3 \times \frac{1}{2}$. **[1]**

(c) State the inequality for h_{15} which corresponds to the inequality for h_7 in part **(b)**. **[1]**

(d) Find an integer n such that $h_n > 10$. **[2]**

(e) Determine, with justification, $\lim_{n \to \infty} h_n$. **[2]**

END OF QUESTION PAPER