Oxford Cambridge and RSA

Practice Paper - Set 1

A Level Further Mathematics B (MEI)
Y421/01 Mechanics Major

MARK SCHEME

Duration: 2 hours 15 minutes

MAXIMUM MARK 120

Final

This document consists of 19 pages

Text Instructions

1. **Annotations and abbreviations**

Annotation in scoris	**Meaning**
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining a result or establishing a given result
dep*	Mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This indicates that the instruction **In this question you must show detailed reasoning** appears in the question.

2. Subject-specific Marking Instructions for A Level Mathematics B (MEI)

a Annotations should be used whenever appropriate during your marking. The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded. For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
If you are in any doubt whatsoever you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more ‘method’ steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation ‘dep*’ is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case please, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be ‘follow through’. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is numerically correct and expressed either in SI or in the units of the question. (e.g. lengths will be assumed to be in metres unless in a particular question all the lengths are in km, when this would be assumed to be the unspecified unit.) We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so. When a value is given in the paper only accept an answer correct to at least as many significant figures as the given value. This rule should be applied to each case. When a value is not given in the paper accept any answer that agrees with the correct value to 2 s.f. Follow through should be used so that only one mark is lost for each distinct accuracy error, except for errors due to premature approximation which should be penalised only once in the examination. There is no penalty for using a wrong value for *g*. E marks will be lost except when results agree to the accuracy required in the question.

g Rules for replaced work: if a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests; if there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others. NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate’s data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question. Marks designated as cao may be awarded as long as there are no other errors. E marks are lost unless, by chance, the given results are established by equivalent working. ‘Fresh starts’ will not affect an earlier decision about a misread. Note that a miscopy of the candidate’s own working is not a misread but an accuracy error.

i If a graphical calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers (provided, of course, that there is nothing in the wording of the question specifying that analytical methods are required). Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

j If in any case the scheme operates with considerable unfairness consult your Team Leader.

Question			Answer	Marks	AOs		Guidance
1			$Y = \frac{Ta}{Ax}$ $[Y] = \frac{\text{MLT}^{-2}\text{L}}{\text{L}^2\text{L}}$ $[Y] = \text{ML}^{-1}\text{T}^{-2}$	B1 M1 A1 [3]	1.1 1.1 2.5	Rearrange correctly to make Y the subject Substitute correctly into their expression for Y	Accept rearrangement and substitution in either order.
2			Energy stored in string $\frac{10mg\left(0.5a\right)^2}{2a}$ $\frac{1}{2}mv^2 = \frac{5}{4}mga$ $v = \frac{1}{2}\sqrt{10ga}$	B1 M1 A1 [3]	1.1 3.1b 1.1	Correct use of $\frac{\lambda x^2}{2a}$ Equate EPE with KE oe	
3	(i)		At top of slide, PE = 45g(6) KE = 0.5(45)(3)^2 At bottom of slide, PE = 0 and KE = 0.5(45)(8)^2 $\left(45g(6) + 0.5(45)(3)^2 - 0.5(45)(8)^2\right) = 15R$ $R = 93.9$	B1 B1 B1 M1 A1 [5]	1.1 1.1 1.1 1.1 1.1	Apply work-energy with all terms present	
3	(ii)		The resistance to motion could be modelled as a variable force (as more likely to increase with the speed)	E1 [1]	3.5c		

Question			Answer	Marks	AOs		Guidance
4	(i)		Driving force = $\dfrac{60000}{v}$	B1	1.1		
			$\dfrac{60000}{v} - 1800 = 1200\dfrac{dv}{dt}$	M1	3.3	N, II with 3 terms – allow D oe for the driving force and a oe for the acceleration	
			$2v\dfrac{dv}{dt} = 100 - 3v$	E1 **[3]**	2.2a	**AG**	Must show sufficient working as **AG**
4	(ii)		When $v = 33$, $t = -\dfrac{2}{3}\left(33 + \dfrac{100}{3}\ln(100-99)\right) + 82$ $= 82 - 22 = 60$	B1	3.1a		Must verify that $v = 33$ at $t = 60$
			$\dfrac{dt}{dv} = -\dfrac{2}{3} - \dfrac{200}{9}\left(\dfrac{1}{100-3v}\right)(-3)$	M1*	1.1	Attempt differentiation using chain rule - must be of the form $A + \dfrac{B}{100-3v}$	A and B must be non-zero
			$\dfrac{dt}{dv} = \dfrac{-200 + 6v + 200}{3(100-3v)} \Rightarrow \dfrac{dv}{dt} = \ldots$	M1dep*	1.1	Combine to a single fraction and take reciprocal	
			$\dfrac{dv}{dt} = \dfrac{100-3v}{2v} \Rightarrow 2v\dfrac{dv}{dt} = 100 - 3v$	E1 **[4]**	1.1	**AG**	Must show sufficient working as **AG**

5	**(i)**		 $\dot{x} = 1 + 2\cos(2t)$ and $\dot{y} = 3\sin(1.5t)$ $\dot{x} = \dot{y} = 0$ $\sin(1.5t) = 0 \Rightarrow t = 0, \frac{2\pi}{3}, \frac{4\pi}{3}, \ldots$ $\cos(2t) = -0.5 \Rightarrow t = \frac{\pi}{3}, \frac{2\pi}{3}, \ldots$ $t_1 = \frac{2\pi}{3}$	**M1*** **A1** **M1dep*** **A1ft** **A1** **[5]**	**1.1a** **1.1** **2.1** **1.1** **2.2a**	Attempt derivative of both x **and** y – must be of the form $\dot{x} = 1 \pm A\cos 2t$ and $\dot{y} = \pm B\sin(1.5t)$ Both correct Sets both derivatives equal to zero Find at least one value of t for each of $\dot{x} = 0$ and $\dot{y} = 0$	Non-zero constants A and B
5	**(ii)**		At $t = 0$, P is at $(0, -1)$ At $t = t_1$, P is at $\left(\frac{2\pi}{3} - \frac{\sqrt{3}}{2}, 3\right)$ Displacement is $\sqrt{\left(\frac{2\pi}{3} - \frac{\sqrt{3}}{2}\right)^2 + (3 - (-1))^2}$ $= 4.18$ (3 sf)	**M1*** **M1dep*** **A1** **[3]**	**1.1** **1.1** **1.1**	Attempt to find the position of P at $t = 0$ and $t = t_1$ Correctly apply distance formula for their two points of P	Note that at $t = t_1$ P is at (1.2283696…, 3) 4.1843628…

6	(i)		$4mu = m(v_B + u)$ $v_B = 3u$	M1 A1 [2]	3.3 1.1	Attempt at Impulse = change in momentum for B	Allow incorrect sign(s) but must be using m - allow use of J/I
6	(ii)		 $-4mu = 3m(v_A - 4u)$ $v_A - v_B = e(u + 4u)$ $\frac{8}{3}u - 3u = -5eu$ $e = \frac{1}{15}$	M1 A1 M1 A1ft A1 [5]	3.3 1.1 3.3 3.4 2.2a	Attempt at Impulse = change in momentum for A oe Attempt at use of restitution equation, must be correct way round Correct substitution of their values of v_A and v_B - dependent on all previous M marks	Allow incorrect sign(s) but must be using $3m$ – allow use of J/I 0.066666…

7			$\ddot{x} = -g\sin 30$, $\ddot{y} = -g\cos 30$	**B1**	**2.1**		
				M1*	**3.4**	Attempt to integrate for both x and y	
			$\dot{x} = 10\cos 30 - gt\sin 30$	**A1**	**1.1**		
			$\dot{y} = 10\sin 30 - gt\cos 30$				
				M1*	**1.1a**	Attempt to integrate both	
			$x = 10t\cos 30 - 0.5gt^2\sin 30$	**A1**	**1.1**		
			$y = 10t\sin 30 - 0.5gt^2\cos 30$				
			$y = 0 \Rightarrow t = ...$	**M1dep***	**3.1b**	Sets $y = 0$ and solve for t	$t = 1.17826\ldots$ or $\frac{100\sqrt{3}}{147}$
			$x = 10(1.178...)\cos 30 - 0.5g(1.178...)^2\sin 30$	**M1**	**3.4**		Dependent on previous M mark
			$\text{OA} = 6.80\text{ m}$	**A1**	**2.2b**		$x = 6.8027210\ldots$
				[8]			

8	**(i)**		$r = a\cos\alpha$	**B1**	**1.1**		
				M1*	**3.3**	Resolving vertically (three terms)	T is the tension and R is the normal contact force
			$T\sin\alpha + R\sin\alpha = mg$	**A1**	**1.1**		
				M1*	**3.3**	Resolving horizontally (3 terms)	Allow r for radius
			$T\cos\alpha - R\cos\alpha = m\left(a\cos\alpha\right)\omega^2$	**A1**	**1.1**		
				M1dep*	**3.4**	Eliminating R and solving for T	
			$T = \frac{m}{2}\left(\frac{g}{\sin\alpha} + a\omega^2\right)$	**A1**	**2.2a**		
				[7]			
8	**(ii)**		$R = \frac{mg}{2\sin\alpha} - \frac{ma\omega^2}{2}$	**M1**	**3.1b**	Using their T to find an expression for R	May substitute given values at any point
			Max value of $\omega^2 = \frac{2g}{a\sqrt{3}}$	**M1**	**2.1**	Sets $R = 0$	
			$v^2 = \frac{2ga\cos^2\alpha}{\sqrt{3}}$	**M1**	**1.2**	Use of $v = (a\cos\alpha)\omega$	
			$v = 0.9950656... < 1$	**E1**	**3.2a**	Must indicate that this value is less than 1 metre per second	
				[4]			

9			$T - mg\sin 30 = 0$	**M1**	**3.1b**	Resolving parallel to the plane at the position of equilibrium	T is the tension in the string
			$\frac{2mg}{8l}e = \frac{mg}{2} \Rightarrow e = \ldots$	**M1**	**1.2**	Use of Hooke's law and attempt to solve for the extension e	
			$e = 2l$	**A1**	**1.1**		
				M1	**3.3**	Apply N,II parallel to the plane (3/4 terms)	Allow T for this mark
			$\frac{mg}{2} - \frac{2mg}{8l}(2l + x) = m\ddot{x}$	**A1ft**	**1.1**	FT their value for e	
			$\ddot{x} = -\frac{g}{4l}x$ hence SHM about equilibrium position	**A1**	**3.2a**		
			$T = 2\pi / \sqrt{\frac{g}{4l}} \Rightarrow T = 4\pi\sqrt{\frac{l}{g}}$	**E1**	**2.2a**	**AG**	
				[7]			

10	**(i)**		$\dot{\mathbf{r}}_P = \mathbf{i} + (100k - 2kt)\mathbf{j}$ Max. height occurs at t = 50 $k(50)(100-50) = 25$ $k = 0.01$	**M1*** **A1** **M1dep*** **A1** **[4]**	**3.1a** **1.1** **1.1a** **1.1**	Attempt differentiation of $\mathbf{r}_P$ - may only see **j** component Substitutes their t into **j**-component of $\mathbf{r}_P$ and sets equal to 25	t = 50 by any valid method gets M1A1
10	**(ii)**		Horizontal distance of 40 km from A $\Rightarrow$ Q launched at t = 60 Initially for Q, $\mathbf{r}_Q = 100\mathbf{i}$ $\mathbf{r}_Q = -\frac{5}{3}t\mathbf{i} + \frac{5}{4}t\mathbf{j}(+\mathbf{c})$ $100\mathbf{i} = -\frac{5}{3}(60)\mathbf{i} + \frac{5}{4}(60)\mathbf{j} + \Rightarrow \mathbf{c} = \ldots$ $\mathbf{r}_Q = \frac{1}{3}(600-5t)\mathbf{i} + \frac{5}{4}(t-60)\mathbf{j}$.	**B1** **B1** **B1** **M1** **E1** **[5]**	**3.4** **3.4** **1.1** **3.4** **2.2a**	Using their values in attempt to find **c** $(\mathbf{c} = 200\mathbf{i} - 75\mathbf{j})$ **AG**	Dependent on third B mark Must show sufficient working as **AG**
10	**(iii)**		$t = 60$	**B1** **[1]**	**2.3**		

10	(iv)		$\frac{1}{3}(600-5t)=t \Rightarrow t=...$ $t=75$ Show that $0.01t(100-t)=\frac{5}{4}(t-60)$ when $t = 75$ and so yes Q intercepts P	M1 A1 E1 **[3]**	2.1 1.1 2.2a	Comparing either **i** or **j** components of $\mathbf{r}_Q$ and $\mathbf{r}_P$ and attempt to solve for t	 Dependent on correct value of k
11	(i)		$\frac{1}{2}mu^2+mga=\frac{1}{2}mv^2+mga\cos\theta$ $v^2=u^2+2ag(1-\cos\theta).$	M1 E1 **[2]**	3.3 1.1	Attempt at conservation of energy – 4 terms (allow sign errors and/or angle confusion) **AG**	Where m is the mass of P Must show sufficient working as **AG**
11	(ii)		$mg\cos\theta-R=\frac{mv^2}{a}$ $mg\cos-R=\frac{m}{a}\left(u^2+2ag(1-\cos\theta)\right)$ $\Rightarrow\ mg\cos\theta-R=\frac{m}{a}\left(\frac{2}{5}ag+2ag(1-\cos\theta)\right)$ $3mg\cos\theta-\frac{12}{5}mg=0\Rightarrow\cos\theta=...$ $\cos\theta=\frac{4}{5}$	M1* A1 M1dep* M1 E1 **[5]**	3.3 1.1 3.4 3.4 2.2a	N,II radially at angle θ (3 terms) – allow any form for the acceleration Substitute given results Set R = 0 and attempt to make $\cos\theta$ the subject **AG**	May set R= 0 at any point Must show sufficient working as **AG**

11	**(iii)**		$v^2 = \frac{2}{5}ag + 2ag\left(1 - \frac{4}{5}\right) \quad \left(= \frac{4}{5}ag\right)$ Horizontal component of velocity of P at the point when P loses contact is $v\cos\theta$ Horizontal distance of P from A = $a - a\sin\theta$ $T = \frac{a - a\sin\theta}{v\cos\theta} = \frac{0.4a}{0.8\sqrt{0.8ag}}$ $= \frac{1}{4}\sqrt{\frac{5a}{g}}$	**B1** **B1** **B1** **M1** **E1** **[5]**	**1.1** **3.1b** **1.1** **3.4** **2.2a**	Use of $s = ut$ horizontally to find T **AG**	Must show sufficient working as **AG**
11	**(iv)**		In reality the hemisphere is unlikely to be smooth	**E1** **[1]**	**3.5b**		

12	(i)		P is at rest before impact and as P moves towards B therefore the impulse at impact is in the direction of BD and hence so is the line of centres	E1 [1]	2.4		
12	(ii)		$\cos\theta = \frac{2}{\sqrt{5}}$	B1*	3.1b	Where θ is the angle that Q makes with the line of centres before impact	
				M1*	3.3	Conservation of linear momentum parallel to line of centres (3 terms)	
			$2mu\cos\theta = 2mu_Q + mu_P$	A1	1.1		
				M1*	3.3	Attempt at use of restitution Equation parallel to line of centres, must be correct way round	
			$u_Q - u_P = e(0 - u\cos\theta)$	A1	1.1		
			$u_Q = \frac{2u}{3\sqrt{5}}(2-e)$	M1dep*	3.4	Substitute for $\cos\theta$ and $u_P = \frac{u\sqrt{5}}{2}$ to find u_Q	u_P may have been substituted earlier
			$\Rightarrow \frac{2u}{3\sqrt{5}}(2-e) - \frac{u\sqrt{5}}{2} = -e\left(\frac{2u}{\sqrt{5}}\right) \Rightarrow e = \ldots$	M1	3.4	Substitute u_P, u_Q and $\cos\theta$ to find e – dependent on all previous M and B marks	
			$e = \frac{7}{8}$	A1 [8]	2.2a		

12	(iii)		$v_Q = u\sin\theta$	B1	1.1	Speed of Q perpendicular to line of centres	
			$u_Q = \frac{2u}{3\sqrt{5}}\left(2-\frac{7}{8}\right)$	M1	3.4	Substitute their value of *e* into their u_Q	
			Speed of Q $= \sqrt{\left(\frac{u}{\sqrt{5}}\right)^2 + \left(\frac{3u}{4\sqrt{5}}\right)^2}$	M1	1.2	Attempt speed of Q with their u_Q and v_Q - dependent on B mark	
			$= \frac{u\sqrt{5}}{4}$	A1	1.1		
			Loss of KE = $\frac{1}{2}(2m)u^2 - \frac{1}{2}m\left(\frac{u\sqrt{5}}{2}\right)^2 - \frac{1}{2}(2m)\left(\frac{u\sqrt{5}}{4}\right)^2$	M1	1.1a	Dependent on both previous M marks	
			$= \frac{1}{16}mu^2$	A1	2.2a		
				[6]			

13	**(i)**	**(a)**	$V=\pi\int_0^a\left(a^2-x^2\right)\mathrm{d}x$	**M1**	**1.1**	Correct integral with one of their terms integrated correctly	Limits not needed for this mark – condone lact of π
			$=\pi\left[a^2x-\frac{x^3}{3}\right]_0^a$	**A1**	**1.1**	Both terms integrated corerctly	
			$=\pi\left(a^3-\frac{a^3}{3}\right)=\frac{2}{3}\pi a^3$	**E1**	**1.1**	**AG**	Must show sufficient working as **AG**
				[3]			
13	**(i)**	**(b)**	$V\bar{x}=\pi\int_0^a x\left(a^2-x^2\right)\mathrm{d}x=\left[\frac{a^2x^2}{2}-\frac{x^4}{4}\right]_0^a$	**M1***	**1.1**	Correct integral with one term integrated correctly	Limits not needed for this mark – condone lact of π
			$V\bar{x}=\frac{1}{4}\pi a^4$	**A1**	**1.1**		
			$\bar{x}=\frac{V\bar{x}}{V}=\frac{\frac{1}{4}\pi a^4}{\frac{2}{3}\pi a^3}=\ldots$	**M1dep***	**1.1**	Use of $\bar{x}=\frac{V\bar{x}}{V}$	
			$\bar{x}=\frac{3}{8}a$	**E1**	**1.1**	**AG**	Must show sufficient working as **AG**
				[4]			
13	**(ii)**			**M1**	**1.1a**	Table of values idea	
			$\frac{3a}{4}\left(\pi a^2\left(\frac{3a}{2}\right)\right)+\left(\frac{3a}{2}+\frac{3a}{8}\right)\left(\frac{2}{3}\pi a^3\right)=\ldots$	**A1** **A1**	**1.1** **1.1**	A1 for each term on lhs	
			$=\left(\frac{3\pi a^3}{2}+\frac{2\pi a^3}{3}\right)\bar{x}$	**A1**	**1.1**		

			$\bar{x} = \frac{57a}{52}$	A1 [5]	1.1		
13	(iii)		$\tan\alpha = \frac{a}{\left(\frac{57a}{52}\right)}$ $\alpha = 42.4$	M1 A1 [2]	3.1b 1.1	Using tan and their $\bar{x}$	 $\alpha = 42.37359...$
13	(iv)		 $\frac{21}{52}Wa\sin 30$ $\frac{21}{52}W(0.2)\sin 30 = 0.1575$ $W = 3.9$	M1* A1 M1dep* A1 [4]	3.3 1.1 3.4 1.1	Attempt moment of the weight about the point of contact with the plane Sets their moment of the weight equal to 0.1575 with correct substitution of a and θ	
13	(v)		The value of W would be the same as the frictional force has zero moment about the point of contact with the plane	E1 [1]	3.5a		

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