Oxford Cambridge and RSA

A Level Further Mathematics B (MEI)

Y421/01 Mechanics Major

Practice Paper – Set 1

Time allowed: 2 hours 15 minutes

You must have:
- Printed Answer Booklet
- Formulae Further Mathematics B (MEI)

You may use:
- a scientific or graphical calculator

INSTRUCTIONS
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** If additional space is required, you should use the lined page(s) at the end of the Printed Answer Booklet. The question number(s) must be clearly shown.
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION
- The total number of marks for this paper is **120**.
- The marks for each question are shown in brackets **[ ]**.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is used. You should communicate your method with correct reasoning.
- The Printed Answer Booklet consists of **20** pages. The Question Paper consists of **8** pages.

Section A (34 marks)

Answer **all** the questions.

1 The tension, T, in a stretched wire is given by $T = \frac{YAx}{a}$, where a is the natural length of the wire, A is the cross-sectional area of the wire and x is the extension of the wire. The quantity Y is called Young's modulus; it depends on the material from which the wire is made. Determine the dimensions of Young's modulus. **[3]**

2

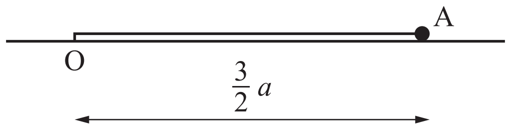

Fig. 2

Fig. 2 shows a particle of mass m which is attached to one end of a light elastic string of natural length a and modulus of elasticity $10mg$. The other end of the string is fastened to a fixed point O on a smooth horizontal table. The particle is initially held at a point A on the table at a distance $\frac{3}{2}a$ from O. The particle is released from rest at A. Find, in terms of g and a, the speed of the particle at the instant the string becomes slack. **[3]**

3 A child descends a vertical distance of 6 m while travelling down a straight water slide of length 15 m. The mass of the child is 45 kg. The child's initial speed is $3\,\mathrm{m\,s^{-1}}$ and the child's final speed is $8\,\mathrm{m\,s^{-1}}$. The resistance to motion is modelled as a constant force of magnitude R N.

(i) By considering the change in energy of the child, calculate the value of R. **[5]**

(ii) State how the model for the resistance to motion could be refined to make it more realistic. **[1]**

4 A car of mass 1200 kg travels along a straight horizontal road. The power developed by the car is constant and equal to 60 kW. The resistance to the motion of the car is constant and equal to 1800 N. At time t seconds the velocity of the car is $v\,\mathrm{m\,s^{-1}}$.

(i) Show that $2v\frac{\mathrm{d}v}{\mathrm{d}t} = 100 - 3v$. **[3]**

After one minute the velocity of the car is $33\,\mathrm{m\,s^{-1}}$.

(ii) Verify that

$$t = 82 - \frac{2}{3}\left(v + \frac{100}{3}\ln(100 - 3v)\right).$$ **[4]**

5 A particle P moves in the x-y plane so that its position at time t is given by

$$x = t + \sin 2t, \quad y = 1 - 2\cos\left(\frac{3t}{2}\right), \quad t \geqslant 0.$$

At time t_1, P first comes momentarily to rest.

(i) Find the exact value of t_1. **[5]**

(ii) Find, correct to 3 significant figures, the distance of P from its original position at time t_1. **[3]**

6 Two small spheres A and B, of masses $3m$ and m respectively, are moving in opposite directions along the same straight line towards each other on a smooth horizontal surface. A has speed $4u$ and B has speed u before they collide. The magnitude of the impulse that A exerts on B is $4mu$.

(i) Find, in terms of u, the speed of B after the collision. **[2]**

(ii) Find the coefficient of restitution between A and B. **[5]**

Section B (86 marks)

Answer **all** the questions.

7 A small ball P is projected with speed $10\,\mathrm{m\,s^{-1}}$ at an angle of elevation of 60° from a point O at the bottom of a plane inclined at 30° to the horizontal. P hits the plane at a point A, where OA is a line of greatest slope, as shown in Fig. 7.

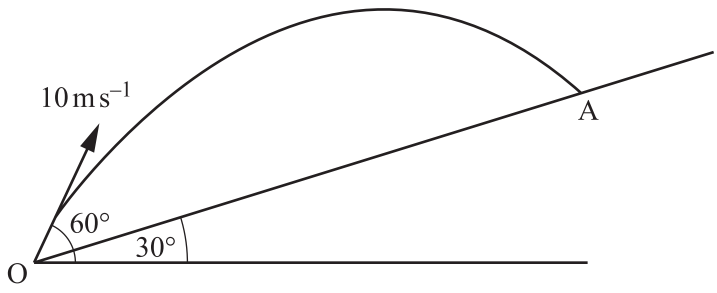

Fig. 7

Find the distance OA. **[8]**

8 A smooth solid cone of semi-vertical angle α, where $\frac{\pi}{4} < \alpha < \frac{\pi}{2}$, is fixed to the ground with its axis vertical. A particle P of mass m is attached to one end of a light inextensible string of length a. The other end of the string is attached to a fixed point O vertically above the vertex of the cone. P rotates in a horizontal circle on the surface of the cone with constant angular speed ω. The string is inclined at a constant angle to the horizontal; this angle is also α, as shown in Fig. 8.

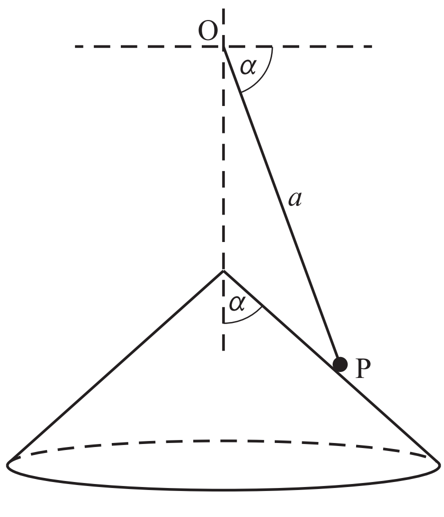

Fig. 8

(i) Find the tension in the string, giving your answer in terms of m, g, a, ω and α. **[7]**

(ii) Given that $\alpha = \frac{\pi}{3}$ and $a = 0.35\,\mathrm{m}$, show that the maximum speed for which P remains in contact with the cone is less than $1\,\mathrm{m\,s^{-1}}$. **[4]**

9 A particle P of mass m is attached to one end of a light elastic string of natural length $8l$ and modulus of elasticity $2mg$. The other end of the string is fixed to a point O on a smooth plane, which is inclined at an angle of 30° to the horizontal. The string lies along a line of greatest slope of the plane and the particle rests in equilibrium on the inclined plane at a point A, as shown in Fig. 9.

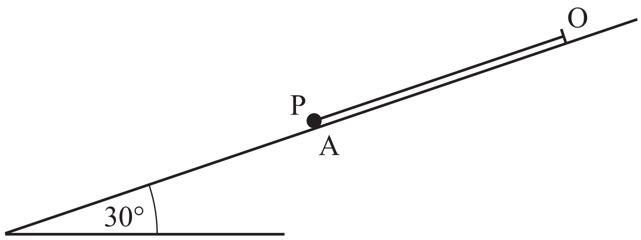

Fig. 9

P is now pulled a further distance $\frac{1}{2}l$ down the line of greatest slope of the plane and released from rest.

Show that P performs simple harmonic motion about A with period $4\pi\sqrt{\frac{l}{g}}$. **[7]**

10 In this question $\mathbf{i}$ is a horizontal unit vector and $\mathbf{j}$ is a vertical unit vector. A missile P is launched from a point O and targeted at a point A which is 100 km horizontally from O. The position vector of P at time t seconds after launch is given by

$$\mathbf{r}_{\mathrm{P}} = t\mathbf{i} + kt(100 - t)\mathbf{j},$$

where k is a positive constant. The maximum height reached by P is 25 km.

(i) Find the value of k. **[4]**

A second missile Q is launched from A with the intention of intercepting P. Q is launched when P is a horizontal distance of 40 km from A. Q moves with a constant velocity of $-\frac{5}{3}\mathbf{i} + \frac{5}{4}\mathbf{j}\ \mathrm{km\,s^{-1}}$.

(ii) Show that the position vector of Q is given by

$$\mathbf{r}_{\mathrm{Q}} = \frac{1}{3}(600 - 5t)\mathbf{i} + \frac{5}{4}(t - 60)\mathbf{j}.$$ **[5]**

(iii) State the minimum value of t for which the expression for $\mathbf{r}_{\mathrm{Q}}$ in part **(ii)** is valid. **[1]**

(iv) Determine whether Q intercepts P. **[3]**

11 The base of a hemisphere of radius a and centre O is fixed in a horizontal plane and touches the foot of a vertical wall at Q. A particle P of mass m is initially at rest at the highest point of the hemisphere, as shown in Fig. 11.

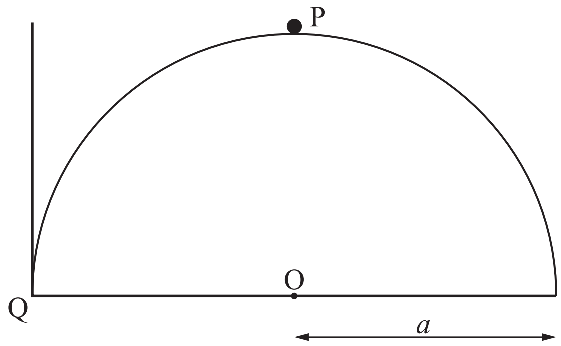

Fig. 11

The hemisphere is modelled as smooth. P is given an initial speed u and starts to slide on the surface of the hemisphere towards Q. While P remains on the surface, OP makes an angle θ with the upward vertical and the speed of P is v.

(i) Show that $v^2 = u^2 + 2ag(1 - \cos\theta)$. **[2]**

(ii) Given that $u^2 = \frac{2}{5}ag$, show that P loses contact with the hemisphere when $\cos\theta = \frac{4}{5}$. **[5]**

P then moves freely under gravity and at time T after leaving the surface strikes the wall at a point vertically above Q.

(iii) Show that $T = \frac{1}{4}\sqrt{\frac{5a}{g}}$. **[5]**

(iv) Explain why the value of $\cos\theta$ is unlikely to be $\frac{4}{5}$. **[1]**

12 A rectangular table ABCD has $\mathrm{AB} = 4a$ and $\mathrm{BC} = 2a$. A small uniform smooth sphere P, of mass m, lies at rest at the centre of the table and a second small uniform smooth sphere Q, of equal radius and mass $2m$, is at rest at the midpoint of AD. Q is projected along the horizontal table towards P with speed u and collides obliquely with P so that, after the collision, P moves directly towards B.

(i) Explain why the direction of the line of centres at impact is DB. **[1]**

The speed of P after the collision is $\frac{u\sqrt{5}}{2}$.

(ii) Find the coefficient of restitution between P and Q. **[8]**

(iii) Find, in terms of m and u, the kinetic energy lost due to the collision. **[6]**

13 A uniform solid hemisphere of radius a is formed by rotating the region bounded by the positive x-axis, the positive y-axis and the curve $x^2 + y^2 = a^2$, where a is a constant, through 2π radians about the x-axis.

(i) Show by integration that

(a) the volume of the solid hemisphere is $\frac{2}{3}\pi a^3$, **[3]**

(b) the x-coordinate of the centre of mass of the solid hemisphere is $\frac{3a}{8}$. **[4]**

A solid S is formed by joining the solid hemisphere to a uniform solid cylinder of the same material and of radius a and height $\frac{3a}{2}$ so that the plane face of the hemisphere coincides with one end of the cylinder, as shown in Fig. 13.

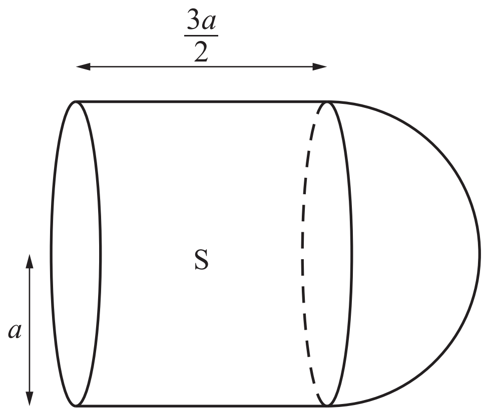

Fig. 13

(ii) Find, in terms of a, the distance of the centre of mass of S from its plane face. **[5]**

S is now placed with its axis vertical and its plane face in contact with a rough horizontal plane. The plane is slowly tilted and S topples when the angle of inclination of the plane is $\alpha°$.

(iii) Find α, correct to 1 decimal place. **[2]**

The solid S has weight W N. It now rests with its hemispherical surface in contact with a smooth horizontal plane and with its axis inclined at an angle of 30° to the vertical, equilibrium being maintained by a couple.

(iv) Given that $a = 20$cm and that the moment of this couple is 0.1575 N m, find the value of W. **[4]**

(v) State whether you would expect the value of W found in part **(iv)** to be larger, smaller or the same if the plane was rough. Give a reason for your answer. **[1]**

END OF QUESTION PAPER